

# Non-Linear Full-Car Modeling and Sky-Hook Control for a Direct-Drive Active Suspension System

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## ABSTRACT

At Eindhoven University of Technology an active suspension system has been developed [1]. This system is superior to other active suspension in terms of bandwidth and power consumption. This active suspension system was tested on a quarter car setup and showed improvements of up to 48% in comfort [2]. In order to implement this suspension in a test vehicle with the same improvements, a non-linear full-car model is developed in this paper which is used to simulate and design various controllers. The non-linear model incorporates non-linear damping, bump stops, actuator saturation and actuator friction. To model the friction in the actuator a combination of Coulomb and viscous friction is used. To model the MacPherson suspension strut, two methods are described and compared. Also the implications of using acceleration sensors which are placed in line with the MacPherson strut are discussed. It is shown that the placement of the acceleration sensors limits the control performance during vehicle acceleration and cornering. To control the suspension system, a sky-hook controller is used. The implemented sky-hook controller showed improvements in a frequency region of 0.7 to 8 Hz. When comparing the nonlinear model and linear model to the measurements conducted on a test vehicle, it is shown that the non-linear model more accurately predicts the accelerations of the sprung mass.

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## I. INTRODUCTION

A suspension system is one of the most important components in a vehicle. It has two main functionalities. One is increasing driving comfort, by isolating the vehicle body from external disturbances, which mainly come from irregular road surfaces. The other one is to maintain a firm contact between road and tires to provide guidance along the track. This is called handling performance. In a conventional (i.e. passive) suspension system, which is composed of passive springs and dampers, a trade-off is required to resolve the conflicting requirements of ride comfort and good handling performance. The reason is that a hard suspension is required for handling and to support the mass of the vehicle, on the other hand, a soft suspension is needed to isolate the disturbances from the road (comfort). The purpose of an active suspension is to simultaneously improve both handling and comfort.

Suspension systems can be divided into three groups: passive, semi-active and active suspension systems. Passive systems have fixed characteristics of the suspension. A semi-

active system is able to alter some of its characteristics, e.g. the amount of damping. The active system is able to change the amount of dissipative power, but it can also supply power to the system, hence it can work in the four quadrants of the force-velocity diagram. This allows for significantly improving the performance of the suspension system with an active suspension. Several (semi-)active suspension systems have already been developed. Examples of semi-active systems are MagneRide [3] which uses magneto-rheological adjustable dampers and the one developed by Magneti Marelli [4], which controls the valves in the damper, thereby changing its characteristics. Examples of active systems are Hydractive [5] used by Citroen, the BMW Anti Roll Control (ARC) [6], the Active Body Control (ABC) by Mercedes [7] and the patented active suspension developed at the Eindhoven University of Technology [8] which is shown in Figure 1.

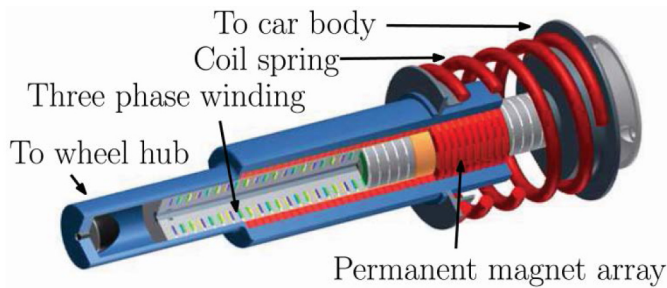


**Figure 1. Active suspension system as developed by Eindhoven University of Technology in close industrial cooperation with SKF [1].**

Hydractive uses spheres filled with nitrogen and a hydraulic fluid separated by a rubber membrane to control the ride. Since continuous pressurization is necessary, the power consumption is high. The ARC uses hydraulic rotary actuators inserted in the front and rear anti-roll bar. Using sensor input the roll is minimized. However the bandwidth of ARC is limited and stiffening the anti-roll bar in the corner while simultaneously hitting a pothole or stone can cause potentially dangerous situations. The ABC system uses a hydraulic actuator in series with a passive spring-damper combination. The bandwidth of ABC is limited to 5 Hz. Therefore, it is mainly used to level the vehicle. Since the operating pressure is 200 bar, its power demand is in the range of 3-5 kW [9].

