

Revisiting the Formulas for Tractive and Braking Energy on the EPA Driving Schedules

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ABSTRACT

A primary **input** to any study of automobile fuel economy is the **tractive energy** required by vehicles to negotiate prescribed driving schedules. This energy depends on both the particular characteristics of a vehicle **and** the detailed velocity profile of the particular schedule. Two schedules of special interest are the **EPA Urban** and **Highway** used in government fuel-economy regulation. In making comprehensive studies of the influence of various vehicle parameters on fuel economy, an ability to readily and accurately predict the tractive and braking energy requirements of vehicles can be very valuable.

The author published such a capability in 1981. (Numbers in brackets denote References at the end of the paper.) Since that time he has used it for addressing several different fuel-economy related issues (2,3,4). Over these same years the **physical** basis for the **near linearity** of **solutions** of the equations for tractive and braking energy has become clearer. This provides **additional** validation for the resultant **linear** energy **equations** that fit those solutions **nearly exactly** ! However, some of this information has not been made available to the technical community at large. This is one purpose of the present publication. In addition, It is hoped that a somewhat different execution of the analysis will facilitate the comprehension of its true nature, and lead to increased appreciation for the significance and value of its results.

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POWERED DRIVING

Any vehicle travelling along the ground must overcome resistances to its motion. For a **level** road, these are its tire rolling resistance (R), its aerodynamic drag (D), and the resistance of its **effective** mass (M_e) to acceleration. While a vehicle's total **static** mass M (**curb** mass **plus** a mandatory **passenger** load for fuel-economy testing) represents the inertia to **linear** motion, M_e additionally accounts for the **rotational** inertia of the 4 rotating wheel assemblies (see [Appendix](#)).

The **tractive force**, F_{TR} , required to overcome these resistances acts on the **axle** of a vehicle's **driving** wheels, and is the **reaction** to the force generated on the road at the tire-road contact patch of those wheels by a **torque** applied to the driving axle (see [Appendix](#)).

On any driving schedule that **prescribes** the variation of vehicle speed (V) with time (t), such as the EPA schedules, Newton's Second Law dictates that the **magnitude** of the **instantaneous** tractive force that a vehicle **requires** for **propulsion** on a level, straight road be

$$F_{TR} - R - D = M_e (dV/dt) \text{ or,}$$

$$F_{TR} = R + D + M_e (dV/dt) \quad (1)$$

The resistances R and D are always positive, while the inertia term is positive in accelerations but negative in decelerations. Consequently, F_{TR} is greater than **zero** in constant-speed driving (cruising), accelerations, and the **portions** of decelerations in which the **magnitude** of the negative inertia term is **less** than that of the corresponding $(R + D)$, i.e., **powered** decelerations.

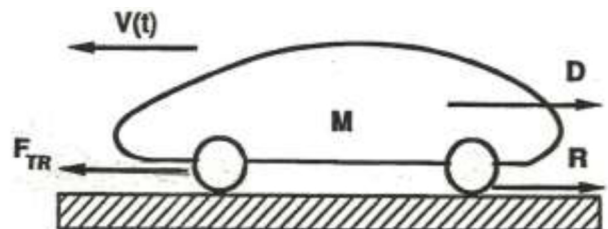


Figure 1. Freebody Diagram for a Tractively Driven Vehicle

The delivery of a **positive** F_{TR} to the driving axle of a vehicle requires a corresponding supply of **tractive energy**, E_{TR} . For travelling an **incremental** distance $ds = V dt$, that energy is

$$d(E_{TR}) = F_{TR} ds = R ds + D ds + M_e (dV/dt) V dt \quad (2)$$

Now, $M_e (dV/dt) V dt = d(M_e V^2/2)$, which is the **increase** in **vehicle kinetic energy** during the incremental powered travel. Using this in Eq.(2),

$$d(E_{TR}) = R ds + D ds + d(M_e V^2/2) \quad (3)$$

Summing over **all** the segments of **positive** F_{TR} in a driving schedule, which are of **total duration** τ , the **total** energy required for vehicle **propulsion** is

$$E_{TR} = \int_{\tau} d(E_{TR}) = \int_{\tau} R ds + \int_{\tau} D ds + \int_{\tau} d(M_e V^2/2) \quad (4)$$

