

An Efficient, One-Dimensional, Finite Element Helical Spring Model for Use in Planar Multi-Body Dynamics Simulation

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ABSTRACT

The helical spring is one of fundamental mechanical elements used in various industrial applications such as valves, suspension mechanisms, shock and vibration absorbers, hand levers, etc. In high speed applications, for instance in the internal combustion engine or in reciprocating compressor valves, helical springs are subjected to dynamic and impact loading, which can result in a phenomenon called “surge”. Hence, proper design and selection of helical springs should consider modeling the dynamic and impact response.

In order to correctly characterize the physics of a helical spring and its response to dynamic excitations, a comprehensive model of spring elasticity for various spring coil and wire geometries, spring inertial effects as well as contacts between the windings leading to a non-linear spring force behavior is required. In practical applications, such models are utilized in parametric design and optimization studies. For that reason, computational efficiency is also a key requirement.

In this paper, a helical spring dynamics model is presented, in which spring coils are modeled by means of one-dimensional curved beams. The displacements of these elastic components are expressed in a “floating” frame, which can undergo large rigid body motions in 2D space. The formulation thus enables modeling of planar displacements of the helical spring and captures the two-dimensional inertial effects associated with the rigid body motion/acceleration of coils but, at the same time, only requires a single degree-of-freedom per spring node. The Finite Element approach is used in order to efficiently discretize these elastic components. As compared to a more commonly used lumped mass-elastic helical spring modeling method, the proposed finite-element approach enables reduction of degrees of freedom required to accurately capture dynamic response of the spring. Furthermore, two different approaches for modeling spring coil contact and clash are presented, i.e. “lumped” and “distributed”. In the simpler, lumped coil clash model, contact forces are expressed as point loads. The distributed coil clash model treats coil contact forces as distributed loads. This results in a more efficient system since it requires fewer contact elements to correctly represent coil closure during spring compression event and, at the same time, eliminates the polygonal effect common to lumped characterization of coil-to-coil interactions.

The model has been implemented within a general-purpose multi-body dynamics analysis tool. In order to validate the model's accuracy, results from numerical simulation were compared to analytical solution of the hyperbolic partial differential equation governing helical spring dynamic response. Furthermore, experimental results from a static and dynamic spring compression tests of an automobile engine valve spring were used and compared to simulation results showing good correlation. Finally, computational efficiency of the presented model was studied in the context of a multi-body simulation of engine valvetrain system.

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INTRODUCTION

The analysis of static and dynamic behavior of helical springs is an important engineering problem. In practice, helical springs are commonly used as structural elements in many mechanical applications, for example in suspension systems, shock and vibration absorbers, engine and compressor valves, hand levers, etc. The primary functions of springs are to absorb energy, to apply a definite force, to support moving components or to isolate vibration. In high speed applications (for instance, in the internal combustion engine valvetrain), the helical spring is usually the softest components with the lowest natural frequency in the system. Its influence on the overall dynamics of the system is significant and cannot be ignored. Operating near the natural frequency of the spring can result in excessive vibration of spring coils. This undesired phenomenon, commonly referred to as “spring surge”, is a response in the form of initiation and propagation of a wave similar to an acoustic wave. Spring surge and a subsequent spring coil clash have an adverse effect on overall dynamics and can compromise the performance of the whole system. Therefore, proper design and selection of helical springs should consider modeling this type of response.

There is a vast amount of literature devoted to helical spring analysis where different mathematical and numerical models are used to study a wide range of spring phenomena including spring deformation, vibration, force and stress developed in spring wire. The study of helical springs dates back to the late 1800s and was pioneered by Thomson and Tait [1]. The analytical solution to the static equations of cylindrical helical springs subjected to small deformations was obtained by Love [2]. Numerous studies have been conducted to understand the dynamic behavior of the spring and to calculate the first vibration modes. Wittrick [3] performed the calculation of vibration modes of helical beams with circular cross-section based on analytical but approximate solutions. Similar computations have been carried out by Mothershead [4] using a special helical finite element and by Ward et al. [5] using the assumed mode method. Pearson et al. [6] and Lee et al. [7] contributed complicated equations describing spring vibration and elastic wave propagation using the Bernoulli-Euler and Timoshenko beam models, respectively.

