

Using the Reciprocal Work Identity to Evaluate the Transmission Loss of Mufflers

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ABSTRACT

Transmission loss (TL) is a good performance measure of mufflers since it represents the muffler's inherent capability of sound attenuation. There are several existing numerical methods, which have been widely used to calculate the TL from numerical simulation results, such as the four-pole and three-point methods. In this paper, a new approach is proposed to evaluate the transmission loss based on the reciprocal work identity. The proposed method does not assume plane wave propagation in the inlet and outlet ducts, and more importantly, does not explicitly apply the anechoic termination impedance at the outlet. As a result, it has the potential of extending TL computation above the plane wave cut-off frequency.

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INTRODUCTION

Mufflers and silencers are devices to attenuate exhaust noise in various environments. Reactive mufflers take advantage of an impedance mismatch due to geometry changes (i.e., area contraction and expansion, side branch resonators). Dissipative mufflers, on the other hand, use sound absorbing materials to convert sound energy to heat. In practice, most mufflers use both mechanisms since reactive elements effectively abate sound at low frequencies, while dissipative mufflers are better suited to addressing high frequency broadband noise [1].

The use of a muffler or silencer is prompted by the need to reduce the radiated noise from a source. In most applications, the final selection is based on trade-offs between the predicted acoustical performance, mechanical strength, volume/weight ratio, and cost [2]. To evaluate the acoustical performance of mufflers, the most frequently used metrics are insertion loss (IL), transmission loss (TL) and noise reduction (NR) [3]. Transmission loss is often preferred since it represents the inherent capability of the muffler to attenuate sound if both the source and termination are assumed to be anechoic. The TL is defined as the difference in dB between the incident sound power upstream (L_{wi}) and the transmitted sound power downstream (L_{wt}) [3]. Accordingly, TL can be expressed as:

$$TL = L_{wi} - L_{wt} \quad (1)$$

where an anechoic termination is assumed.

Several analytical approaches have been developed for muffler and silencer analysis. Certainly, the most ubiquitous technique is the transfer matrix approach detailed by Munjal in his classic text [3]. Other approaches have been developed that are ideal for analyzing complex systems of interconnecting ducts. For instance, Glav and Åbom [4] proposed a general formalism for analyzing acoustic 2-port network systems using a scattering matrix approach. Kar and Munjal [5] posed an alternative matrix approach.

Numerical simulation is also commonly used to evaluate the performance of mufflers. Commonly used approaches include acoustic finite [6,7,8,9,10] and boundary [11,12,13,14] element methods. Numerical simulation is especially useful when the plane wave cutoff frequency is exceeded in the muffler.

If plane wave analysis is used, transmission loss is normally calculated directly from the four-pole parameters [3]. If numerical simulation is used, the four-pole parameters can be determined by selecting appropriate boundary conditions [11,15,16,17]. Alternatively, the four-pole parameters can be determined from an impedance matrix [11]. In that case, a single analysis can be used to determine the TL if the muffler or silencer is symmetric.

Other approaches have abandoned an explicit determination of the four-pole parameters. For example, Munjal [3] utilized a two-port approach to calculate the transmission loss from the sound pressure at the source and

termination, and particle velocity at the source. Alternatively, a three-point method, which uses wave decomposition, can be used to determine the incident and transmitted power. Both the two-port and three-point approaches assume an anechoic termination.

In this paper, an alternative approach for determining transmission loss is developed based on the reciprocal work identity and wave decomposition theory. Plane waves are not assumed in the inlet/outlet ducts and no explicit anechoic termination impedance is applied at the outlet. Below the cutoff frequency, the proposed method reduces to the conventional four-pole method. A relatively complex muffler is used to demonstrate the proposed method and then verify the equivalency between it and the conventional four-pole method below the plane-wave cut-off frequency.

REVIEW OF CURRENT TECHNIQUES

Conventional Four-Pole Method

One of the most common methods to evaluate the TL is the conventional four-pole method. Below the cut-off frequency of the inlet/outlet ducts, a muffler like that shown in [Figure 1](#) can be described by using the four-pole parameters A , B , C and D [11]:

$$\begin{Bmatrix} p_1 \\ v_1 \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} p_2 \\ -v_2 \end{Bmatrix} \quad (2)$$

where p_1 and p_2 are the sound pressures, and v_1 and v_2 the normal particle velocities at the inlet and the outlet, respectively. The four-pole parameters can be determined directly from the transfer matrix.