The instantaneous **total** rolling resistance, R , of the **4** wheels of a vehicle is determined by the tire rolling-resistance **coefficient**, r , and the total **weight** of the vehicle, Mg , where g is the acceleration of gravity, i.e.,

$$R = r Mg \quad (5)$$

In an exhaustive computer and on-road evaluation of the impact of tire rolling resistance on vehicle fuel economy the Michelin tire company (5) considered the various factors that influence the coefficient r , and concluded that “for **usual** driving conditions, the influence of speed on rolling resistance can be **neglected**”. It follows that $r = r_0$, where r_0 is a dimensionless **constant**, is **adequate** for the present analysis. The friction of wheel bearings is very small compared to tire rolling resistance and is assumed to be included in r_0 . Additionally, the contribution of aerodynamic **lift** to the tire-patch **vertical load**, the other determinant of rolling resistance, is considered to be **negligible**. Consequently,

$$\int_{\tau} R ds = \int_{\tau} (r_0 Mg) V dt = r_0 Mg \int_{\tau} V dt \quad (6)$$

A vehicle's instantaneous aerodynamic drag is

$$D = C_D A (V^2/2) \rho \quad (7)$$

where C_D is the aerodynamic drag **coefficient** of a vehicle's body, A its maximum cross-sectional area, and ρ the density of ambient air. It is **implicit** in the driving schedules used for fuel-economy regulation that driving occurs in the **absence** of ambient wind. Consequently, the relevant velocity for aerodynamic drag is the **vehicle** speed V . It is conventional to assume that C_D is **constant** in the speed range of driving schedules. Consequently,

$$\int_{\tau} D ds = \int_{\tau} [C_D A (V^2/2) \rho] V dt = C_D A (\rho/2) \int_{\tau} V^3 dt \quad (8)$$

Using Eqs. (6) and (8) in Eq. (4),

$$E_{TR} = r_0 Mg \int_{\tau} V dt + C_D A \rho/2 \int_{\tau} V^3 dt + M_e \int_{\tau} d(V^2/2)$$

Normalizing with a vehicle's static mass, M , and the total distance, S , travelled in a schedule,

$$E_{TR}/MS = [g/S \int_{\tau} V dt] r_0 + [\rho/2S \int_{\tau} V^3 dt] (C_D A/M) + [1/S \int_{\tau} d(V^2/2)] (M_e/M) \quad (9)$$

Let

$$g/S \int_{\tau} V dt \equiv I_1 \quad (10)$$

$$\rho/2S \int_{\tau} V^3 dt \equiv I_3 \quad (11)$$

$$1/S \int_{\tau} d(V^2/2) \equiv I_2 \quad (12)$$

Using these in Eq. (9), its **representation** is simplified to

$$E_{TR}/MS = I_1 r_0 + I_3 (C_D A/M) + I_2 (M_e/M) \quad (9')$$

Since the development from Eq.(1) has been **mathematically** rigorous, it follows that Eq.(9) represents the **valid solution** for the tractive energy, E_{TR} , that a vehicle requires to drive a prescribed $V(t)$ schedule. The issue, then, is to determine the specific **nature** of the integral coefficients I_1 , I_3 and I_2 .

The **integrands** in these coefficients and the value of S are dependent **only** on the specific nature of the $V(t)$ of a schedule. They are **independent** of the vehicle parameters r_0 , C_D , A , M , M_e . The **range** of integration τ , however, is **dependent** on them, as will be shown.

