

Vibration Analysis of Tire Circumferential Mode Under Loaded Axle

Masami Matsubara, Nobutaka Tsujiuchi, Takayuki Koizumi and Yuya Hirano
Doshisha Univ

ABSTRACT

Road noise is one of the main vehicle interior noises. To minimize this, it is necessary to reveal the vibration characteristics of a rolling tire. Tire vibration has complex behavior due to tire contact with the road and rolling. In an earlier study, we clarified the effect of contact patch restriction for tire vibration characteristics in the non-rolling condition using the tire dynamical model. However, mode shapes were identified with circumferential wave number. Therefore, it is difficult to clarify the effect for tire vibration in the contact and rolling condition. In this paper, we will apply the receptance method, which is used as an analysis of the rotating disc and gear pair for the tire model toward the tire vibration analysis in the contact and rolling condition. Furthermore, the validity of the approach using this method will be verified from comparison with the result of an earlier study.

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INTRODUCTION

With the rapid progress of motorization, the environmental problem of vehicle noise has arisen. Typical sources of noise in the engine and intake-exhaust system have been improved with advanced noise regulation. As a result, tire contribution to vehicle interior noise has increased relatively, and is a serious problem that must be addressed. Therefore, many studies have been conducted on vehicle NVH performance prediction in the case where the tire is built as a partial structure [1]. It is important to reconstruct the input into the body from road to spindle and suspension for accurate prediction [2]. One of the problems is how to express the tire vibration characteristics in the operational condition, which are complex. It is difficult to construct a tire model that expresses the vibration characteristics under the contact and rolling condition because of the rolling effect, contact patch restriction, and the property of rubber.

In an earlier study, we clarified the effect of contact patch restriction for tire vibration characteristics in the non-rolling condition using the tire dynamical model [3]. However, mode shapes were identified with circumferential wave number. Therefore, it is difficult to clarify the effect for tire vibration in the contact and rolling condition. In this paper, we will apply the receptance method, which is used as an analysis of the rotating disc and gear pair for the tire model toward the tire vibration analysis in the contact and rolling condition. Furthermore, the validity of the approach using this method

will be verified from comparison with the result of an earlier study.

VIBRATION EXPERIMENT

Experiment Overview

A radial tire, 195/65R15, was prepared for experiment. The experiment was performed under the non-contact and contact conditions. The boundary condition is to clamp the wheel center only in the translational direction. Under the contact condition, a vertical load of 4200 N was added to the wheel center. The tire was pressed into the road. The internal pressure was 230 kPa. [Figure 1](#) shows a schematic view of the experimental system. The tread center of the tire's upper side was hammered in the z direction, and the driving-point response was measured in the radial direction by accelerometer. The driving-point accelerance (frequency response function) was calculated, and the natural frequency was comprehended. In addition, the response points in the tire radial direction were set on the tread center in the circumferential direction every 30 degrees, and the total number of points was 12. The mode shape of each natural frequency was comprehended from experimental modal analysis.

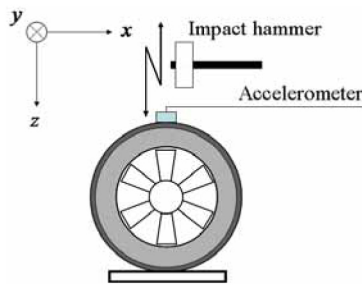


Figure 1. Test schematic

Experimental Result

Figure 2 shows the frequency response function under the non-contact and contact conditions. The resonance peaks are shown between 50-200 Hz, and the frequency that shows each resonance peak is increased by contact. Under the assumption that damping is small, the frequency that shows the resonance peak is assumed to be the natural frequency. Figure 3 shows the comparison of natural frequency under each condition. From the result, the natural frequency is increased by contact and with increasing mode number.

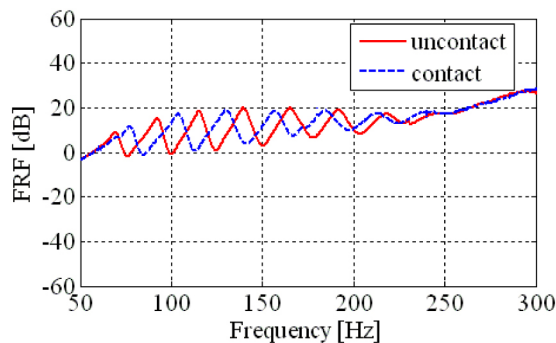


Figure 2. Comparison of frequency response function in non-contact and contact conditions

