

Extended Steady State Lap Time Simulation for Analyzing Transient Vehicle Behavior

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ABSTRACT

The extended steady state lap time simulation combines a quasi steady state approach with a transient vehicle model. The transient states are treated as distance dependent parameters during the calculation of the optimal lap by the quasi steady state method. The quasi steady state result is used afterwards to calculate a new dynamic behavior, which induces in turn a different quasi steady state solution. This iteration between the two parts is repeated until the dynamic states have settled. An implementation of the extended quasi steady state simulation is built up to determine the capabilities of the approach. In addition to pure steady state simulation abilities, the method is able to judge the influence of the transient or time variant vehicle states on lap time. Sensitivity studies are generated to analyze the influence of basic parameters like mass, but also the influence of parameters with transient interaction like vertical damping or tire temperature. The extended quasi steady state lap time simulation approach provides an accurate possibility to integrate dynamics into the quasi steady state approach.

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INTRODUCTION

Minimizing the lap time for a given race track is the main development goal in motorsports. A simulation model which predicts the vehicle's performance, enables one to judge the effect of various vehicle parameters on the lap time early in the development process.

Present lap time minimization algorithms are mainly based in quasi steady state approximation, control theory or numerical optimization like optimal control.

The quasi steady state approach calculates the maximal vehicle force potential independently in every time step for a given curvature. The optimal lap is generated as a combination of optimal braking and acceleration phases that are developed coming from the apexes. Milliken and Milliken [1] described the development process of the quasi steady state simulations since 1950. The first lap time simulations worked with a point mass representation and the knowledge of the maximal longitudinal and lateral acceleration potential of the car. A simulation based on the Milliken moment method is developed by Milliken and Milliken [1] to calculate the lap time for bicycle or two track models. Candelpergher et al. [2] used a speed related g-g diagram as

shown in [figure 1](#) that describes the maximum car acceleration potential to develop a lap time simulation tool for setup optimization on the race track. The pre-calculated g-g diagram is used as a lookup table to interpolate the maximal car potential depending on the current vehicle state for example, vehicle speed.

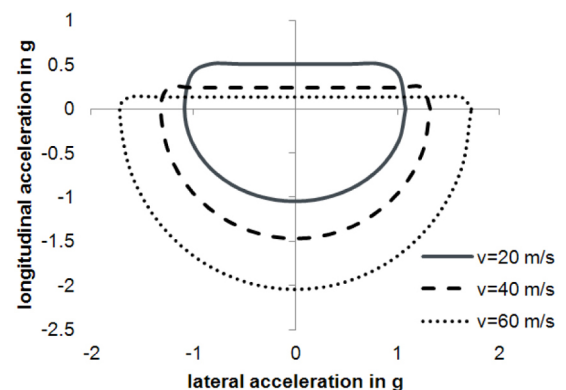


Figure 1. Speed related g-g diagram. The maximal car acceleration potential is described for different speeds.

Brayshaw [3] also used the performance envelope description of a two track model with 7 degrees of freedom (DOF) in the form of a speed related g-g diagram to compare different car configurations. Brayshaw and Harrison [4] applied this method to analyze the effects of different weight distributions on the optimal driving line, which have little interaction with each other according to the authors' analysis.

Instead of pre-calculating a g-g diagram, the maximum car potential can be calculated online in every time step as Siegler [5] used it to analyze simple corner maneuvers. He also developed an optimization process for the control input to calculate the minimal maneuver time for a transient vehicle. The results of the comparison of steady state, quasi steady state and transient cornering maneuvers are summarized by Siegler et al. [6]. The steady state simulation did not produce sufficient results, while the quasi steady state and the transient approach had been on a comparable level.

Quasi steady state simulations are based on a given curvature profile. To get a full trajectory optimization, the quasi steady state approach is combined with additional optimization algorithms as described by Mühlmeier and Müller [7]. They found the optimal path for a point mass model representation for different car configurations with the help of a genetic optimization algorithm.

