

Multi-Point Mesh Modeling and Nonlinear Multi-Body Dynamics of Hypoid Geared System

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ABSTRACT

A multi-point hypoid gear mesh model based on 3-dimensional loaded tooth contact analysis is incorporated into a coupled multi-body dynamic and vibration hypoid gear model to predict more detailed dynamic behavior of each tooth pair. To validate the accuracy of the proposed model, the time-averaged mesh parameters are applied to linear time-invariant (LTI) analysis and the dynamic responses, such as dynamic mesh force, dynamic transmission error, are computed, which demonstrates good agreement with that predicted by single-point mesh model. Furthermore, a nonlinear time-varying (NLTV) dynamic analysis is performed considering the effect of backlash nonlinearity and time-varying mesh parameters, such as mesh stiffness, transmission error, mesh point and line-of-action. Simulation results show that the time history of the mesh parameters and dynamic mesh force for each pair of teeth within a full engagement cycle can be simulated. This capability enables the analysis of durability of the gear pair and more accurate prediction of the system response.

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INTRODUCTION

Hypoid and spiral bevel gears are widely used in the rear axle power-train systems of vehicles. The dynamic behavior of hypoid and spiral bevel gear pair is important mainly in two reasons: (1) the vibration and noise problems are related to the dynamic mesh force, which is believed to be excited by gear transmission error (TE) and parametric variation of mesh characteristics, (2) the prediction of dynamic mesh force is critical to the study of gear fatigue and gear life. Moreover, the vibration of the meshing gear pair is transmitted through shaft-bearing assembly into gear housing, which affects the overall system dynamics of driveline system.

An accurate mesh model is critical for the prediction of dynamic mesh force. The spring-damper model is usually used to represent the mesh characteristics of gear pairs. Over the past 20 years, there are a number of studies assuming a time and spatial-invariant spring damper model. With the development of tooth surface contact program, there are more studies recently based on the 3-dimensional gear tooth geometry results [1, 2, 3, 4, 5]. In their models, the time-varying mesh characteristics, including mesh stiffness, transmission error, mesh point and line-of-action (LOA), are obtained from the contact results of quasi-static loaded tooth contact analysis (LTCA), and the contact results are synthesized to a single spring-damper coupling, which

represents the gear mesh. Besides, Wang [3] also investigated the multiple spring-damper coupling and compared the mesh parameters and dynamic responses with those by assuming single spring-damper coupling.

The basic frame work of hypoid and spiral bevel gear dynamics is established by Cheng and Lim [1], a nonlinear time-varying (NLTV) model is applied, both backlash nonlinearity and time-varying mesh parameters are considered. Later, Peng [4] derived the coupled multi-body dynamic and vibration model by including the large shaft rotation. Yang [6-7] further investigated the nonlinear dynamic response of hypoid gearbox with time-varying bearing stiffness and elastic housing.

Previous hypoid gear mesh model based on single-point coupling may lose accuracy when gear teeth are large and contact ratio is high. The multi-point coupling mesh model is believed to be able to catch the true mesh characteristics with less assumption. In addition, the lack of the detailed analysis of the time history of dynamic mesh force on each engaging tooth pair, limited the further study on gear surface wear and fatigue model. In this paper, a multi-point hypoid gear mesh model based on 3-dimensional loaded tooth contact analysis is incorporated into a coupled multi-body dynamic and vibration hypoid gear model to predict more detailed dynamic behavior of each tooth pair. Linear dynamic responses are

compared between single-point and multi-point mesh coupling to validate the model. The nonlinear dynamic analysis is also performed considering the effect of backlash nonlinearity and time-varying mesh parameters. More detailed dynamic responses on each engaging gear tooth pair are examined, thus it can be applied to investigate the interaction between gear dynamics and surface wear [8].