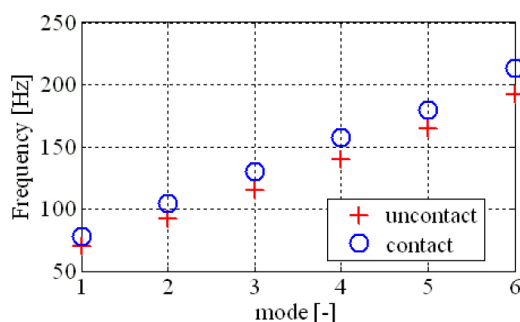


Figure 3. Comparison of natural frequency non-contact and contact conditions

Figures 4 and 5 show the mode shapes comprehended from the experimental modal analysis under the non-contact and contact conditions. Under the contact condition, the vibration response point, which corresponds to the contact

part (shown by $\uparrow$ in Figure 5), is plotted as no vibration. The radial tire is in excited circumferential mode, in which radial vibration significantly appears due to the high-stiffness belt on the tread in the circumferential direction [4]. From the result in Figure 4, the circumferential wave number corresponds to the mode number under the non-contact condition. However, that is not so under the contact condition. Figure 6 shows the circumferential degree under the contact condition as the x-coordinate and vibration amplitude in the vertical direction as the y-coordinate. The position of the vertical bottom part is set as 0 degrees. This result shows that the circumferential wave number that corresponds to each mode increases.

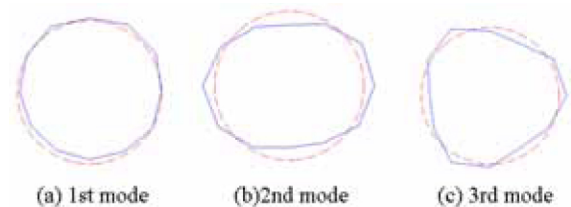


Figure 4. Mode shape in non-contact condition based on experiment

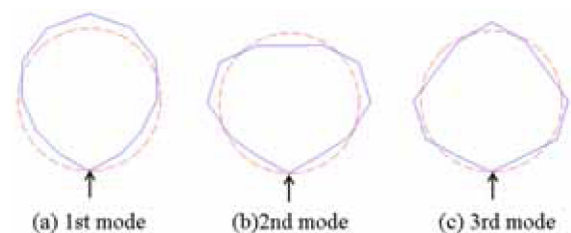


Figure 5. Mode shape in contact condition based on experiment

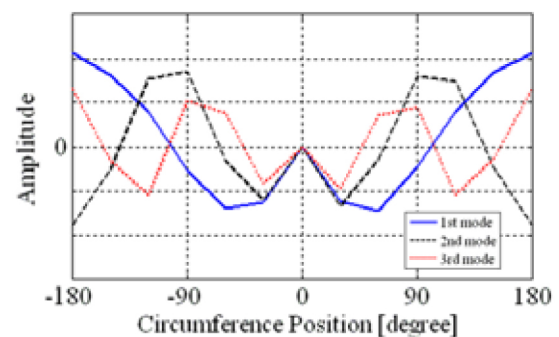


Figure 6. Vibration of radial direction in contact condition

From the above, changing from the non-contact to contact condition, it is clear that the tire natural frequency is increased and, when each mode shape is estimated as a

circumferential wave number, the wave number is also increased.

TIRE VIBRATION MODEL IN ROLLING CONDITION

Tire Vibration Model

First, we model the vibration in the non-contact and rolling condition [5]. A tire model is used in which the tread is expressed as a thin cylindrical ring (tread ring). [Figure 7](#) shows the tire vibration model. The system of the spindle to the wheel is set as a rigid body because it has higher rigidity than the tire. The tread ring is connected to the wheel by springs, which shows the tire sidewall. The parameters of this model are: the radius R and cross-section area A of the tread ring, the tread ring's modulus of longitudinal elasticity E and mass density ρ , and the radial and tangential sidewall stiffness K_θ and K_r , respectively. The location of this model is described using polar coordinate (θ, z) . The central surface of the tread ring section is $z = 0$, and the displacements of arbitrary point on the central surface are (v, w) in relation to (θ, z) . In addition, it is supposed that the deformation of tread follows the inextensional deformation assumption. If the tread is inextensible in the circumferential direction, the following relation between the radial and tangential displacement applies.

