

Effect of the Tooth Surface Waviness on the Dynamics and Structure-Borne Noise of a Spur Gear Pair

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ABSTRACT

This article studies the effects of tooth surface waviness and sliding friction on the dynamics and radiated structure-borne noise of a spur gear pair. This study is conducted using an improved gear dynamics model while taking into account the sliding frictional contact between meshing teeth. An analytical six-degree-of-freedom (6DOF) linear time varying (LTV) model is developed to predict system responses and bearing forces. The time varying mesh stiffness is calculated using a gear contact mechanics code. A Coulomb friction model is used to calculate the sliding frictional forces. Experimental measurements of partial pressure to acceleration transfer functions are used to calculate the radiated structure-borne noise level. The roles of various time-varying parameters on gear dynamics are analyzed (for a specific example case), and the predictions from the analytical model are compared with prior literature.

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INTRODUCTION

The static transmission error is a well-recognized source, and thus tooth modifications are often undertaken to minimize the excitation at specific loads to reduce gear whine noise [1]. Yet, at high torque loads, noise levels are still relatively high [2]. This suggests that tooth surface waviness and sliding friction could manifest itself as an alternate noise source. For instance, Mark [3] found that the surface waviness is related to the machining process kinematics. Furthermore, the experiments conducted by Mitchell [4] and Amini et al. [5] on gears and by Othman et al. [6] on rotating disks show an increase in the sound level with an increase in the surface waviness amplitude. The role of tooth surface waviness on gear noise is not well understood, especially for structure-borne noise source(s) or path(s), due to the complexity in modeling micro-surface characteristics. This article employs a six degree of freedom (6DOF) linear time-varying (LTV) model of a spur gear pair to quantify the structure-borne noise source and to illustrate a relationship between waviness amplitude and wave number to gear dynamics source and resulting sound radiation. The overall procedure for predicting the sound pressure level (L) is previewed in Fig. 1, where θ is the angular displacement of the pinion or gear, while x and y are the translational motions along the line-of-action (LOA) and the off-line-of-action (OLOA) directions, respectively. Subscripts p and g represent

pinion and gear, respectively. (Also, refer to the list of symbols for the identification of variable and parameters.)

LINEAR TIME-VARYING SPUR GEAR MODEL

The proposed 6DOF LTV model is schematically shown in Fig. 2. The gear and pinion are considered rigid discs of polar moments of inertia J_p and J_g with external torques T_p and T_g . Here, $h_p(t)$ and $h_g(t)$ represent tooth surface waviness with respect to perfect involute profiles. The governing equations are described by torsional and translational motions. The effective shaft-bearing stiffness elements are given by k_{pSx} and k_{gSx} in the X direction (LOA) and k_{pSy} and k_{gSy} in the Y direction (OLOA). The time-varying mesh stiffness ($k(t)$) is calculated for a range of torques by using a well-known gear contact mechanics code (Load Distribution Program or LDP) [7].

The parameters of the unity gear pair example used in this study are as follows: Number of teeth = 28, outside diameter = 94.95 mm; root diameter = 79.73 mm; diametral pitch = 0.315 m^{-1} ; center distance = 88.9 mm; pressure angle = 20° ; face width = 6.35mm; tooth thickness = 4.851 mm; and elastic modulus = 206.9 kN/mm^2 . Further, it is assumed that the bearings of the gear and pinion are frictionless, and the waviness amplitude is independent of the load.