A high performance patented active suspension strut has been developed by the Eindhoven University of Technology in close cooperation with industrial partner SKF [1], [10]. The active suspension is shown in Figure 1 and is composed of a coil spring in parallel with a patented tubular permanent magnet actuator with integrated electromagnetic damping [8]. This suspension has a bandwidth in the order of hundreds of hertz. Due to the fact that the coil spring supports the sprung mass, no continuous pressurization is necessary. Hence, it also has much lower average power consumption than a hydraulic system. In this paper this actuator is used as front suspension.

Numerous papers have been written on the control of (semi-) active suspension systems. Many of these papers, however, only consider the theory and not practical results. Moreover, the criterion for describing comfort is different in most publications. In [11], an experimental verification on a quarter car setup of a robust controller in combination with a hydraulic actuator is conducted. The robust controller was designed to reduce the body vibrations at 1.5 Hz without amplifying the body acceleration and tire force at other frequencies. To describe comfort, only the body acceleration between 1 and 5 Hz is taken into account. This resulted in a reduction of 50%. In [12], an  $H_\infty$  robust controller was tested on a test vehicle with a semi-active suspension. It uses the same criteria as in [8], but the frequency range is 1-8 Hz. A reduction of 64% was measured in vertical body acceleration.

The actuator developed in [1] is used in a quarter car setup to test a linear quadratic (LQ) controller [13] and a robust controller [2]. To describe comfort, the sprung mass acceleration was weighted to emphasize the frequencies for which humans are sensitive. The measurements show that LQ and robust control can improve comfort with 35% and 48%, respectively. To implement such a controller in the test vehicle, it was recommended to do additional research on full-car modeling. Therefore, a full-car model is developed and evaluated in this paper. In this paper a 7 DoF (degree of freedom) model from the literature is taken as a basis and several components are added, such as friction, bump stops and a MacPherson strut. The difficulties caused by the sensor placement on the MacPherson strut are also addressed. Finally, a sky-hook controller is used to improve ride comfort according to three different evaluation criteria.

This paper is structured as follows: in section II, the model of the car and the actuator is described. Also the implication of using a MacPherson strut is discussed. In section III, the problems which arise from the acceleration sensors on the MacPherson strut are addressed. Section IV gives a description of the road, and section V shows the design aspects of the controller. In section VI results of the used controller are given. Finally, in section VII conclusions and recommendations are presented.

## II. NON-LINEAR FULL-CAR MODEL INCLUDING ACTIVE SUSPENSION

In this paper, the test vehicle used to implement the active suspension is a BMW 530i. In Figure 2 the car is shown with the active suspension. The actuator is a tubular permanent magnet actuator as described in [10]. The model used to describe the suspension system is based on a model described in literature [14]. This section presents the (non-linear) modifications that have been applied to the model with special emphasis on the MacPherson strut.



**Figure 2. Close-up view of the active suspension system installed on the BMW test vehicle [6, 8].**

## Car Model

Several different car models have been described in the literature. From very simple 1 DoF models [15] up to nonlinear large number of DoF models [16]. Tests have already been conducted at the Eindhoven University of Technology with controllers based on a quarter car model [15]. Since a quarter car model does not take several aspects of a full-car into account, a 7 DoF full-car model is considered here. This model is more complex than the quarter car model, especially the linkage between the four wheels is modeled more accurately. For the design of the controller, a linear model of the car is used. To evaluate the controller, a non-linear model is used, which has non-linear elements, such as a bump stop, damping and friction from the linear bearings in the actuator.

