

Managing the Computational Cost of Monte Carlo Simulation with Importance Sampling by Considering the Value of Information

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ABSTRACT

Importance Sampling is a popular method for reliability assessment. Although it is significantly more efficient than standard Monte Carlo simulation if a suitable sampling distribution is used, in many design problems it is too expensive. The authors have previously proposed a method to manage the computational cost in standard Monte Carlo simulation that views design as a choice among alternatives with uncertain reliabilities. Information from simulation has value only if it helps the designer make a better choice among the alternatives. This paper extends their method to Importance Sampling. First, the designer estimates the prior probability density functions of the reliabilities of the alternative designs and calculates the expected utility of the choice of the best design. Subsequently, the designer estimates the likelihood function of the probability of failure by performing an initial simulation with Importance Sampling. For this, the designer uses the Edgeworth expansion, which requires estimates of the mean value, standard deviation and the skewness of the failure probability. Finally, the designer selects the best design for each outcome, and the expected utility of the decision. The value of information from the simulation is the increase in the Certainty Equivalent of the reliability resulting from the use of information from the simulation.

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INTRODUCTION

Design decision making under uncertainty requires evaluation of the reliability of many design alternatives using large-scale models. Designers often use Monte Carlo simulation for this purpose. Although efficient simulation methods are available, the computational cost is an important consideration.

One way to manage the computational cost is to predict the number of replications required to determine the best design among alternatives before running the simulation. A designer accomplishes that by *pre posterior analysis*: she predicts all the possible outcomes of a simulation and the value of the best design for each outcome. The value is measured by the expected utility. If the designer repeats this analysis for different numbers of replications, then the designer can estimate the effect of this number on the value of the information from the simulation. Nikolaidis et al. [1]

proposed a method that uses the above concept to measure the value of information from standard Monte-Carlo simulation in selecting a design among alternatives with uncertain reliabilities. This paper extends this method to Importance Sampling (IS). In the proposed method, we estimate the probability of failure using a different Probability Density Function (PDF) than the true one. We choose this sampling PDF to maximize the accuracy of the results for a given number of replications.

Information enables people to make better decisions. *Information economics* is the systematic study of measuring the value of information in support of a decision. Information economics is a valuable tool for supporting decision making by helping designers manage testing and computing costs [2, 3].

The basic concept is that the value of information from a Monte Carlo simulation is equal to the gain from using this information to select the best design among alternatives. In

the following, we ignore the cost of simulation. We would have to subtract the cost of the simulation from the gain in order to determine the net gain. This method relies on two principles:

- 1). The value of information depends on the particular decision.
- 2). The value of information is equal to the increase in *certainty equivalent* of a decision resulting from the use of the information to make a more informed choice:

$$\text{Value of information} = \text{Certainty equivalent of the decision with the information} - \text{Certainty equivalent without the information}$$

(1)

Certainty equivalent of a decision with uncertain payoff is the certain amount that the decision maker considers equivalent to the uncertain payoff. We consider both the objective and subjective interpretations of the probability of failure. This probability is either the long-term frequency of failure, or a subjective probability. The latter expresses the degree of belief that the design could fail. This probability is equal to the probability of a device, such as a well-balanced and oiled wheel of fortune, which makes the decision maker indifferent between two tickets (A and B) that pay the same prize (e.g., \$100) if the arrow of the wheel of fortune stops in a predetermined sector (ticket A), and the design fails (ticket B). We consider that a subjective probability of failure is unique. In principle, we can estimate it by a suitable elicitation procedure [4].

Next section explains the problem formulation and the method for measuring the value of information from Monte Carlo simulation with IS. The succeeding section demonstrates the method on the selection between two designs with uncertain reliabilities, and the last section concludes.

METHOD

Problem Formulation

A designer wants to select a design among the alternative options $A, B, \dots$ to maximize the expected utility, $E\{U(P_A), E(U(P_B)), \dots$

where $P_A, P_B, \dots$ are the design attributes of options A and B , and $U(P_A), U(P_B)$ their utilities.

Ideally, a manufacturer seeks to maximize the profit derived from a product. However, this is difficult to predict, especially in the early design stages [5]. To circumvent this issue, manufacturers evaluate a design based on surrogate attributes such as weight, reliability, and rigidity.

This paper focuses on decisions in which the reliability R is the only important attribute of a design. The designer does not know the exact value of reliability because of the uncertainties in the probability distributions of the random variables and errors in the predictive models. She calculates reliability using IS, and is uncertain about this attribute

because of the limited number of replications that she can perform. As a result, the designer cannot identify with certainty the most reliable design, but she can increase her confidence in selecting it by performing more replications. The objective of the proposed method is to measure the value from performing these replications.

Calculating the Value of Information

Figure 1 shows the process of selecting the most reliable design among alternatives and estimating the value of information by using Bayesian updating, and IS. For simplicity, this paper illustrates this process for two alternatives. First, the designer estimates the prior PDFs of the true failure probabilities of the two designs based on judgment and an initial simulation. In addition, the designer determines the variance and skewness of the sample from this simulation. The next five steps are repeated NS times. In step 2, sample values of the probabilities of the two designs are drawn. The designer performs a simulation with IS (step 3) and updates the PDFs of the failure probabilities of designs A and B in view of the observed failures (step 4). In steps 5 and 6, she calculates the expected utilities of the alternative designs and selects the best. The expected utility of the decision equals to the expectation of the utilities from the NS replications:

$$EU = \frac{1}{NS} \sum_{i=1}^{NS} \max(E(U(p_{A_i})), E(U(p_{B_i}))) . \quad (2)$$

Utility is a *value function* - not a *value difference function*. Therefore, we can only determine if the value of a decision increases with the replications - but not by how much, from the increase in the expected utility. Instead, we should use the increase in the Certainty Equivalent (CE) reliability of the design. CE is defined such that a design with certain reliability $R = CE$ is equivalent to the design with uncertain reliability:

$$U(CE) = E(U) . \quad (3)$$

The term “equivalent designs” means that the designer would exchange happily one for the other.

The Edgeworth expansion [6]

Consider a sample $X_1, \dots, X_N$. The objective is to find the probability distribution function of the estimator of $E(X)$, which is the average:

$$\bar{X} = \frac{1}{N} \sum_{i=1}^N X_i . \quad (4)$$

For large sample sizes, the probability distribution of the above estimator approaches the normal one. Here, we seek a better approximation.