Quasi steady state lap time simulations are precise, numerically efficient and robust, but do not account for transient or time variant effects. The identification of the optimum path and speed profile that correlates with a minimum lap time for a full transient vehicle model has the need for a high-order nonlinear optimization. The direct transcription optimal control methods allow transfer of the ordinary differential equations (ODE) of the dynamic system as an optimization constraint, so the problem can be solved with nonlinear programming. Casanova [8] used a direct optimal control method to calculate the minimal maneuver time for a vehicle model with seven DOF on a complete racing circuit. Casanova used the method to analyze the effects of yaw inertia [9] and weight distribution [10]. For complete lap optimizations, calculation times of over 24 hours are stated with a lap time tolerance of about 0.1%.

Velenis [11] derived a semi analytic approach for trajectory planning from the optimal control solution to increase robustness and reduce calculation time. The simplification reduced accuracy compared to the optimal control planned trajectory but allowed the solution to be used in online guidance schemes.

More detailed car models that increase the size or nonlinearity of the problem reduce the numerical efficiency and robustness of the numerical optimization. Therefore Kelly [12] applied a finite horizon approach, which did not need all vehicle states for optimization, to analyze a more complex vehicle model. Hence Kelly was able to analyze the effects of tire temperature and vertical dynamics. The drawback of the method is the reduced efficiency. Kelly also introduced some stability constraints to avoid nonlinear vehicle states and to increase robustness of the solution.

Mavroudakakis [13] used the same approach as Kelly in comparison with a speed related g-g diagram to calculate the lap time effects of a decoupled suspension system. The author came to the conclusion that the additional information justifies the higher complexity and numerical effort of the optimal control approach compared to the results from the quasi steady state simulation.

To reduce the computational effort of the lap time simulation algorithm, driver models based on control theory are used. Plöchl and Edelmann [14] provide a general overview over the different types of driver models and the diverse research areas of driver model application.

The lap time simulation calculation effort is reduced by using simplified or constrained vehicle models for trajectory planning in combination with a driver model that guides the full vehicle along the pre-calculated trajectory. Gordon et al. [15] generated a vector field of the desired speed profile at every point on the track. They suggested taking the vector field as an input for the driver to find the minimal time maneuver. König [16] used a quasi steady state lap time simulation to plan the speed profile and a controller based on exact linearization to guide the full vehicle along the planned path. Butz [17] planned the minimal time trajectory for a double lane test with a simple bicycle model and direct optimal control. A nonlinear position controller is used to stabilize the full vehicle while driving on the optimized trajectory.

The simplification in the trajectory planning leads to deviation from the planned path when the full vehicle model is guided along the planned path. The deviations lead to reduced accuracy and higher lap time tolerances. Therefore the optimization algorithm has to work with the full vehicle model to increase accuracy. The control algorithm that guides the vehicle along a given trajectory reduces the order of the optimization problem. The driver model controls the nonlinearities of the vehicle and the optimization process calculates the minimal time trajectory.

Völkl et al. [18] used a controller that keeps the vehicle near the performance limit in combination with a simple optimization algorithm that varied the braking points to find the minimal maneuver time on a given driving line. Butz and Ehmann [19] applied nonlinear programming to iteratively optimize the speed profile and driving line, which are then used as reference for the driver model as described in [17]. The authors stated calculation times from 1.5 to 7 hours depending on the optimization solver type.

The usage of the full model feedback for the optimization improves the results compared to a driver model that follows an optimization result, but the application of a driver model still changes the dynamics of the controlled system. The optimal vehicle condition is not generated by the optimization algorithm but by the driver model. Consequently the design and application of the driver model has a major influence on the lap time result, the optimality of the solution is more difficult to prove and the method is exposed to a lack of precision compared to the optimal control or quasi steady

state solution. On the other hand the vehicle dynamics simulation with driver model is numerically efficient, robust and the vehicle model can be extended with minor effort. The optimal control approach in contrast becomes numerically inefficient and less robust to find feasible solutions for complex vehicle models on long tracks. Lap time simulation needs high robustness and precision combined with numerical efficiency. Quasi steady state solution fulfils these requirements, but as already explained, the quasi steady state (QSS) approach does not allow the evaluation of transient or time variant effects.