Figure 3 shows a graphical representation of the general full-car model as described in [14] including the active suspension. Here,  $m_s$  is the sprung mass of the vehicle,  $m_{ui}$  is the unsprung mass of corner  $i$ , with  $i \in \{fl, fr, rl, rr\}$  denoting the front left, front right, rear left and rear right, respectively. The mass  $m_s$  has a longitudinal moment of inertia,  $I_x$ , and a lateral moment of inertia,  $I_y$ .

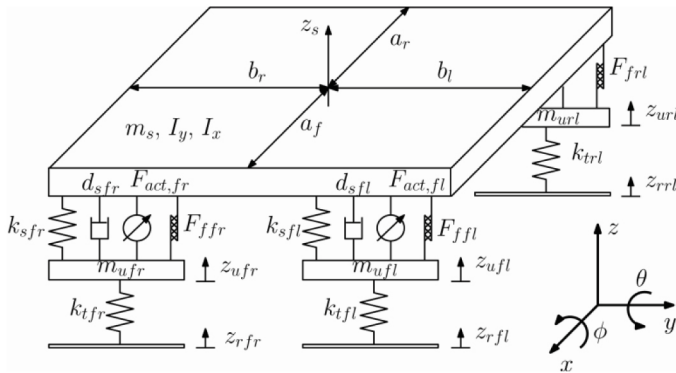


**Figure 3. Full-car model of a vehicle including the active suspension and friction.**

The stiffness and damping of the  $i^{th}$  corner suspension are denoted by  $k_{si}$  and  $d_{si}$ , respectively. The tire stiffness of corner  $i$  is indicated by  $k_{ti}$ . Because the damping of a rolling tire is much smaller than the damping of the suspension, the damping of the tire is ignored. The actuator force is denoted by  $F_{act,i}$ . Finally, the friction of the actuator is introduced by  $F_{fi}$ .

The degrees of freedom are the displacement of the sprung,  $z_s$ , and unsprung masses,  $z_{ui}$ , the body roll,  $\phi$ , and body pitch,  $\theta$ . The displacement of the road is denoted by  $z_{ri}$ .

The BMW test vehicle has an active anti-roll bar in the front and in the rear, with a torsional stiffness of  $k_{Rf}$  and  $k_{Rr}$ , respectively. The anti-roll bar is used to reduce the amount of roll of the vehicle. When using the active suspension the anti-roll bar is obsolete, because the actuators can be used instead. During testing the test vehicle is driven on a straight road,

hence, minimal roll is introduced. Therefore, the anti-roll bar has not been taken into account for the model.

The full-car model has seven degrees of freedom with the following equations of motion:

$$\begin{aligned} m_s \ddot{z}_s &= \sum_i -k_{si} z_i - d_{si} \dot{z}_i + F_{act,i} - F_{fi}, \\ I_y \ddot{\theta} &= \sum_i a_i (k_{si} z_i + d_{si} \dot{z}_i - F_{act,i} + F_{fi}), \\ I_x \ddot{\phi} &= \sum_i b_i (k_{si} z_i + d_{si} \dot{z}_i - F_{act,i} + F_{fi}), \\ m_{ui} \ddot{z}_{ui} &= k_{si} z_i + d_{si} \dot{z}_i - F_{act,i} - k_{ti} (z_{ui} - z_{ri}) + F_{fi}, \end{aligned} \quad (1)$$

with:

$$\begin{aligned} z_i &= z_s - z_{ui} - b_i \phi - a_i \theta, \\ \dot{z}_i &= \dot{z}_s - \dot{z}_{ui} - b_i \dot{\phi} - a_i \dot{\theta}, \\ a_i &= \begin{cases} a_f & i \in \{fl, fr\} \\ -a_r & i \in \{rl, rr\} \end{cases}, \\ b_i &= \begin{cases} -b_l & i \in \{fl, rl\} \\ b_r & i \in \{fr, rr\} \end{cases}. \end{aligned} \quad (2)$$

In the rest of this work  $\ddot{z} = a$  and  $\dot{z} = v$  Using Eq. 1 and Eq. 2, it is possible to apply a force of the active suspension system on the front wheels.