Numerical simulation can also be used to determine the four-pole parameters. For this paper, the boundary element method (BEM) will be assumed. Due to the inward normal direction used in most BEM software, a negative sign is added to v_2 in [Eq. \(2\)](#). By imposing two different sets of boundary conditions, the four-pole parameters A , B , C and D can be obtained from

$$\begin{aligned} A &= \left. \frac{p_1}{p_2} \right|_{v_2=0, v_1=1} & B &= \left. \frac{p_1}{-v_2} \right|_{p_2=0, v_1=1} \\ C &= \left. \frac{v_1}{p_2} \right|_{v_2=0, v_1=1} & D &= \left. \frac{v_1}{-v_2} \right|_{p_2=0, v_1=1} \end{aligned} \quad (3a,b,c,d)$$

Two separate BEM runs must be carried out in order to obtain the four-pole parameters at each frequency.

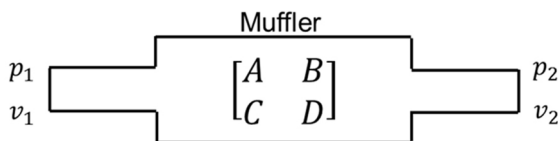


Figure 1. Schematic showing a muffler with four-pole parameters.

Once the four-pole parameters A , B , C and D are available, the transmission loss of the muffler can be evaluated by

$$TL = 20 \log_{10} \left(\frac{1}{2} \left| A + \frac{B}{\rho c} + \rho c C + D \right| \right) + 10 \log_{10} \frac{S_i}{S_o} \quad (4)$$

where S_i and S_o denote the cross-sectional areas of the inlet and outlet, respectively, ρ is the density and c the speed of sound.

Wu et al. [11] pointed out that since the two sets of boundary conditions are distinct, the two BEM runs do not share the same BEM coefficient matrix. Hence, the matrix decomposition needs to be performed twice at each frequency, which makes the conventional four-pole method an impractical choice for computing the transmission loss.

Improved Four-Pole Method

Instead of computing the four-pole parameters directly, an improved method was proposed by Wu et al. [11]. The method also requires two BEM runs but in a more efficient way. The four-pole matrix in [Eq. \(2\)](#) can be rearranged to define an impedance matrix expressed as:

$$\begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{bmatrix} A^* & B^* \\ C^* & D^* \end{bmatrix} \begin{Bmatrix} v_1 \\ -v_2 \end{Bmatrix} \quad (5)$$

where

$$\begin{aligned} A^* &= p_1|_{v_1=1, v_2=0} & B^* &= p_1|_{v_1=0, v_2=-1} \\ C^* &= p_2|_{v_1=1, v_2=0} & D^* &= p_2|_{v_1=0, v_2=-1} \end{aligned} \quad (6a,b,c,d)$$

Like the conventional four-pole method, two separate BEM runs are still required to compute the impedance matrix. Nonetheless, the two BEM runs share the same coefficient matrix because the two sets of boundary conditions are both velocity boundary conditions. Therefore, the second BEM run only requires a trivial back-substitution procedure. The impedance matrix can be easily converted into the four-pole parameters by

$$\begin{aligned} A &= \frac{A^*}{C^*} & B &= B^* - \frac{A^* D^*}{C^*} \\ C &= \frac{1}{C^*} & D &= -\frac{D^*}{C^*} \end{aligned} \quad (7a,b,c,d)$$

and TL can be calculated by [Eq. \(4\)](#).

Three-Point Method

A three-point method [18,19] can be used to determine the TL directly if the four-pole parameters are not desired. The primary advantage of the three-point method is that TL can be determined from a single BEM run. To do so, v_1 is

applied at the inlet and an anechoic termination impedance is applied at the outlet.

