

Comparative Assessment of Multi-Axis Bushing Properties Using Resonant and Non-Resonant Methods

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ABSTRACT

Shaped elastomeric joints such as engine mounts or suspension bushings undergo broadband, multi-axis loading; however, in practice, the elastomeric joint properties are often measured at stepped single frequencies (non-resonant test method). This article helps provide insight into multi-axis properties with new benchmark experiments that are designed to permit direct comparison between system resonant and non-resonant identification methods of the dynamic stiffness matrices of elastomeric joints, including multi-axis (non-diagonal) terms. The joints are constructed with combinations of inclined elastomeric cylinders to control non-diagonal terms in the stiffness matrix. The resonant experiment consists of an elastic metal beam end-supported by elastomeric joints coupling the in-plane transverse and longitudinal beam motion. The dynamic stiffness and loss factors of the elastomeric cylinders are measured in a non-resonant commercial elastomer test machine in shear, compression, and inclined configurations and a coordinate transformation is used to estimate the kinematic non-diagonal stiffness terms. Strong agreement is found for both dynamic stiffness and loss factors between the resonant and non-resonant methods at small displacements.

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INTRODUCTION

Elastomeric joints are widely used in vehicle isolation systems and their geometries are shaped to provide favorable properties in certain directions based on the diagonal terms [1, 2]. Nonetheless, non-diagonal (coupling) terms are often unknown though they are intrinsic to the design of complex automotive assemblies [3, 4]. Elastomeric joints present many challenges in modeling such as amplitude and frequency dependent properties. Direct measurement of the multidimensional dynamic properties of practical elastomeric joints is not possible, and these properties are further complicated when an elastomeric component is integrated into a sub-system assembly. Typically, the dynamic properties of assemblies may differ from those of components due to preload and boundary condition effects; therefore, there is a need to develop improved experimental methods to examine these issues [4].

Identification methods that utilize computational and experimental modal analyses may be integrated into some applications as long as the structures behave in a linear manner [5, 6]. For instance, Kim et al. [7] used a modal-based technique to characterize the dynamic stiffness of beam supports, each modeled by a lumped transverse spring. This prior method [7] has recently been extended by Noll et al. [8]

to include the identification of multidimensional matrices. The method uses the physical system matrices developed from a discretized model (lumped parameter or finite element) without joints, and then measured natural frequencies, modal loss factors, and mode shapes are utilized to extract the joint parameters. This article introduces a resonant identification method [8] and compares the results with the conventional dynamic stiffness properties measured using a non-resonant test method.

PROBLEM FORMULATION

The scope of the resonant test is limited to an elastic beam structure connected to ground through two elastomeric joints, where each joint is comprised of two cylinders. The assembled system is assumed to be linear time invariant and self-adjoint; the damping of the elastomeric joints is structural, and the beam is assumed to be proportionally viscous damped. Further, the joint mass is known *a priori*, and the dynamic properties of a joint can be represented at a point by structural damping **h** and stiffness **k** matrices. The specific objectives of this article are as follows. 1. Design a tractable resonant beam experiment that allows a direct comparison with the non-resonant test of an elastomeric component. 2. Use a recently extended resonant formulation

of the authors [8] to identify fully populated joint dynamic matrices given limited modal measurements on the resonant beam. 3. Compare the dynamic joint properties identified with resonant and non-resonant test methods; having consistency between the two different methods would suggest a heuristic validation though these two methods are rarely compared in the scientific literature.

The experiments are illustrated in Fig. 1: elastomeric cylinders (Fig. 1a) are inclined to permit multi-axis loading of the components in a non-resonant test method; An elastic beam system is supported by the same inclined elastomeric cylinders at each end, with each elastomeric joint loaded in multiple directions. Kinematic coupling to the structure is introduced through inclined attachment of the joints which couples the transverse $y(x,t)$ and longitudinal $u(x,t)$ motions of the beam. The modal parameters of the unconstrained (free-free) beam are selected so that the first elastic flexural mode is representative of the first elastic mode of real-world automotive subframe structures, say between 100 to 200 Hz. Further, the first 3 rigid body and 3 elastic modes (up to 1024 Hz) are examined, as both rigid and elastic modes are of concern in practical assemblies. Elastomeric joints are intentionally selected even though they exhibit excitation amplitude and frequency dependent properties, as well as sample to sample variations (say 10 to 15%). Given the amplitude-dependent properties, comparisons must be made with a non-resonant method. Both resonant and non-resonant methods are employed in industry, from system and component perspectives, respectively.