Therefore an approach is suggested in this work to extend the quasi steady state method with a representation of the transient vehicle states. The transient states are treated as distance dependent parameter during the calculation of the optimal lap by the quasi steady state method. The quasi steady state result is afterwards used to calculate a new dynamic solution, which induces in turn a different quasi steady state solution. This iteration between the two parts is repeated until the dynamics converge.

First the application of the method in its general form is demonstrated on a simple time minimal maneuver problem. Subsequently the method is applied to the lap time minimization problem of a vehicle guided on a given driving line. The different model parts and the exchange method between the quasi steady state and the dynamic models are described. Convergence behavior and precision are identified and used to rate the range of application of the method. Different sensitivity studies are performed to visualize the capabilities of the method. Finally the results of the method are discussed resulting in a summary.

DESIGN AND IMPLEMENTATION

The minimal maneuver time for an object on a given path is generated by maximizing the average velocity. In the quasi steady state procedure, maximal average velocity is achieved by using maximal acceleration potential in every time step coming from the slowest points. The speed in the slowest points is calculated from the maximal lateral potential. Above this speed the vehicle gets uncontrollable. The time steps are independent in a quasi steady state model description, so the control strategy is developed from different starting points forwards or backwards and speed minima or maxima are identified before the solution is calculated. The identification of the speed minima/maxima is convenient, because the minima are used as the starting points of the calculation.

The transient states are treated like a distance dependent parameter during the QSS simulation in the utilized approach. At the beginning, the quasi steady state optimal solution is calculated with transient states that are based on an initial guess. The transient states are refined afterwards with the result from the optimal solution in a separate model. The states influence the acceleration potential therefore the minimal maneuver time. Hence the refined transient states are used for the calculation of a new optimal speed solution,

which induces in turn a different transient behavior. This iterative calculation of the optimal solution and the transient states is repeated until the dynamic states have settled. The described procedure is feasible if the dynamic system is stable and the dynamics have no high feedback on the control, otherwise the iteration is not converging. The stability is checked in the dynamic model and constraints keep the system in stable conditions. The feedback gain depends on the interaction of the model and its controls and is checked for every model separately.

The method is explained through a simple example based on the task of landing a space rocket in minimal time on the ground. The space rocket is described with the following dynamics. The acceleration $\dot{v}$ depends on the gravity g , the actual mass m and the thrust α .

$$\dot{v} = g + \frac{\alpha(t)}{m(t)} \quad (1)$$

The mass is changing with the used fuel, described by the consumption factor c .

$$\dot{m} = -c\alpha(t) \quad (2)$$

The following parameters are used:

Thrust $0\text{N} \leq \alpha(t) \leq 5\text{N}$,

gravity force $g = -1 \frac{\text{m}}{\text{s}^2}$,

starting height $h_{\text{start}} = 10\text{m}$,

mass $m_{\text{start}} = 1 \text{ kg}$,

starting velocity $v_{\text{start}} = -10 \frac{\text{m}}{\text{s}}$

consumption factor $c = 0.05$.

The changing mass keeps the solution from being calculated with the pure quasi steady state approach. The mass is therefore fixed while calculating the optimal solution. The solution is generated rearwards from the ground with maximal braking force and forward from the starting point with maximal acceleration. The point of intersection of the two speed traces is the start of the braking phase. With the finished optimal solution the mass change is calculated. Based on the distance dependent new mass, a new optimal solution is calculated. The iteration stops once the difference between actual and previous speed trace lies within a specified tolerance. In this case the iteration is finished after nine iterations. [Figure 2](#) shows first, second and last iterations of the problem. The figure demonstrates how the iteration works. The mass history of the second iteration is generated by the controls of the first calculation, which is based on an initial guessed mass. The solution of the second iteration is generated with the mass history of the second solution, but its controls will induce a different mass history. The calculated mass induces a new different solution, until the dynamics are settled.