

Development of Analytical Models for the Identification of Dynamic Parameters in a Double Wishbone Front Suspension

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ABSTRACT

The main objective of this study is to apply a dynamic parameter identification methodology to a double-wishbone type front suspension of a race car. Two methodologies for identification are presented: the first one is based on the Singular Value Decomposition (SVD) and elimination by means of the relative standard deviation of each parameter, while the second comprises three consecutive steps: the elimination of those less contributive parameters, the application of SVD method and elimination by relative standard deviation. The physical feasibility of the obtained parameters is taken into account. Both methodologies are validated with collected data of virtual simulation in a computational package program.

CITATION: Mejía, L., Valero, F. and Mata, V., "Development of Analytical Models for the Identification of Dynamic Parameters in a Double Wishbone Front Suspension," *SAE Int. J. Passeng. Cars - Mech. Syst.* 6(1):2013, doi: 10.4271/2013-01-0709.

INTRODUCTION

In the dynamic models used for the simulation and control of systems, the values of certain parameters such as the terms of inertia, masses and the locations of centres of masses are fundamental for obtaining accurate results ([Blundell and Harty 2004](#)). The precise values of these dynamic parameters are difficult to obtain because they usually require experimental measurements that have a very high cost. Depending on the size and complexity of the system, it may not be a practical methodology. For these reasons, there are more and more application fields in which parameter identification is used.

Initially, the dynamic parameter identification procedure was developed for serial robots ([Khosla and Kanade 1985](#), [Atkenson et al. 1986](#), [Gautier and Khalil 1988](#)), followed by closed chain systems such as parallel robots ([Bhattacharya et al. 1997](#), [Guegan et al. 2003](#) and [Grotjahn et al. 2004](#)) and later for vehicle systems ([Jo and Beale 1998](#), [Venhovens and Naab 1999](#), [Russo et al. 2000](#) and [Chen and Beale 2003](#)) and biomechanics ([Kraus et al. 2005](#), [Gordon and Hopkins 1997](#), and [Silva et al. 1997](#)). An area such as automotive engineering is not indifferent to the advantages that the identification process contributes to the design and completion of vehicles.

At present, the dynamic simulation of vehicles is an important tool for vehicle manufacturers, since it allows to limit the testing of real prototypes. However, the validity of the results depends largely on the accuracy of the dynamic parameters used, which gives an idea of the importance associated with their acquisition and validation.

The main objective of this study is to apply the dynamic parameter identification methodology, regularly used in robotics, to vehicle systems. To this end, the results are validated with collected data of virtual simulation in the *MSC Adams®* computational package program.

This paper is organized as follows. In the first section, a dynamic model of a push-rod front suspension is presented. Then, a dynamic model for parameter identification is obtained. This model is rewritten in a linear form with respect to the parameters and is reduced to obtain the so-called base parameters set. The comparisons made between symbolic and numerical models, as well as the results of the identification process are depicted in the fourth section. In the last section the most significant conclusions are presented.

1. DYNAMIC MODEL

The double-wishbone type front suspension analyzed in this paper characterizes by having push-rods and torsion bars instead of coil springs. The components and joints of the

systems are presented in Figure 1 and Table 1. In order to validate the analytical model obtained, it is necessary, by means of MBS (multibody systems analysis software), to build an equivalent model required to compare and validate the theoretical results.

Lagrange-Euler dynamic formulation (Shabana 2001, Samin and Fiset 2003) is considered in order to obtain the equations of motion of the mechanical system. Local reference systems are attached to each link, and Euler parameters are used to describe the orientation of the elements. In this way, the equation of motion for a system modelled with a set of dependent coordinates $\vec{q}$ will look as follows:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\vec{q}}} \right) - \frac{\partial L}{\partial \vec{q}} + \frac{\partial D}{\partial \dot{\vec{q}}} + \frac{\partial V_E}{\partial \vec{q}} - \vec{\tau} - C_q^T \cdot \vec{\lambda} = 0 \quad (1)$$

Where L is the Lagrangian function, V_E is the elastic potential energy, D is the dissipative energy, $\vec{\tau}$ is the vector of generalized forces, C_q^T is the constraint Jacobian matrix and $\vec{\lambda}$ is the vector of Lagrange multipliers (García de Jalón and Bayo, 1994).