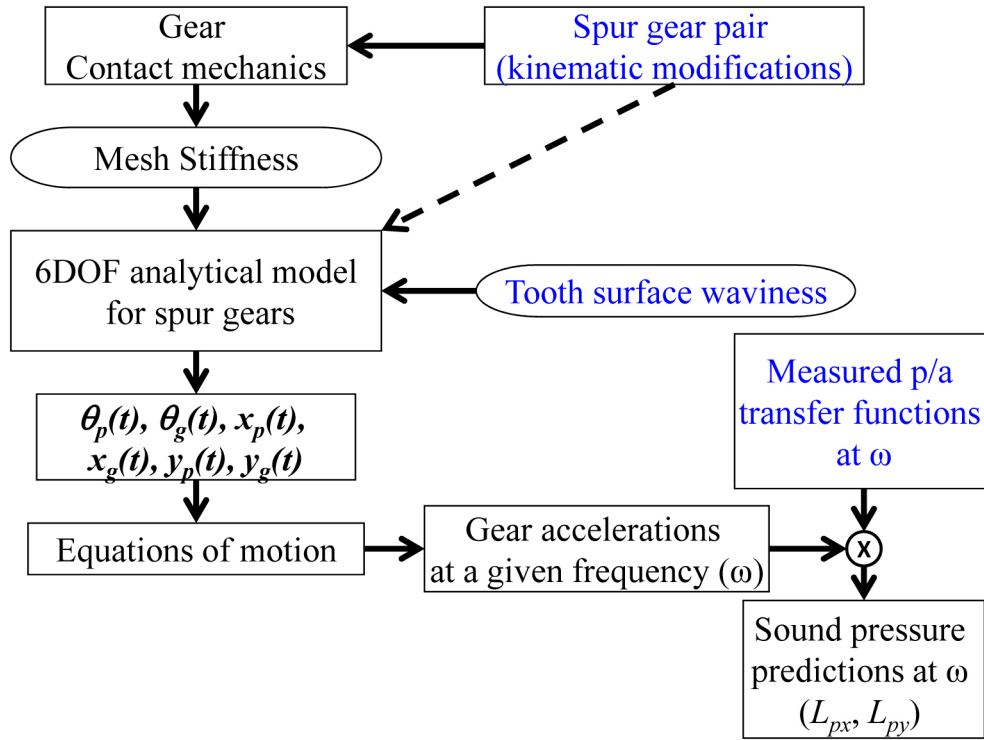


Fig. 1. Procedure for predicting gear accelerations and sound pressure levels.

With reference to the geared system model of Fig. 2, the governing equations for $\theta_p(t)$ and $\theta_g(t)$ are:

$$J_p \ddot{\theta}_p(t) = T_p + \sum_{i=0}^n \chi_{pi}(t) F_{pfi}(t) - \sum_{i=0}^n R_p N_{pi}(t), \quad (1)$$

$$J_g \ddot{\theta}_g(t) = -T_g + \sum_{i=0}^n \chi_{gi}(t) F_{gfi}(t) + \sum_{i=0}^n R_g N_{gi}(t). \quad (2)$$

The time-varying moment arms $\chi_{pi}(t)$ and $\chi_{gi}(t)$ for the i^{th} meshing pair with a contact ratio (σ) are the following with $n = \text{floor}(\sigma)$:

$$\begin{aligned} X_{pi}(t) &= L_{XA} + (n-i)\lambda + \text{mod}(\Omega_p R_p t, \Lambda), \\ X_{gi}(t) &= L_{YC} + i\lambda - \text{mod}(\Omega_g R_g t, \Lambda). \end{aligned} \quad (3 \text{ a, b})$$

Here, Ω_p and Ω_g are the nominal speeds (in rad/s); Λ is the base pitch; and L_{AP} , L_{XA} , and L_{YC} are the geometric length constants as defined by He et al. [8]. The normal loads (N_p and N_g) and friction forces (F_{pfi} and F_{gfi}) are defined as follows, where $\mu(t)$ is the time-varying coefficient of friction and $\Delta h(t) = h_p(t) - h_g(t)$:

$$\begin{aligned} N_{pi}(t) = N_{gi}(t) &= k_i(t) \left[\begin{aligned} &R_p \theta_p(t) - R_g \theta_g(t) \\ &+ x_p(t) - x_g(t) - \Delta h(t) \end{aligned} \right] \\ &+ c_i(t) \left[\begin{aligned} &R_p \dot{\theta}_p(t) - R_g \dot{\theta}_g(t) \\ &+ \dot{x}_p(t) - \dot{x}_g(t) - \Delta \dot{h}(t) \end{aligned} \right] \end{aligned} \quad (4)$$

$$\begin{aligned} F_{pfi}(t) &= \mu_i(t) N_{pi}(t), \\ F_{gfi}(t) &= \mu_i(t) N_{gi}(t). \end{aligned} \quad (5 \text{ a, b})$$

Equations of translational motion along X and Y are given by the following, where ζ is modal damping ratio:

$$m_p \ddot{x}_p(t) + 2\zeta_{pSx} \sqrt{k_{pSx} m_p} \dot{x}_p(t) + k_{pSx} x_p(t) + \sum_{i=0}^n N_{pi}(t) = 0 \quad (6)$$

$$m_g \ddot{x}_g(t) + 2\zeta_{gSx} \sqrt{k_{gSx} m_g} \dot{x}_g(t) + k_{gSx} x_g(t) - \sum_{i=0}^n N_{gi}(t) = 0 \quad (7)$$

$$m_p \ddot{y}_p(t) + 2\zeta_{pSy} \sqrt{k_{pSy} m_p} \dot{y}_p(t) + k_{pSy} y_p(t) - \sum_{i=0}^n F_{pfi}(t) = 0 \quad (8)$$

$$m_g \ddot{y}_g(t) + 2\zeta_{gSy} \sqrt{k_{gSy} m_g} \dot{y}_g(t) + k_{gSy} y_g(t) - \sum_{i=0}^n F_{gfi}(t) = 0$$