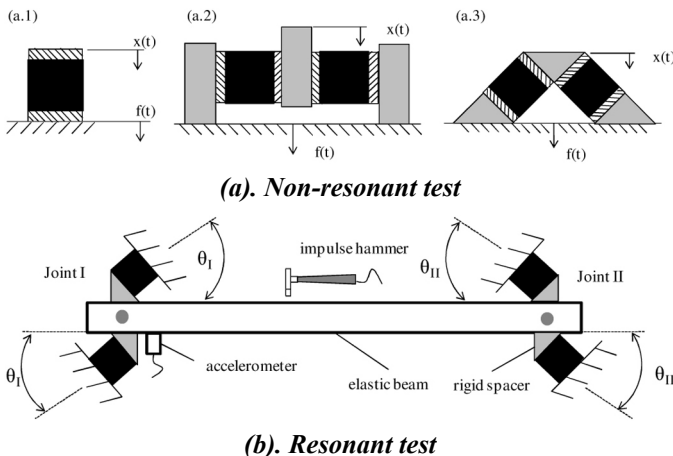


Fig. 1. Schematic of the non-resonant and resonant tests: (a) elastomeric cylinders loaded in: 1. compression; 2. shear; 3. angled; (b) elastic beam is end supported by two joints, where each joint is comprised of two inclined elastomeric cylinders.

JOINT STIFFNESS MATRIX OF AN ELASTOMERIC CYLINDER

Analytical Joint Stiffness Matrix

Each elastomeric cylinder in the resonant experiment has a transverse eccentricity ε from the neutral axis of the beam and an inclination angle θ from the beam's nodal coordinate system as shown in Fig. 2. Both the eccentricity and the inclination angle introduce a form of kinematic coupling that must be considered such that a suitable transformation can be made from the local coordinate system of the cylinder to the beam's nodal coordinate system.

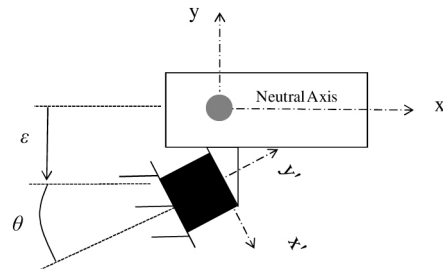


Fig. 2. Details of transverse eccentricity, ε , and angle of inclination, θ .

A uniform circular cylinder when grounded at one end exhibits a stiffness matrix in a local coordinate system of the following form:

$$\mathbf{k}_{cyl} = \begin{bmatrix} a & 0 & d \\ 0 & b & 0 \\ d & 0 & c \end{bmatrix} \quad (1)$$

where a is the shear stiffness, b is the compressive stiffness, c is the rotational stiffness, and d is the elastic coupling between rotation and shear. Assuming that the point stiffness at the end of the cylinder follows a rigid connection to the beam node, the effective stiffness matrix of the elastomeric cylinder in the beam nodal coordinate system can be computed via the following transformations:

$$\mathbf{k}_{eff} = \mathbf{T}_\varepsilon \mathbf{T}_\theta^{-1} \mathbf{k}_{cyl} \mathbf{T}_\theta \mathbf{T}_\varepsilon^{-1} \quad (2)$$

where $\mathbf{T}_\theta$ and $\mathbf{T}_\varepsilon$ are defined as:

$$\mathbf{T}_\theta = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{T}_\varepsilon = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\varepsilon & 0 & 1 \end{bmatrix}.$$