The aim of this paper is to present an efficient, yet still accurate model of helical spring for use in multi-body dynamics (MBD) simulations. In practical MBD applications, such models are utilized in parametric design and optimization studies. It is for that reason that computational efficiency is a key requirement. Spring models most commonly used in the context of such applications include the dual-mass spring model [8, 9], the surge-mode model [8, 9, 10] and the lumped multi-mass model [8, 9, 10, 11]. The dual-mass spring model represents the lowest level of refinement. It consists of two masses connected via a force element with a function representing the nominal (static)

stiffness characteristics of the spring. This model does not consider any internal dynamics of the spring and is therefore not suited for the detailed dynamic investigation of mechanical systems operating at high speed. The surge-mode model is an efficient and accurate model for a helical spring operating in its linear range and is used extensively in MBD simulations. The model takes advantage of the fact that the elastic medium fastened at both ends can be described in terms of normal vibration modes. The drawback of the surge-mode model is that it does not consider any nonlinearity caused by the interaction between spring windings. The multi-mass spring model is based on the lumped discretization of the spring. Each spring winding is represented with a series of massed connected using spring and damper elements. As opposed to the dual-mass and surge-mode modeling techniques, the multi-mass formulation also considers contact/clash between coils. Coil-to-coil contact forces are modeled as point loads using nonlinear contact elements. As a result of lumped modeling approach, the multi-mass model requires a large number of masses in order to achieve sufficient accuracy. The high number of degrees of freedom (leading to stiff equations of motion) combined with the nonlinear characteristics of coil contact forces presents a significant solution effort.

The helical spring dynamics model described in this work uses one-dimensional curved beams to represent spring coils. The finite element approach is used in order to discretize these elastic components and, therefore, enables reduction of degrees of freedom required to accurately capture dynamic response of the spring. A novel feature of the proposed formulation is that the displacements of one-dimensional beam elements are expressed in a “floating” coordinate system, which can undergo large rigid body motions in 2D space. The formulation enables modeling of planar displacements of helical spring and accounts for inertial effects associated with the rigid body motion/acceleration of coils but, at the same time, requires a single degree of freedom per spring node. In addition to a traditional, lumped representation of coil contact and clash, a new method for modeling of coil-to-coil interactions is presented where contact forces are treated as distributed loads. This approach is more efficient since it requires fewer contact elements to correctly represent (progressive) coil closure during spring compression event and, at the same time, it eliminates the polygonal effect common to lumped characterization of coil contact.

Several numerical examples are analyzed to validate model's accuracy. First, results from numerical simulation are compared to analytical solution of the hyperbolic partial differential equation governing helical spring dynamic response. Next, experimental results from a static and dynamic spring compression test of an automobile engine valve spring are compared to simulation results showing good correlation. Finally, computational efficiency of the presented model was studied and compared to the equivalent multi-

mass model in the context of a multi-body simulation of engine valvetrain system.

HELICAL SPRING MODEL

The following section contains the description of the proposed model of helical spring.

Geometric Description

Two coordinate systems are introduced to aid in the geometric presentation of the spring helix and shown in [Figure 1](#). The Cartesian coordinate system denoted by (x, y, z) is position such that its origin coincides with the central axis of the spring helix and one end of the spring lies in the xy-plane at Z=0. The x-axis is parallel to the central axis of the spring. The position vector of any point *p* belonging to the middle fiber of the spring's beam is defined in this coordinate system by

$$\underline{r}_o = \begin{bmatrix} r(\theta) \cos \theta \\ r(\theta) \sin \theta \\ h(\theta) \end{bmatrix} \quad (1)$$

where $r(\theta)$ and $h(\theta)$ are respectively the radius and height of the helix curve and θ is the polar angle around the helix. The curvilinear coordinate *s* measures the length of spring wire in its undeformed configuration and is related to the angle θ by the relation

$$ds = \sqrt{r(\theta)^2 + \left[\frac{dr(\theta)}{d\theta}\right]^2 + \left[\frac{dh(\theta)}{d\theta}\right]^2} d\theta \quad (2)$$