As stated earlier, a linear model and a non-linear model are used. The linear model uses the equations of motion to create a state space representation of the model. This means that the parameters are constant. For the non-linear model the equations of motion are implemented in Matlab/Simulink and the parameters can, therefore, become variable.

## Vehicle Parameters

To obtain accurate results for the model, the parameters have to be carefully investigated and measured. Some parameters are given by the manufacturer of the car and some are calculated. The parameter values selected in this work are listed in Table 1. The parameter values for front right and front left are considered equal. The same holds for the rear. As can be seen, the mass distribution has a near to perfect 50% ratio, resulting in a center of gravity which lies exactly between the wheels. This means  $a_f = a_r = 1.44$  m and  $b_l = b_r = 0.78$  m. The only unknown parameters are the inertias of the test vehicle and the strut characteristics, namely the spring constant, the damping and friction.

**Table 1. Vehicle parameters of the BMW 530i.**

| Parameter                              | Value          | Unit |
|----------------------------------------|----------------|------|
| Sprung mass, $m_s$                     | 1615           | kg   |
| Unsprung mass front, $m_{uf}$          | 48.3           | kg   |
| Unsprung mass rear, $m_{ur}$           | 45             | kg   |
| Tire stiffness front/rear, $k_{tf}$    | $(3.1-3.7)e^5$ | N/m  |
| Mass distribution front-rear           | 50.5-49.5      | %    |
| Mass distribution left-right           | 49.4-50.6      | %    |
| Track width, $W$                       | 1.558          | m    |
| Wheel base, $L$                        | 2.888          | m    |
| Height of center of gravity, $h_{cog}$ | 0.544          | m    |

### Inertia of the Vehicle Chassis

Inertia is the resistance of any physical object to a change in its state of motion or rest. To make an estimation of the vehicle inertias, a 1<sup>st</sup> order approximation [17] is used:

$$\begin{aligned} 0.14m_s W^2 &< I_x < 0.19m_s W^2, \\ 0.2m_s L^2 &< I_y < 0.25m_s L^2, \end{aligned} \quad (3)$$

where  $W$  and  $L$  are the width and the length of the test vehicle, respectively. This equation only gives a rough estimation and is used as a starting point, where it is updated through optimization. This optimization is done manually, such that the simulations closely represent the measurements for a frequency range of 0.1 till 15 Hz.

### Spring Characteristics

The spring is elastic, which means that the force depends on the stroke,  $z$ . To prevent the strut from reaching the end of its stroke when highly compressed, specially shaped rubber modules called bump stops are used. These bump stops gradually increase the elastic force for high compression, mainly for comfort and material lifetime reasons. The spring and bump stop force for the front are shown in Figure 4. Since the spring is linearly dependent on the stroke and the bump stop is non-linear dependent, only the spring is modeled in the linear model, while in the non-linear model also the bump stop is considered. The spring constants of the front and rear are 30 kN/m and 31.5 kN/m, respectively.

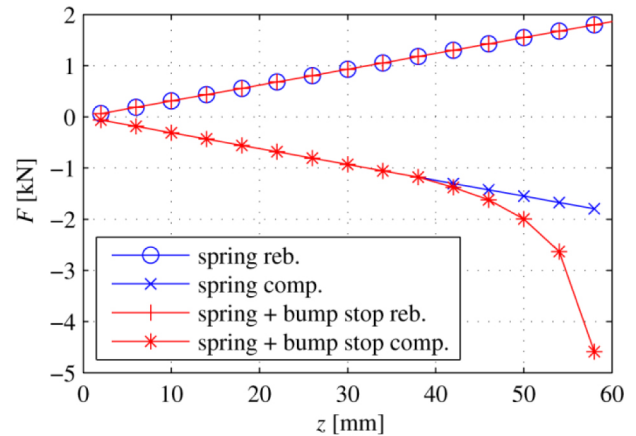


**Figure 4. Compression and rebound spring characteristics of the active suspension system.**

### Damping Characteristics

The damping force depends on the velocity. In [18], measurements are performed on the suspension strut, as shown in Figure 5. The non-linear dependency of the damping value in relation to damper velocity is taken into account in the nonlinear model. In the linear model, a rear damping of 4000 Ns/m and a front damping of 1450 Ns/m are used as explained in [12].