MESH AND DYNAMIC MODEL

Gear Mesh Model

The hypoid gear mesh model, including mesh point, line-of-action, mesh stiffness and kinematic TE can be derived from the load distribution results calculated by a 3-dimensional quasi-static loaded tooth contact analysis program for hypoid and spiral bevel gears [9]. This program combines the semi-analytical theory with finite element method [10], which can solve the gear tooth contact problem very efficiently.

A hypoid gear pair with multiple contact interfaces is shown in Figure 1(a), and Figure 1(b) shows contact cells on each tooth pair. The position vector of the contact cell in the mesh coordinate system is $r_i(r_{ix}, r_{iy}, r_{iz})$, the contact force is f_i , and the normal vector is $n_i(n_{ix}, n_{iy}, n_{iz})$.

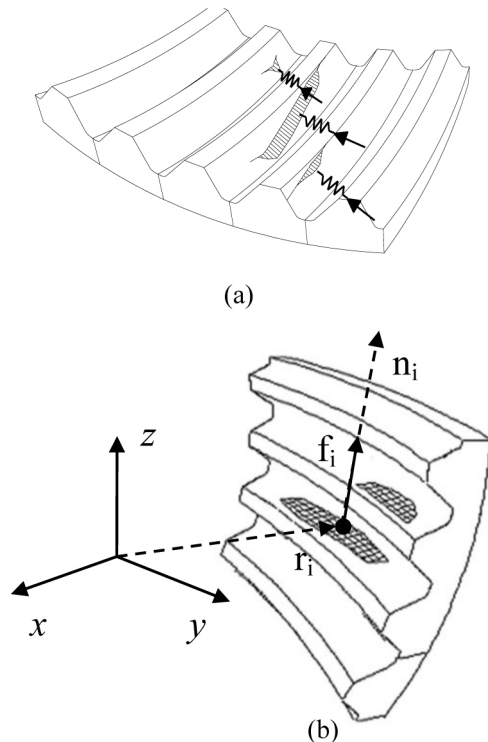


Figure 1. Illustrations of: (a) multi-point coupling of hypoid gear pair, and (b) contact cells on engaging tooth surface.

The total contact force is calculated by summing the contact forces on each contact cell (assuming there are N contact cells)

$$F_j = \sum_{i=1}^N n_{ij} f_i, \quad F = \sqrt{\sum_{j=x}^z F_j^2}, \quad (j = x, y, z), \quad (1)$$

The line-of-action vector $L(n_x, n_y, n_z)$ can be obtained from

$$n_j = F_j / F, \quad (j = x, y, z). \quad (2)$$

The total contact moment is given by

$$M = \sum_{i=1}^N f_i \cdot (r_i \times n_i) \quad (3)$$

The mesh point $R(x_m, y_m, z_m)$ can be obtained from

$$y_m = \frac{\sum_{i=1}^N r_{iy} f_i}{\sum_{i=1}^N f_i} \quad (4a)$$

$$x_m = (M_z + F_x y_m) / F_y, \quad (4b)$$

$$z_m = (M_y + F_z x_m) / F_x, \quad (4c)$$

The translational loaded and unloaded transmission errors e_L and e_0 are the projections of corresponding angular transmission error along the line of action.

Finally, the mesh stiffness is defined by

$$k_m = F / (e_L - e_0). \quad (5)$$

Above formulation is applied to each contact interface, thus the multi-point mesh parameters in one mesh cycle can be obtained and applied to the dynamic model. The number of contact tooth pairs at any given time is a function of load, pinion roll angle and tooth geometry.

Dynamic Formulation

The coupled multi-body dynamic and vibration model, proposed by Peng [4], is employed in this paper. The pinion, gear, engine and load component are considered as rigid rotors. The engine and load component each has only one rotational coordinate. The shaft-bearing assembly stiffness is used to represent the flexibility at both pinion and gear lumped support. Multi-point mesh coupling is applied between the engaging gear teeth.

The equations of motion in matrix form can be written as