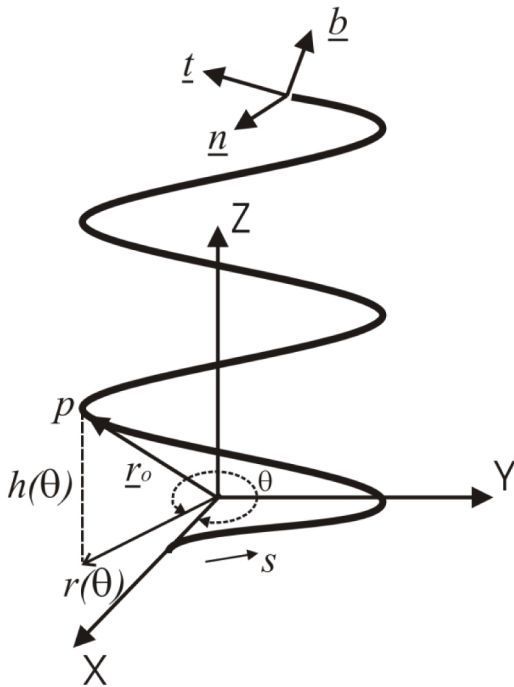


Figure 1. Geometric description of the helical spring and its two coordinate systems

The second, local coordinate system is formed by a set of three mutually orthogonal unit vectors, i.e. tangent $\underline{t}$, normal $\underline{n}$ and binormal $\underline{b}$ as depicted in [Figure 1](#). This orthonormal basis is called *Frenet's triad*, and associates with each point of the curve a rotation tensor

$$\underline{R}(s) = [\underline{t}(s) \quad \underline{n}(s) \quad \underline{b}(s)] \quad (3)$$

Vector $\underline{t}$ is normal to the spring wire cross-section at any point around the helix curve.

Spring Coil Kinematics

To determine the displacement field, the following simplifying assumptions are made. The spring is assumed to be a onedimensional structure, i.e. the only non-zero displacement component is in the direction of spring helix axis (z-axis in [Figure 1](#)). In accordance with the small displacement theory, spring coil displacements and, consequently, strains are assumed to be small. In order to maintain the above assumptions, but also to be able to account for the large twodimensional motion (translation and rotation) of the spring structure, spring coil displacements are expressed in a “floating” coordinate system denoted by unit vectors ($\underline{e}_1$, $\underline{e}_2$). This coordinate frame, which can undergo large rigid body motions in 2D space, effectively carries the spring rigid body information. The inertial position of any point *p* belonging to the spring after the spring has deformed and undergone a planar translation and rotation can be expressed as

$$\underline{X}_p + \underline{x}_p = \underline{X}_F + \underline{x}_F + \underline{R}_F(\underline{r}_0 + \underline{u}) \quad (4)$$

where $\underline{X}_p$ and $\underline{x}_p$ are respectively the initial position and displacement of point *p* belonging to the spring and expressed in the inertial frame, similarly $\underline{X}_F$ and $\underline{x}_F$ are the inertial initial position and displacement of the “floating” frame; $\underline{r}_0$ is the position vector of point *p* when the spring is in its reference configuration with its components expressed in the local “floating” frame; $\underline{u}$ (is the local displacement vector of point *p* with its only non-zero component *u* in the direction of $\underline{e}_1$; $\underline{R}_F$ is the two-dimensional rotation tensor defining the instantaneous orientation of the “floating” frame. The schematic of helical spring in its reference and deformed configuration with all inertial and local components of displacements labeled can be seen in [Figure 2](#).

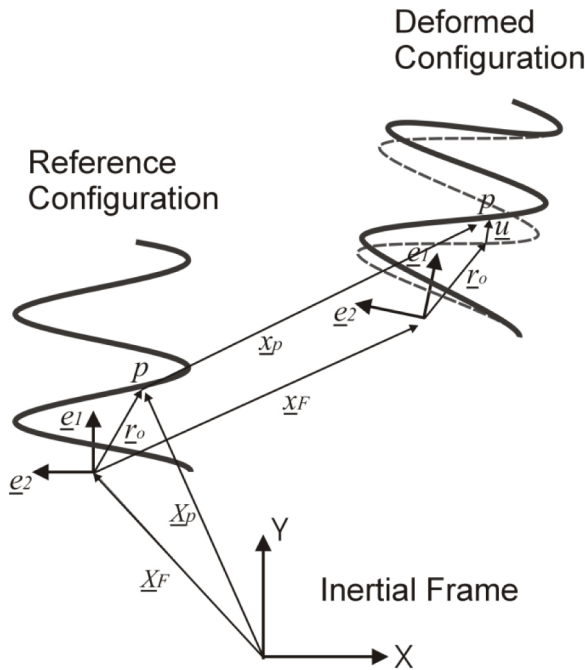


Figure 2. Helical spring configuration

Since the helical spring is a one-dimensional structure, the strain tensor reduces to