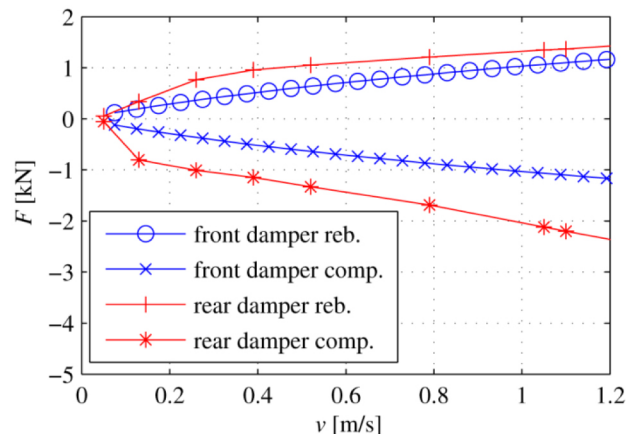


**Figure 5. Compression and rebound damping characteristics of the active suspension system (front) and passive suspension system (rear).**

## Friction

Friction occurs in all mechanical systems. Friction models can be divided into two types of models, namely dynamic models and static models [19]. In this paper only static models are described, where in the active suspension the friction is considered to be non-linear.

In Figure 6(a)-(c), three commonly used types of friction are shown. In Figure 6 (a) Coulomb friction is shown. Coulomb friction is independent of the magnitude of the velocity. This type of friction can be written as:

$$F_{fa} = F_c \operatorname{sgn}(v). \quad (4)$$

To describe the friction force caused by the viscosity of lubricants the term viscous friction is used, which is described as:

$$F_{fb} = d_v v. \quad (5)$$

Viscous friction and Coulomb friction are often combined as shown in Figure 6(b). This combination can be written as:

$$F_{fc} = F_c \operatorname{sgn}(v) + d_v v. \quad (6)$$

In all these friction models, static friction or stiction is not taken into account. Stiction describes the friction force at rest. The idea is that stiction is higher than the Coulomb friction level. Stiction counteracts external forces below a certain level and thus prevents an object from moving. Stiction cannot be described as a function of only velocity. It has to be modeled using the external force  $F_e$ .

$$F_{fd} = \begin{cases} F_e, & \text{if } v = 0 \text{ and } |F_e| < F_s, \\ F_s \operatorname{sgn}(F_e), & \text{if } v = 0 \text{ and } |F_e| \geq F_s. \end{cases} \quad (7)$$

A combination of Coulomb friction, viscous friction and stiction is shown in Figure 6(c). With this friction model the external force because of the spring, dampers and linkage between each tire is to be determined. The friction model used in this paper is shown in Figure 6(d). Figure 6(d) is basically the continuous model of the discontinuous friction model in Figure 6(b). The discontinuous friction model results in discontinuous outputs of the car model. For small changes in velocity around zero velocity, the friction changes from  $\pm F_c$  to  $\mp F_c$ . This leads to discontinuous outputs of the car model when the velocity changes sign. By replacing the sharp transition around zero speed by a sine wave in a region of  $\pm \epsilon_v$  m/s this effect is reduced. In this work,  $\epsilon_v = 5$  mm/s,  $F_c = 240$  N and  $d_v = 200$  Ns/m are used. The value  $\epsilon_v$  is large enough to reduce the discontinuous output and small enough such that the friction is still mainly the Coulomb and viscous friction.