In addition to the total durations of acceleration and cruise, the powered-driving duration τ includes the duration of **powered decelerations**. In decelerations, vehicle inertia force $M_e(dV/dt)$ is **negative** and therefore **assists** vehicle motion, rather than **resisting** it as it does in accelerations, with the corresponding energy coming from the **kinetic energy** of a vehicle. During the **initial** phase of a deceleration the magnitude of this assist is **smaller** than a vehicle's corresponding **resistance** to motion, $(R + D)$. Consequently, **tractive force** is required to make up the shortfall. With **increasing time** in an initial phase the resistance **decreases** due to the decrease in D with decreasing vehicle velocity, while the inertial assist remains constant or increases. This

makes the inertial **shortfall**, and hence the need for tractive force, **decrease** with time. Eventually, the inertial shortfall is reduced to **zero**, **ending** this **powered** portion of the deceleration. Since both $M_e(dV/dt)$ and $(R + D)$ are **dependent** on **vehicle parameters** this end point, and hence the **duration** of this powered deceleration, is **also** so dependent. Depending on the **specifics** of $V(t)$ in the **end** region of a deceleration, a segment of powered driving can also occur **there**. The inclusion of these powered decelerations in τ makes **that** duration **vehicle dependent**. The nature and magnitude of this dependency on the values of the integral coefficients therefore needs to be examined.

For any driving schedule the **prescribed** $V(t)$ specifically identifies and quantifies the total duration of both its acceleration and its constant-speed segments. Their sum, τ' , is therefore **independent** of vehicle parameters. Consequently, for **this** range of integration all three integral coefficients in Eq. (9) are **constants** of the prescribed $V(t)$. Therefore, if the **results** of Eq.(9) for different vehicles were plotted with E_{TR}/MS as the **dependent** variable and $C_D A/M$ as the **independent** one, the curves for different values of $r_0 = C$ would be **linear**, and all would have the **same** slope, I_3 . While acceleration and cruise constitute **most** of the powered driving, the key question that needs to be addressed is: what, if anything, does the inclusion of **powered decelerations** do to this linearity?

To examine this question, **computations** of Eq.(9) were made for vehicles with broad ranges of r_0 and $C_D A/M$:

$$6 \leq r_0 \times 10^3 \leq 11 \quad 3 \leq C_D A/M \times 10^4 \leq 8.$$

For typical automobiles, M_e is **not** greatly different from the static mass M ; eg, $1 < M_e/M < 1.04$. For **initial** calculations, the entire field of vehicles was represented by the single value of $M_e/M = 1$.

The numerical algorithm used for the evaluations had the following characteristics.

1. **Velocity derivatives** were determined by $dV/dt)_j = (V_{j+1} - V_{j-1})/(2 \, dt)$, which is of order $(dt)^2$ accuracy. The time step was the same as that used in defining the velocity schedules, i.e. 1 second.
2. Definite integrals were evaluated by the trapezoidal rule.
3. The **limits** of the integrals were determined by the **zero crossings** of the tractive power $P_{TR} = F_{TR} V$. These generally occur **within** time steps, so **contributions** to E_{TR} are produced by these steps. Consequently, for **such** time steps the trapezoidal rule was replaced by evaluations of the individual **triangular areas** that represent E_{TR} contributions.

For the chosen field of vehicle characteristics, the **results** for both the Urban and Highway schedules are shown in Figure 2. Additional results were the total distance, S , travelled on each schedule:

$$S_U = 11,990 \, [m] \quad S_H = 16,510 \, [m]$$

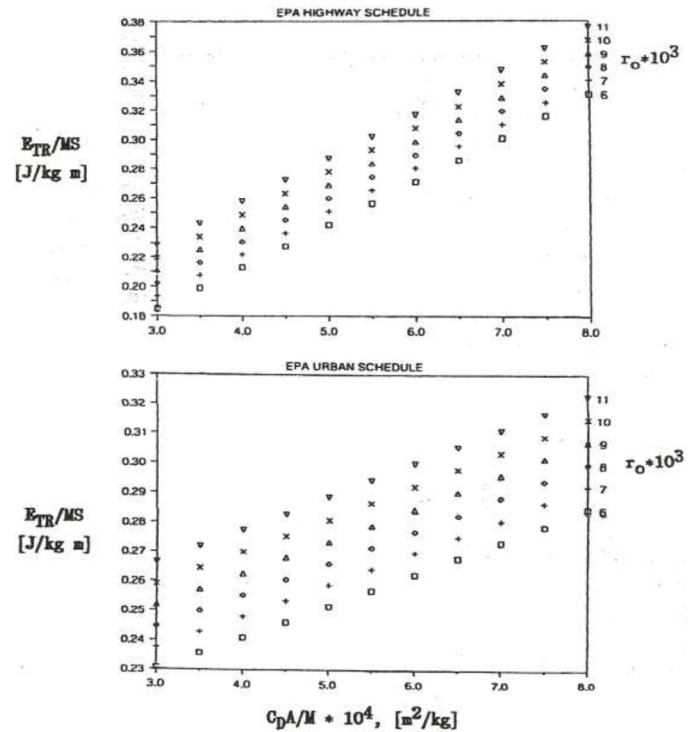


Figure 2. Tractive-energy Requirements for the EPA Driving Schedules

The **linearity** of the $r_0 = \text{constant}$ lines that exists for acceleration-plus-cruise is only **very slightly** compromised ! Consequently, the **total** results for each driving schedule were **fitted**, using the least-squares method, with a **family** of **linear** curves of the form

$$E_{TR}/MS = \alpha r_0 + \beta (C_D A/M) + \gamma (M_e/M)$$

(13)

where α , β , and γ are dimensional **constants** for any given driving schedule, but **different** between schedules. The peripheral regions of Figure 2 were excluded from the fitting in order to avoid **biasing** by vehicles outside the realm of common practice. The fitted range was $7 \leq r_0 \times 10^3 \leq 10$ and $4 \leq C_D A/M \times 10^4 \leq 7$. The values of the coefficients and the degree of fit to the data are shown in Table 1.

The r^2 values of the **linear fits** to the computational results are essentially **unity**, indicating that for **practical engineering** purposes the family of equations represented by Eq.(13) are valid **solutions** of the Tractive-Energy Equation ! A very close examination of the data in Figure 2 reveals that the $r_0 = \text{constant}$ lines are **not** truly linear, each generally having a very slight upward concavity. The linear equations that fit them so well do so by intersecting each curve twice.

Table 1. Tractive-Energy Coefficients for the EPA Driving Schedules

Coefficient	Urban	Highway
α [m/s ²]	7.43	9.074
β [kg/ms ²]	110.0	293.6
γ [m/s ²]	0.1515	0.0403
Fit: r^2	0.99991	0.99997
S [m]	11,990	16,510
M = [kg] ; $M_e/M = 1.0$; A = [m ²] ; $E_{TR} = [J]$		

The integral coefficients in Eq.(9) were predicted to be vehicle **dependent**, and this is confirmed by Figure 3. It can be seen that each **varies** over the vehicle range.

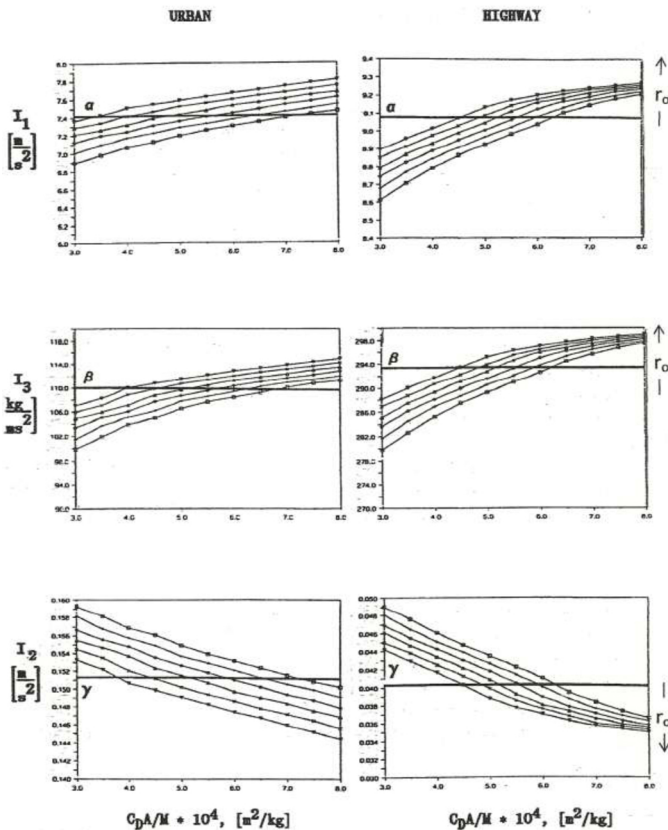


Figure 3. Tractive Integrals for the EPA Driving Schedules; $r_0 \times 10^3 = 6, 7, 8, 9, 10, 11$.