$$\varepsilon = u' \quad (5)$$

where ε represents strain and $(\blacksquare)'$ denotes a derivative with respect to the curvilinear coordinate s .

Equations of Motion

The governing equations of motion of the static problem can be obtained by means of the principle of virtual work

$$\int_0^L F^* \delta \varepsilon ds = \delta W_{ext} \quad (6)$$

where F^* is spring force in the direction of center axis, L is the spring wire length in the reference configuration, δW_{ext} is the virtual work done by external loads. The variation in small strain is

$$\delta \varepsilon = \delta u' \quad (7)$$

For purposed of further derivation, it is convenient to write the expression for variation in small strain using vector notation (note that only the first component of the strain vector is non-zero),

$$\delta \underline{\varepsilon} = \delta \underline{u}' \quad (8)$$

Furthermore, the planar virtual displacement of point p belonging to the spring can be obtained by computing the variation of Equation 4,

$$\delta \underline{x}_p = \delta \underline{x}_F + \delta \underline{R}_F (\underline{r}_0 + \underline{u}) + \underline{R}_F \delta \underline{u} \quad (9)$$

Using Equations 7, 8 and 9 in Equation 6 results in

$$\int_0^L \delta \underline{u}'^T F^* ds = \int_0^L \delta \underline{x}_p^T \underline{f} ds \quad (10)$$

where $\underline{f}$ are the externally applied loads per unit length in the reference configuration. Integrating by parts yields the governing equation of the problem

$$F^* \underline{u}'' = -\underline{f} \quad (11)$$

Equation 11 can be extended to model dynamic problems by setting the externally applied loads to

$$\underline{f} = \underline{f} - m \ddot{\underline{x}}_p \quad (12)$$

where m is the mass of spring wire per unit length in the reference configuration and $\ddot{\underline{x}}_p$ is the planar acceleration vector of point p belonging to the spring, which can be obtained by differentiating Equation 4 with respect to time twice. By combining Equations 11 and 12, performing the necessary vector operations and rearranging terms, the final equations of motion become

$$m \begin{bmatrix} \ddot{\underline{x}}_F \\ \ddot{\underline{\theta}}_F \\ \ddot{\underline{u}} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ F^* \underline{u}'' \end{bmatrix} = \begin{bmatrix} \underline{f}_F \\ \underline{t}_F \\ \underline{f}_u \end{bmatrix} + \begin{bmatrix} \underline{b}_F \\ \underline{b}_F \\ \underline{b}_u \end{bmatrix} \quad (13)$$

where vector $\ddot{\underline{x}}_F$ contains the inertial components of translational acceleration of the “floating” frame, $\ddot{\underline{\theta}}_F$ is the angular acceleration of the “floating” frame, $\ddot{\underline{u}}$ is the translational acceleration of point p belonging to the spring with its components expressed in the “floating” frame, $\underline{f}_F$ and $\underline{t}_F$ are the external forces and moments applied to the “floating” frame degrees of freedom, $\underline{f}_u$ are the external loads applied to point p and expressed in the “floating” frame, $\underline{m}$ is the matrix containing spring mass information, $\underline{b}_F$, $\underline{b}_F$ and $\underline{b}_u$ are the non-linear components of body forces and moments acting on the “floating” frame and point p , respectively.

Constitutive Law

Assuming linear-elastic behavior of the spring material, the spring force F^* in the direction of center axis is given by the following constitutive law

$$F^* = F_0^* + k\varepsilon + \beta k\dot{\varepsilon} \quad (14)$$

where F_0^* is the initial pre-compression, ε is the strain as defined in Equation 5, k is the sectional stiffness, β is a