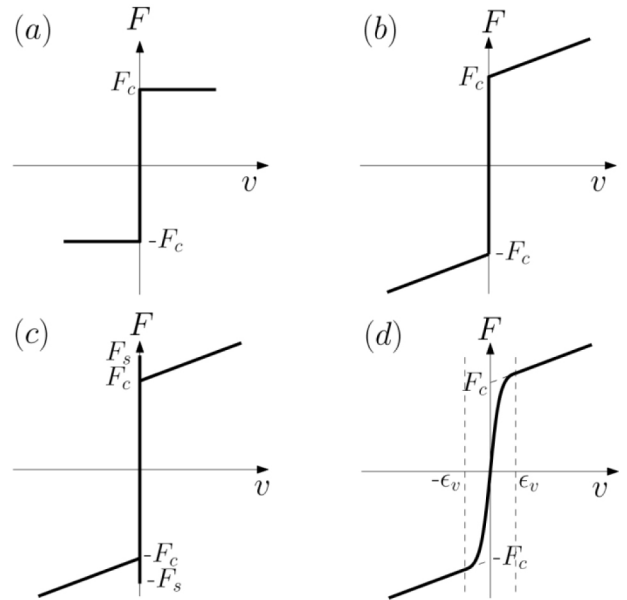


Figure 6. (a) Coulomb friction, (b) Coulomb and viscous friction, (c) Coulomb, viscous and stiction and (d) continuous Coulomb and viscous friction

## Friction Reduction

The non-linearity in the model depends on the friction that occurs within the actuator, where a high friction reduces the suspension efficiency. For this reason the original actuator has been altered. Initially, friction is reduced by applying Teflon grease instead of Teflon spray. Further, the contact area between the bearings and the actuator is reduced by removing most of the bearing along the length of the actuator.

In Figure 7 the suspension deflection on the right side is shown when only the right actuator is powered. In the measurements, the output force of the actuator was a sine wave with an amplitude of 1000 N for three different frequencies (0.1, 1 and 10 Hz). It can be seen that the Teflon grease significantly improved the movement of the actuator at 0.1 Hz. During this frequency the actuator does not have much speed, so it can be concluded that the Teflon grease mainly decreases stiction and Coulomb friction.

For higher frequencies, between 1 and 10 Hz, the effect of shortening the bearings becomes visible. In this range, the actuator has a higher speed, hence shortening the bearings decreases the viscous friction. This can be clearly seen in Figure 7 for the case of 10 Hz, where no improvement is gained when only the Teflon grease is used. When the bearings are shortened an increase in stroke is shown, meaning less viscous friction.