As either $C_D A/M$ or r_0 **increases**, powered driving extends **further** into the deceleration segments. This directly **increases** I_1 and I_3 by **extending** their range of integration.

At the same time, there is a corresponding **decrease** in the **final** velocity of the corresponding powered-driving segments. This **reduces** the **overall increase** in V^2 in these segments, thereby **reducing** a vehicle's overall gain in kinetic energy during powered driving. This **reduces** I_2 . These are the trends evident in Figure 3 for both the Urban and Highway schedule.

Since the variation of I_2 with $C_D A/M$ is of **opposite** sign to that of I_1 and I_3 , there is a degree of **self-compensation** in the **net** effect of their variations on the **linearity** of E_{TR} . This contributes to the **nearly** $r^2 = 1$ values for the linear curve-fitted equations.

For reference, horizontal lines at the values of α , β , and γ are shown in Figure 3. In spite of the approximations that they make of their respective **fields** of I values, Eq.(13) fits the **total** results of Eq.(9) in Figure 2 **extremely well**. Since Eq.(9) is an **exact** representation of E_{TR}/MS and Eq.(13) an **extremely good fit** of its values it follows, **essentially**, that for **any** set of vehicle parameters

$$[I_1 - \alpha] r_0 + [I_3 - \beta] (C_D A/M) + [I_2 - \gamma] (M_e/M) = 0 \quad (14)$$

Figure 3 shows that, **in general**, $[I_1 - \alpha] \neq 0$, $[I_3 - \beta] \neq 0$, $[I_2 - \gamma] \neq 0$. Consequently, the differences in Eq.(14) must be **mutually compensating**. This requires that they not all be of the same sign. For example, for the vehicle combination $C_D A/M \times 10^4 = 4$ and $r_0 \times 10^3 = 6$, the data of Figure 3 for the Urban schedule show that $I_1 < \alpha$, $I_3 < \beta$, but $I_2 > \gamma$. Furthermore, there are vehicles for which one of the difference terms in Eq.(14) is zero, and even cases where all three are nearly zero.

The preceding developments indicate that Eq.(13) for E_{TR}/MS can be used with considerable **confidence** for predicting the **tractive-energy requirement** of any vehicle. However, since any or all of the following inequalities - $\alpha \neq I_1$, $\beta \neq I_3$, $\gamma \neq I_2$ - may exist for a vehicle, the **distribution** of that energy into its component parts (rolling resistance, aerodynamic drag, vehicle kinetic energy) can be somewhat **distorted**.

The **relationship** between the Urban and Highway tractive-energy requirements defined by the coefficients in Table 1 is more apparent when the information is in **graphical** form, as in Figure 4. For clarity, only one value of r_0 is shown. The lines for **both** schedules have decreasing values of E_{TR}/MS as $C_D A/M$ decreases, and the most striking feature is that the lines **intersect**.

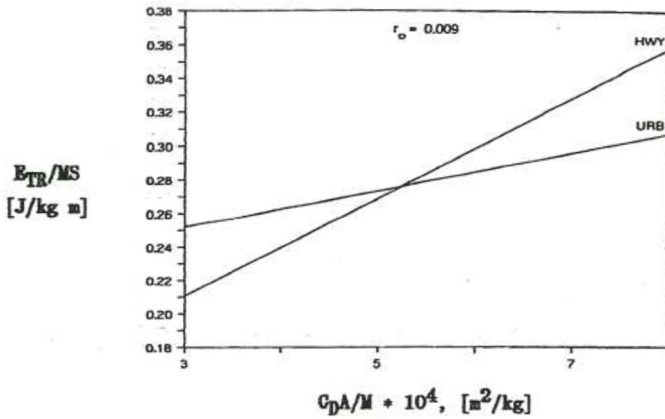


Figure 4. Comparison of the Tractive-Energy Requirements for the EPA Urban and Highway Schedules.