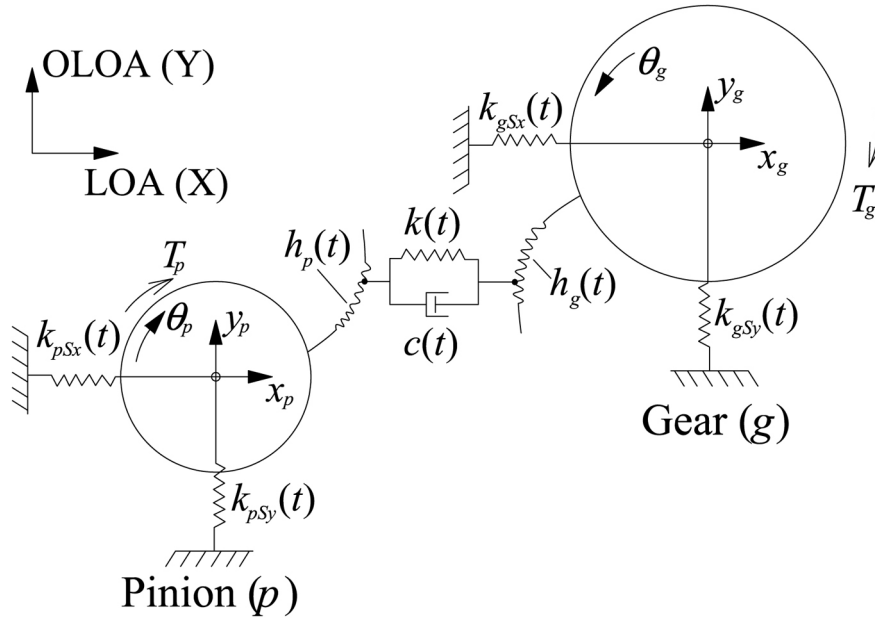


Fig. 2. Proposed 6DOF linear time-varying gear dynamics model with prescribed tooth surface waviness given by $h_p(t)$ and $h_g(t)$.

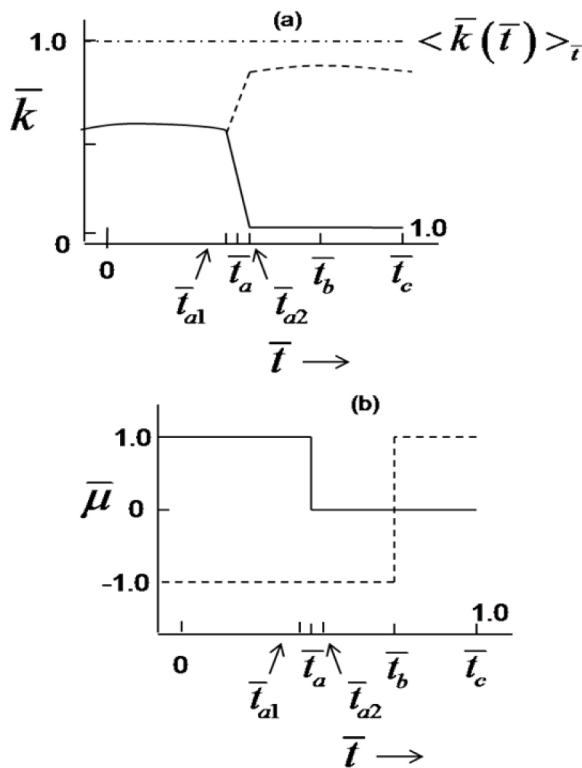


Fig. 3. Normalized time-varying parameters along a mesh cycle. (a) Mesh stiffness, $\bar{k}$; (b) Coefficient of friction, $\bar{\mu}$.

(solid line) and at t_b for tooth pair # 1 (dotted line). Here, t_a represents the time from two teeth in contact to the first tooth leaving contact, t_b represents the time from two teeth in contact to the pitch point where the sliding velocity changes its direction, and t_c represents the gear mesh period. The mesh stiffness parameters at a mean torque are calculated by using LDP [7], given the kinematics.

Assuming that the gear tooth surface is one-dimensional given in terms of mesh locations (s), the involute coordinates s_p and s_g are defined as follows in the involute coordinate system, where α is the roll angle, and j denotes the tooth index:

$$s_{pj}(t) = \frac{1}{2} R_p [\alpha_{p0} + \alpha_{pm}]^2 = \frac{1}{2} R_p [\alpha_{p0} + \Omega_p t]^2 \quad (10a)$$

$$s_{gj}(t) = \frac{1}{2} R_g [\alpha_{gL} - \alpha_{gm}]^2 = \frac{1}{2} R_g [\alpha_{gL} - \Omega_g t]^2 \quad (10b)$$

Expressions for known motion inputs to the geared system are stated below in terms of tooth surface waviness $h_p(s)$ and $h_g(s)$, as shown in Fig. 2:

$$h_{pj}(s) = H_p \sin[\kappa_p s_{pj} + \phi_{pj}] = H_p \sin\left[\frac{2\pi}{\lambda_p} s_{pj} + \phi_{pj}\right] \quad (11a)$$

The normalized time-varying mesh stiffness and coefficient of friction are shown in Fig. 3 for tooth pair #0