$$\frac{\partial v}{\partial \theta} + w = 0 \quad (1)$$

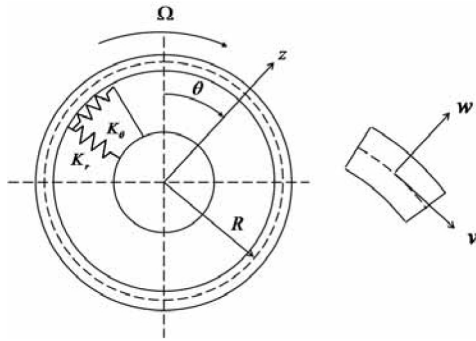


Figure 7. Tire vibration model based on thin shell theory

Motion Equation

The kinetic energy considering circumferential and vertical directions based on each velocity component and initial small displacement due to the centrifugal force is the following.

$$T = \int_0^{2\pi} \frac{1}{2} \rho A \left[\left(\frac{\partial v}{\partial t} + \Omega(R + w_0 + w) \right)^2 + \left(\frac{\partial w}{\partial t} - \Omega \right)^2 \right] R d\theta \quad (2)$$

The potential energy is considered as bending and tension based on the cylindrical shell theory. The energy is shown below adding the elastic energy of the sidewall spring.

$$U = \int_0^{2\pi} \left[\frac{EI}{2R^4} \left(\frac{\partial^2 w}{\partial \theta^2} + w \right)^2 - \frac{S_0 + S_{rot}}{2R^2} \left(\frac{\partial^2 w}{\partial \theta^2} + w \right) w + \frac{1}{2} K_\theta v^2 + \frac{1}{2} K_r (w + w_0)^2 \right] R d\theta \quad (3)$$

S_0 represents the tension generated by the internal pressure, and S_{rot} represents the tension generated by the rotation.

$$S_{rot} = \rho A R^2 \Omega^2 \quad (4)$$

From each derived energy and Hamilton's principle, the motion equation is the following, organized in terms of w .

$$\begin{aligned} & \frac{\partial^4 w}{\partial t^2 \partial \theta^2} - \frac{\partial^2 w}{\partial t^2} + 4\Omega \frac{\partial^2 w}{\partial t \partial \theta} - \frac{S_0 + S_{rot}}{\rho A R^2} \left(\frac{\partial^4 w}{\partial \theta^4} + \frac{\partial^2 w}{\partial \theta^2} \right) \\ & + \frac{EI}{\rho A R^4} \left(\frac{\partial^6 w}{\partial \theta^6} + 2 \frac{\partial^4 w}{\partial \theta^4} + \frac{\partial^2 w}{\partial \theta^2} \right) - \frac{1}{\rho A} \left(K_\theta w - K_r \frac{\partial^2 w}{\partial \theta^2} \right) - \Omega^2 \left(\frac{\partial^2 w}{\partial \theta^2} - w \right) = 0 \end{aligned} \quad (5)$$

Parameter Identification

In this section, we performed parameter identification to unknown quantities, which are bending term, tension term and spring terms in the motion equation with natural frequency in the non-contact and non-rolling condition. There, the solution of the motion equation is assumed in the following as free vibration [6].

$$w(\theta, t) = \sum_{n=1}^{\infty} \alpha_n(t) \cos(n\theta) + \beta_n(t) \sin(n\theta) \quad (6)$$

The second order simultaneous differential equation for α_n and β_n can be derived by substituting [expression \(6\)](#) into [equation \(5\)](#) and making use of the orthogonality of trigonometric functions.

$$\begin{bmatrix} \ddot{\alpha}_n \\ \ddot{\beta}_n \end{bmatrix} + \begin{bmatrix} 0 & -X \\ X & 0 \end{bmatrix} \begin{bmatrix} \dot{\alpha}_n \\ \dot{\beta}_n \end{bmatrix} + \begin{bmatrix} Y & 0 \\ 0 & Y \end{bmatrix} \begin{bmatrix} \alpha_n \\ \beta_n \end{bmatrix} = 0 \quad (7)$$

where

$$X = \frac{4n\Omega}{n^2 + 1} \quad (8)$$

$$Y = \frac{S_0 + S_{rot}}{\rho A R^2} \frac{n^2(n^2 - 1)}{n^2 + 1} + \frac{EI}{\rho A R^4} \frac{n^2(n^2 - 1)^2}{n^2 + 1} + \frac{1}{\rho A} \frac{K_\theta + n^2 K_r}{n^2 + 1} - \Omega^2 \quad (9)$$

From [equation \(7\)](#), the formula for natural frequency in rolling and non-rolling conditions is the following.