The **primary** determinant of a vehicle's fuel consumption is the amount of energy, E_{TR} , that its **powerplant** must deliver to its driving wheels for **propulsion**, **not** its normalized value E_{TR}/MS . The relationship between them is

$$E_{TR} = [E_{TR}/MS] MS$$

For both schedules, E_{TR}/MS is reduced by **reducing** C_{DA}/M . This can be accomplished by **reducing** C_{DA} , **or** by **increasing** M . The former reduces E_{TR} but the latter does **not**, because E_{TR}/MS does not decrease as fast as M increases.

BRAKING

During the **decelerations** of a prescribed driving schedule the **negative** vehicle inertia force $M_e (dV/dt)$ **assists** vehicle motion. In the early phases of any deceleration this assist is **smaller** than the corresponding **resistance** to vehicle motion, $(R+D)$, that has to be overcome. Consequently, a **tractive force**, F_{TR} , is required to make up the shortfall. However, the shortfall **decreases** with decreasing vehicle velocity in the deceleration, eventually reducing to zero and **ending** the need for tractive force. At **subsequent** velocities the inertial assist is **greater** than the corresponding $(R+D)$ that has to be overcome. Consequently, a vehicle **braking force**, F_{BR} , is required to offset the **excessive propulsion**. The detailed $V(t)$ in the **end** region of a deceleration determines **when** this force is **no longer** required, whether **before** the end of the deceleration, or **at** it.

The required magnitude of the **instantaneous** braking force is determined by the following form of Eq.(1).

$$F_{BR} + R + D = -M_e (dV/dt)$$

or,

$$F_{BR} = -M_e (dV/dt) - R - D \quad (15)$$

The **energy** transfers dictated during an **incremental** travel $ds = Vdt$ are,

$$\begin{aligned} d(E_{BR}) &= F_{BR} ds = [-M_e dV/dt] Vdt - R ds - D ds \\ &= -d(M_e V^2/2) - R ds - D ds \end{aligned} \quad (16)$$

The first term on the right is the **vehicle kinetic energy** that must be **removed** during the incremental travel. The next two terms are the kinetic-energy removals produced by the **inherent** rolling and aerodynamic resistances. The **net** value of the right side is then the kinetic energy that must be removed by some type of vehicle **braking** force.

Summing over all the driving segments in which $F_{TR} < 0$, which are of total duration $(T - \tau)$ in a driving schedule of total duration T ,

$$\begin{aligned} E_{BR} &= \int_{T-\tau} d(E_{BR}) \\ &= -M_e \int_{T-\tau} d(V^2/2) - \int_{T-\tau} R ds - \int_{T-\tau} D ds \end{aligned} \quad (17)$$

Although $(T-\tau)$ includes the duration of any **idling** segments in a $V(t)$, this is of no consequence because there are **no** energy transfers during idle. This equation is analogous to Eq.(4) for E_{TR} . Using the **same** process that transformed that equation into Eq.(9), the present equation for E_{BR} can be transformed into

$$\begin{aligned} E_{BR}/MS &= [-1/S \int_{T-\tau} d(V^2/2)] (M_e/M) - \\ &[g/S \int_{T-\tau} V dt] r_0 - [\rho/2S \int_{T-\tau} V^3 dt] (C_D A/M) \end{aligned} \quad (18)$$

Let

$$g/S \int_{T-\tau} V dt \equiv I_1^* \quad (19)$$

$$\rho/2S \int_{T-\tau} V^3 dt \equiv I_3^* \quad (20)$$

$$-1/S \int_{T-\tau} d(V^2/2) \equiv I_2^* \quad (21)$$

Using these in Eq. (18) to simplify its **representation**,

$$E_{BR}/MS = I_2^* (M_e/M) - I_1^* r_0 - I_3^* (C_D A/M) \quad (18')$$

In this equation, which is the analog of Eq.(9') for E_{TR} , the inertia term $I_2^* (M_e/M)$ represents the **total** vehicle kinetic energy that must be **removed** during the braking segments of decelerations, while the tire and aerodynamic terms represent their respective and **inherent** contributions to