Non-Resonant Joint Characterization

A non-resonant test employs a uniaxial hydraulically actuated, close loop servo-controlled elastomer test machine. Typically a sinusoidal displacement, $x(t)$, is applied to the top side of the specimen, and the transmitted force, $f(t)$, at the bottom of the specimen is measured at the same frequency. The dynamic stiffness can be defined as:

$$\tilde{k}_d(\omega) = \frac{f}{x} e^{i\delta} = |\tilde{k}_d| \angle \delta = k + ih \quad (4)$$

where δ is the loss angle. The loss factor, $\gamma = \tan \delta$, is defined as:

$$\tilde{k}_d(\omega) = k(1 + i\gamma) \quad (5)$$

The elastomeric cylinders are characterized in three different configurations: (1) compression; (2) shear; and (3) angled as depicted in Fig. 1. The compression configuration consists of a single cylinder. The shear and angled configurations are conducted with a pair of elastomeric cylinders to ensure appropriate boundary conditions. The diameter and height of each cylinder is 25.4 mm. Metallic caps are bonded to both sides of each cylinder to prevent damage to the elastomer during disassembly. The nonresonant measurements correspond with specific linear combinations of the elements of the cylinder stiffness matrix of Eqn. (1) as: 1. shear = 2a, 2. compression = b, and 3. angled = a+b.

The dynamic characterization is conducted at an initial preset of 0.5 mm from 5 to 200 Hz at 5 Hz increments. Under each configuration, the peak-to-peak amplitude is specified at 0.1 mm, 0.05 mm, or 0.01 mm. The dynamic stiffness of elastomeric cylinders exhibit considerable amplitude and frequency dependence in the non-resonant tests as shown in Table 1. The highest sensitivity to frequency dependence is observed below 50 Hz. The measured $|\tilde{k}_d|$ increases for each configuration as the amplitude of the excitation decreased by a factor of 2.1, 1.5, and 2.0 for the compression, shear, and the angled configurations, respectively. The loss factor (Table 2) is nearly constant with respect to frequency; however, it decreases by half when the peak-to-peak amplitude of the excitation is reduced from 0.1 mm to 0.01.

Validation of Superposition Assumption

Since the elastomer test machine permits only uniaxial measurements, the specimens are oriented in several configurations to measure relevant diagonal elements of the dynamic stiffness matrices. Using Eq. (2), $|\tilde{k}_d|$ is calculated in Table 3 showing the angled measurement against the computed value using the measured data from the shear and compression measurements shown in Table 1. This comparison demonstrates that superposition holds near the operating points though the component is inherently nonlinear. Since there is such a strong agreement, it is

expected that the computed values for the kinematic coupling terms between longitudinal and transverse directions can be similarly estimated by using Eq. (2).

Table 1. Elastomeric joint characterization using non-resonant test method, dynamic stiffness magnitude (N/mm), at 100 Hz

Configuration	Peak-to-peak amplitude		
	0.1 mm	0.05 mm	0.01 mm
Compression	1112	1469	2368
Shear	503	618	782
Inclined at 45 deg	1380	1811	2727

Table 2. Elastomeric joint characterization using non-resonant test method, loss factor at 100 Hz

Configuration	Peak-to-peak amplitude		
	0.1 mm	0.05 mm	0.01 mm
Compression	0.36	0.31	0.14
Shear	0.26	0.19	0.15
Inclined at 45 deg	0.36	0.28	0.13

Table 3. Comparison of the dynamic stiffness magnitude (N/mm) of two elastomeric cylinders angled at 45° at 100 Hz

Peak-peak amplitude	Theory (Eqn. 2)	Measured	Difference (%)
0.1 mm	1364	1380	1.2%
0.05 mm	1778	1811	1.8%
0.01 mm	2759	2727	1.2%

RESONANT BEAM EXPERIMENT

Proposed Experiment

A rigid body formulation alone will not permit a sufficient number of equations to uniquely identify the two different joints in the resonant beam experiment; thus an elastic body formulation is required. Further, the elastomeric cylinders are selected to have similar stiffness properties to automotive bushings and ratio of joint to structure stiffness. To maintain the lumped joint stiffness assumption, the ratio of the contact surface dimension to beam length is kept below 0.05. With these guidelines, a steel beam (with Young's modulus, $E=207$ GPa, Poisson's ratio, $\nu = 0.3$, and mass density, $\rho = 7850$ kg m⁻³) is selected. The beam is 914 mm in length (L) with a rectangular cross-section of 25.4 × 50.8 mm where the thickness is 25.4 mm in the y direction for the desired flexural stiffness in the experiment. Modal measurements are made on the beam while it is suspended by very compliant springs.

The beam is supported near each end by a combination of two angled elastomeric cylinders, specified as Neoprene with a Shore A hardness of 70. To introduce symmetric coupling, angled cylinders with an aspect ratio of 7 are inclined at 45° from above and below the beam. The angled cylinders induce