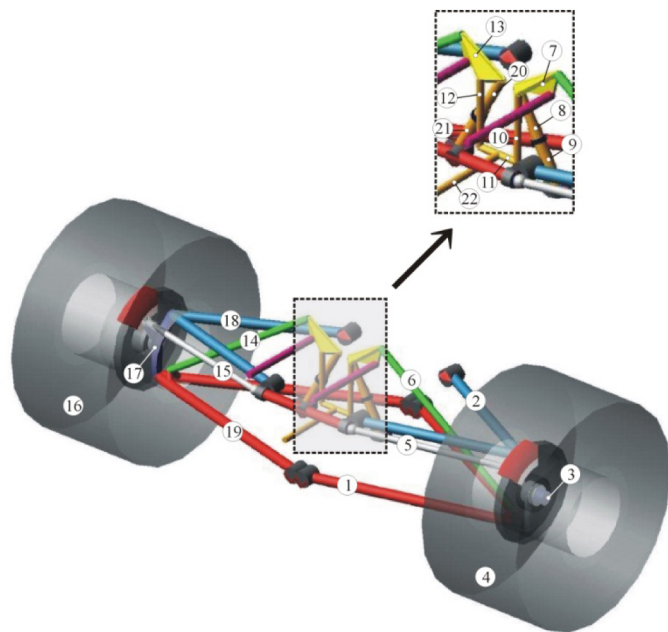


Figure 1. Model of the front suspension, analyzed with MBS (MSCAdams).

If the body coordinate system of an element does not coincide with the mass center, the equation of motion can be written in a format that is linear in dynamic parameters (Farhat et al. 2008). A coordinate partitioning is considered to handle the system as an open chain, removing the Lagrange multipliers.

Considering the terms corresponding to inertial and gravitational generalized forces in the equation of motion,

and regrouping them in a linear form with respect to the set of dynamic parameters to be identified, the following equation will be obtained,

$$K\vec{\Phi} - \vec{\tau} + C_q^T \vec{\lambda} = 0 \quad (2)$$

Where K is the coefficient matrix that relates the inertial parameters with inertial and gravitational forces. $\vec{\tau}$ is the vector of generalized forces, that includes externally applied forces and those due to springs and dampers.

Table 1. Components and joints of the analyzed double wishbone front suspension

Components	Joint
Chassis (0), left lower control arm (1)	Revolute
Chassis (0), left upper control arm (2)	Revolute
Left lower control arm (1), left upright (3)	Spherical
Left upper control arm (2), left upright (3)	Spherical
Left upright (3), left wheel (4)	Revolute
Left upright (3), left tie rod (5)	Spherical
Left tie rod (5), chassis (0)	Hooke
Left lower control arm (1), left push rod (6)	Spherical
Left push rod (6), left rocker (7)	Hooke
Left rocker (7), left lower strut (8)	Spherical
Left damper lower strut (8), left damper upper strut (9)	Translational
Left upper strut (9), chassis (0)	Hooke
Left rocker (7), chassis (0)	Revolute
Left rocker (7), left couple bar (10)	Hooke
Left couple bar (10), antiroll coupler (11)	Spherical
Antiroll coupler (11), right couple bar (12)	Spherical
Right couple bar (12), right rocker (13)	Hooke
Right rocker (13), chassis (0)	Revolute
Right rocker (13), right push rod (14)	Hooke
Right push rod (14), right lower control arm (19)	Spherical
Right lower control arm (19), chassis (0)	Revolute
Right lower control arm (19), right upright (17)	Spherical
Right upright (17), right upper control arm (18)	Spherical
Right upright (17), right wheel (16)	Revolute
Right upper control arm (19), chassis (0)	Revolute
Right upright (17), right tie rod (15)	Spherical
Right tie rod (15), chassis (0)	Hooke
Chassis (0), right upper strut (20)	Hooke
Right damper upper strut (20), right damper lower strut (21)	Translational
Right damper lower strut (21), right rocker (13)	Spherical
Antiroll coupler (11), antiroll bar (22)	Revolute

The vector of dynamic parameters is made up of inertia terms, the product between the mass and the mass centre, and the mass of each element of the mechanical system. Thus,

$$\bar{\Phi} = [\bar{\Phi}_1 \quad \bar{\Phi}_2 \quad \cdot \quad \cdot \quad \bar{\Phi}_n]^T,$$

Where $\bar{\Phi}_i$ is,

$$\bar{\Phi}_i = [Ixx_i \quad Ixy_i \quad Ixz_i \quad Iyy_i \quad Iyz_i \quad Izz_i \quad mx_i \quad my_i \quad mz_i \quad m_i]^T$$

K for each element i of the system can also be obtained from the equation of motion, determined by means of the following equation:

$$K_i = \begin{bmatrix} 0 & {}^0R_i {}^i\tilde{\eta}_i & \ddot{r}_i - \vec{g} \\ {}^i\hat{\eta}_i & {}^0R_i^T \tilde{g} {}^0R_i - {}^0R_i^T \tilde{r}_i {}^0R_i & 0 \end{bmatrix} \quad (3)$$