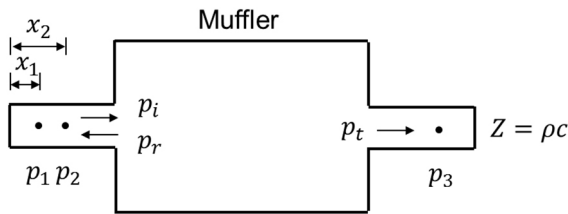


Figure 2. Schematic showing the three-point method.

With reference to Fig. 2, let x_1 and x_2 be the coordinates of two field points in the inlet duct. The corresponding sound pressures p_1 and p_2 can be written as

$$\begin{aligned} p_1 &= p_i e^{-jkx_1} + p_r e^{+jkx_1} \\ p_2 &= p_i e^{-jkx_2} + p_r e^{+jkx_2} \end{aligned} \quad (8a,b)$$

where p_i and p_r denote the incident and reflected waves respectively, and k is the wavenumber.

The incident wave p_i can be extracted by solving Eqs. (8a,b) simultaneously. In that case,

$$p_i = \frac{p_1 e^{+jkx_2} - p_2 e^{+jkx_1}}{j2 \sin(k(x_2 - x_1))} \quad (9)$$

provided that $\sin(k(x_2 - x_1)) \neq 0$. The transmitted sound pressure can be found anywhere in the outlet duct. The TL then can be determined by

$$TL = 20 \log_{10} \frac{|p_i|}{|p_t|} + 10 \log_{10} \frac{S_i}{S_o} \quad (10)$$

Two-Port Method

Like the three-point method, the two-port method [3] requires a single BEM run at each frequency with $v_1 = 1$ at the inlet and an anechoic termination impedance at the outlet. Without using any field points inside the muffler, the method relies only on solutions at the inlet and the outlet. Below the cut-off, the anechoic termination implies

$$p_2 = -\rho c v_2 \quad (11)$$

If the above condition is substituted into the four-pole matrix in Eq. (2) and rearranged, the following relationships can be expressed:

$$A + \frac{B}{\rho c} = -\frac{p_1}{\rho c v_2} \quad (12)$$

$$\rho c C + D = -\frac{v_1}{v_2} \quad (13)$$

Accordingly, the TL in Eq. (4) can be rewritten as

$$TL = 20 \log_{10} \left(\frac{1}{2} \left| \frac{p_1 + \rho c v_1}{\rho c v_2} \right| \right) + 10 \log_{10} \frac{S_i}{S_o} \quad (14)$$

Like the three-point method, the two-port method cannot produce the four-pole matrix as a by-product.

METHOD OF RECIPROCAL WORK IDENTITY

Reciprocal Work Identity

In a homogeneous medium, the sound propagation is governed by the Helmholtz equation

$$\nabla^2 p + k^2 p = 0 \quad (15)$$

where p is the sound pressure. For an arbitrary muffler system shown in Fig. 1, we may apply different sets of boundary conditions. For each different set of boundary conditions, the sound field is expected to be different. Let p_A and p_B represent two different sound fields corresponding to the two boundary condition sets A and B , respectively. Both sound fields, p_A and p_B satisfy the Helmholtz equation,

$$\nabla^2 p_A + k^2 p_A = 0 \quad (16)$$

$$\nabla^2 p_B + k^2 p_B = 0 \quad (17)$$

The Green's second identity is then applied to relate these two sound fields

$$\int_{\Omega} (p_A \nabla^2 p_B - p_B \nabla^2 p_A) dV = \int_{\partial\Omega} \left(p_A \frac{\partial p_B}{\partial n} - p_B \frac{\partial p_A}{\partial n} \right) dS \quad (18)$$

where n is the normal direction, Ω the acoustic domain and $\partial\Omega$ the boundary.

Substituting Eqs. (16) and (17) into the above identity, we obtain

$$\int_{\partial\Omega} \left(p_A \frac{\partial p_B}{\partial n} - p_B \frac{\partial p_A}{\partial n} \right) dS = 0 \quad (19)$$

In linear acoustics, the momentum equation relates the normal derivative of sound pressure to the normal particle velocity by

$$\frac{\partial p}{\partial n} = -j\rho\omega v \quad (20)$$

where ω is the angular frequency and v is the particle velocity in the normal direction. Eq. (19) become