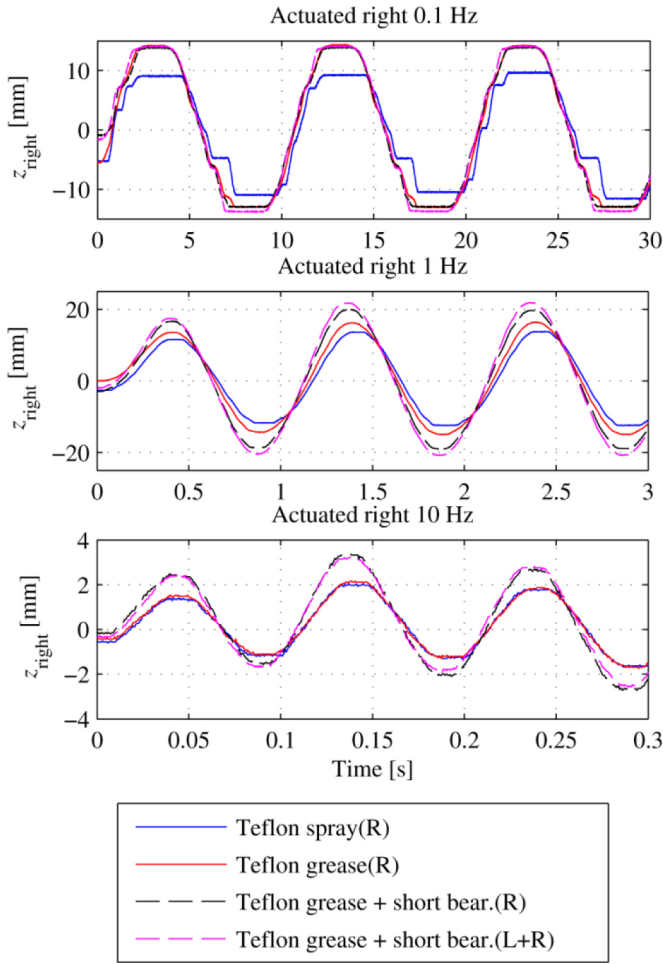


**Figure 7. Measurements after friction reduction (only right actuator is powered).**

## MacPherson Strut Modeling

The front suspension of the BMW test vehicle (a MacPherson suspension) is shown in [Figure 8\(a\)](#) and its equivalent scheme in [Figure 8\(b\)](#). It can be seen that the strut makes an angle with respect to the tire and is not connected directly to the center of the tire. The car model of [Figure 3](#) does not take this into account, it assumes no angle,  $\alpha = 0$  and  $g = h$ . To incorporate the MacPherson strut correctly, the effective spring stiffness and damping constants need to be calculated. This paper considers two methods to describe the MacPherson strut.

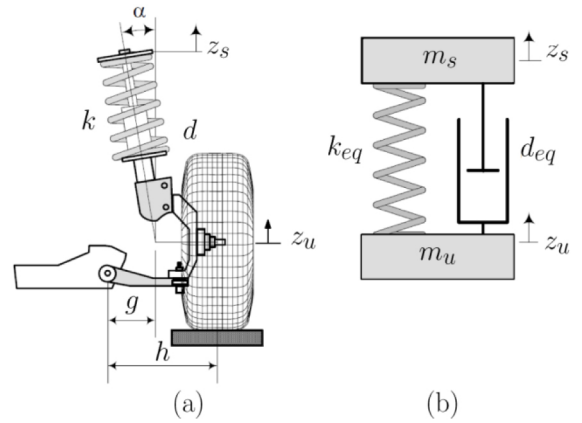


**Figure 8. Representation of the MacPherson suspension system of the BMW test vehicle [13].**

### Method I

The first method to describe the MacPherson strut can be found in [14]. First, it considers a tilted mass-spring system such that the spring makes an angle  $\alpha$  with the axis of mass translation as shown in [Figure 9\(a\)](#). The spring has a stiffness,  $k_s$ . If the motion of  $m_u$  is small, any change in  $\alpha$  can be ignored. The spring elongation,  $\delta_{t,l}$ , is:

$$\delta_{t,l} \approx -z_u \cos \alpha. \quad (8)$$

The spring force  $F_k$  is:

$$F_k = -k_s \delta_{t,l} \approx k_s z_u \cos \alpha. \quad (9)$$

The  $z$  component of this force,  $F_z$ , is the force responsible for moving the mass,  $m_u$ , in the  $z$ -direction and is calculated by:

$$F_z = F_k \cos \alpha \approx (k_s \cos^2 \alpha) z_u. \quad (10)$$

It is now possible to substitute the tilted spring with an equivalent spring,  $k_{eq}$ , on the  $z$ -axis that needs the same force,  $F_z$ , to elongate the same amount as the mass moves

$$\begin{aligned} F_z &= k_{eq,t,l} z_u, \\ k_{eq,t,l} &\approx k_s \cos^2 \alpha. \end{aligned} \quad (11)$$

Second, a displaced spring is considered. The mass,  $m_u$ , is attached to the tip of a massless bar with length,  $h$ . The bar is pivoted to the wall and a spring,  $k_s$ , is attached to the bar at a distance,  $g$ , from the pivot as is shown in [Figure 9\(b\)](#). When the mass oscillates with a small displacement, the elongation,  $\delta_{d,l}$ , of the spring is:

$$\delta_{d,l} \approx -\frac{g}{h} z_u. \quad (12)$$