Where ${}^i\tilde{\eta}_i = \tilde{\omega}_i \cdot \tilde{\omega}_i + \tilde{\dot{\omega}}_i$ and ${}^i\hat{\eta}_i = \hat{\omega}_i + \tilde{\omega}_i \hat{\omega}_i$

The operators $\hat{a}$ and $\tilde{a}$ are defined as:

$$\hat{a} = \begin{bmatrix} a_x & 0 & 0 & a_y & a_z & 0 \\ 0 & a_y & 0 & a_x & 0 & a_z \\ 0 & 0 & a_z & 0 & a_x & a_y \end{bmatrix}$$

$$\tilde{a} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

The angular velocities are expressed in terms of the Euler parameters,

$$\omega_i = G_i \vec{\beta}_i, \quad \tilde{\omega}_i = G_i \dot{G}_i^T, \quad \dot{\omega}_i = G_i \ddot{\beta}_i$$

The Euler parameter vector is,

$$\vec{\beta}_i = [\beta_{0_i}, \beta_{1_i}, \beta_{2_i}, \beta_{3_i}]^T,$$

$$\beta_{0_i} = \cos\left(\frac{\theta_i}{2}\right) \quad \beta_{1_i} = v_{1_i} \cdot \sin\left(\frac{\theta_i}{2}\right)$$

Where,

$$\beta_{2_i} = v_{2_i} \cdot \sin\left(\frac{\theta_i}{2}\right) \quad \beta_{3_i} = v_{3_i} \cdot \sin\left(\frac{\theta_i}{2}\right)$$

v_1, v_2 and v_3 are the components of the unit vector along the instantaneous axis of rotation, while θ is the angle of rotation. The G matrix is defined by

$$G = 2 \begin{bmatrix} -\beta_1 & \beta_0 & -\beta_3 & \beta_2 \\ -\beta_2 & \beta_3 & \beta_0 & -\beta_1 \\ -\beta_3 & -\beta_2 & \beta_1 & \beta_0 \end{bmatrix}$$

The rotation matrix, ${}^0R_i$, establishes the orientation of the i -th link with respect to the fixed reference system. $\ddot{r}_i$ is the acceleration of the origin of the i -th reference system, while the gravity vector is denoted by $\vec{g}$.

Regrouping the terms related to the dependent and independent coordinates of equation (2), it can be written

$$K^d \bar{\Phi} - \tau^d + C_q^{dT} \bar{\lambda} = 0 \quad (4)$$

$$K^i \bar{\Phi} - \tau^i + C_q^{iT} \bar{\lambda} = 0 \quad (5)$$

Solving equation (4) for the vector of Lagrange multipliers $\bar{\lambda}$, and replacing it in (5), a linear dynamic model with respect to the dynamic parameters is obtained,

$$(K^i - X^T K^d) \bar{\Phi} = \tau^i - X^T \tau^d \quad (6)$$

Where

$$X = C_q^{dT-1} \cdot C_q^i$$

It is necessary to consider the inverse of the dependent Jacobian matrix; consequently the selection of coordinates must guarantee a matrix C_q^{dT} of full rank in the columns. Since dynamic model (6) is not determined (the number of unknown variables is larger than the number of equations), its solution can be obtained only by enlarging the linear system. That is, it is necessary the insertion of equations corresponding to different configurations of the mechanical system in order to obtain an overdetermined linear system:

$$\begin{bmatrix} K_1^i - X^T K_1^d \\ \cdot \\ \cdot \\ \cdot \\ K_n^i - X^T K_n^d \end{bmatrix} \cdot \bar{\Phi} = \begin{bmatrix} \tau_1^i - X^T \tau_1^d \\ \cdot \\ \cdot \\ \cdot \\ \tau_n^i - X^T \tau_n^d \end{bmatrix} \quad (7)$$

Which can be written as:

$$W(\vec{q}, \vec{\dot{q}}, \vec{\ddot{q}}) \cdot \bar{\Phi} = \vec{\tau} \quad (8)$$

Where W is the global observation matrix.

Depending on the topology of the mechanical system, an observation matrix W of deficient rank could be obtained, and consequently the linear system (8) cannot be solved by the Least Square Method. This rank deficiency could arise from:

- Those parameters that do not influence the dynamic behaviour. They are associated to the null columns of the observation matrix.
- Those parameters that contribute to the dynamic behaviour of the system in linear combination. This corresponds to a linear dependence among columns of the observation matrix.

Moreover, the Euclidean norms of each column of the observation matrix, allows to characterize the contribution of each parameter to the dynamic behaviour of mechanical system.

Taking into account the later observations, it is possible to obtain an observation matrix of complete rank to determine a