

Application of Laminated Composite Materials in Vehicle Design: Theories and Analyses of Composite Beams

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ABSTRACT

With rising fuel prices, lightweight structures and materials (like composites) are receiving more attention. Composite materials offer high stiffness to weight and strength to weight ratios when compared with traditional metallic materials. Traditionally, composite materials were generally costly which made them only attractive to very limited industries (e.g., the defense industry). Advances in their manufacturing and new innovations have brought the cost of these materials down and made them reasonably competitive. They have gained more and more usage in the last 3 decades in the aerospace industry and have recently been gaining more usage in the automotive industry. In automotive design, they yield lighter structures which have positive impact on attributes like fuel economy, emission and others. Proper modeling and analyses need to be performed to make sure that other attributes (e.g. durability, noise, vibration and harshness or NVH) are assessed properly and remain competitive. Composite materials offered for real applications in the automotive industry vary from thermo-plastics to laminated and fiber-reinforced structures. This paper will focus on laminated beams and their applications in the auto industry.

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INTRODUCTION

There are many components in the vehicle that are made of beams and can be made of composite materials. These include elements like the bumper beam, the side frame, vehicle sub-frame and others ([Figure 1](#)). Proper modeling is needed to make sure that accurate analyses are performed and an optimal beam design is delivered. While beams can be of box shape cross section (main frame) or simple layered rectangular cross section, this paper focuses on the latter.

Literature on composite beam research can be found in many conference proceedings and journals. Kapania and Raciti [1] reviewed research on laminated beams and plates vibration and wave propagation in 1989. Rosen [2] focused on pretwisted rods and beams. Chidamparam and Leissa [3] reviewed literature on the vibrations of planar curved bars, beams, rings and arches. Also, a recent book [4] dedicated portions to composite beams, plates and shells. This book gives a clarification of all symbols and equations used here.

Beams are three dimensional (3D) bodies bounded by four, relatively close surfaces. The 3D equations of elasticity

are unnecessarily complicated when written for a beam. Researchers simplify such equations by making certain assumptions for particular applications. Almost all beam theories reduce the 3D elasticity problem into a one dimensional (1D) problem. Laminated beams may offer (depending on the lamination sequence) additional phenomena such as stretching-bending coupling, stretching-twisting coupling and bending-twisting coupling.

We will focus here on beams moving in their plane. There are several issues typically encountered for 1D analysis of beams. The first issue is how to include the various couplings (stretching bending, bending twisting and others) that are ignored in reducing 3D equations to 1D. A suitable approach for inclusion of coupling parameters is to redefine stiffness parameters such that it includes other couplings. To overcome coupling problem, it is proposed that instead of normal definition of A_{11} , B_{11} , and D_{11} , one can use equivalent modulus of elasticity of each lamina to find the equivalent ABD parameters. That is why different definitions of stiffness parameters are presented first.



Figure 1. Examples of beams in vehicle structures

STIFFNESS PARAMETERS

Rectangular Cross Section

Kaw [5] suggested to define the stiffness parameters based on the flexibility matrix (the ABD matrix inverse). Rios and Chan [6] proposed other formulation for ABD parameters. Ecsedi and Dluhi [7] analyzed the static and dynamic behavior of nonhomogeneous curved beams and closed rings. Instead of ABD terms, the modulus was treated as a function in cylindrical coordinates. Some other researchers [8,9,10,11] treated the problem of laminated composite beams to that of isotropic homogeneous beams with effective bending (EI), torsional (GJ), and axial stiffness parameters (EA). Hajianmaleki and Qatu [12] showed that using equivalent modulus of elasticity of each lamina, one can get accurate results for static and dynamic analyses of generally laminated beams with any kind of coupling. The equivalent modulus of elasticity of each lamina is found using [12, 13]

$$\frac{1}{E_x^k} = \frac{\cos^4(\theta_k)}{E_{11}} + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_{11}} \right) \cos^2(\theta_k) \sin^2(\theta_k) + \frac{\sin^4(\theta_k)}{E_{22}} \quad (1)$$

The above equation allows the engineer to use an equivalent modulus for each layer in the x-direction. As noted, the modulus elasticity will be closer to E_{11} when the fiber angles are close to 0 degrees. It should be stated here that only beams with symmetry around a vertical axis are treated. In addition, only in-plane vibration is considered. Equivalent A_{11} , B_{11} and D_{11} using the following equations would be used for a curved beam [14]:

$$\begin{aligned} A_{11} &= R \sum_{k=1}^N b E_x^{(k)} \ln \left(\frac{R+z_k}{R+z_{k-1}} \right) \\ B_{11} &= R \sum_{k=1}^N b E_x^{(k)} \left[(z_k - z_{k-1}) - R \ln \left(\frac{R+z_k}{R+z_{k-1}} \right) \right] \\ D_{11} &= R \sum_{k=1}^N b E_x^{(k)} \left[\frac{1}{2} \left((R+z_k)^2 - (R+z_{k-1})^2 \right) \right. \\ &\quad \left. - 2R(z_k - z_{k-1}) + R^2 \ln \left(\frac{R+z_k}{R+z_{k-1}} \right) \right] \end{aligned} \quad (2)$$

and for a straight beam, they reduce to [15-16]

$$\begin{aligned} (A_{11})_e &= \sum_{k=1}^N b E_x^k (h_k - h_{k-1}) \\ (B_{11})_e &= \sum_{k=1}^N b E_x^k \frac{(h_k^2 - h_{k-1}^2)}{2} \\ (D_{11})_e &= \sum_{k=1}^N b E_x^k \frac{(h_k^3 - h_{k-1}^3)}{3} \end{aligned} \quad (3)$$

The above equations are the stiffness parameters for the beam undergoing both extension and bending at the same time. Note here the coupling term B_{11} . The inclusion of shear deformation in the analysis of beams was first made by Timoshenko [e.g. 4]. Hence, theories considering shear deformation are called Timoshenko beam theories. In this regard, the beam theories are classified based on the order of polynomial for approximation of displacements through the thickness. Figure 2 shows a free body diagram of a differential beam element. Suppose that the displacement u can be expressed as

$$u = u_0 + z \left[c_0 \frac{\partial w}{\partial x} + c_1 \phi(x, t) \right] \quad (4)$$

and ϕ is the rotation of a line element perpendicular to the original direction. For this equation, special cases are defined as the classical beam theory (CBT) where $c_0 = -1$, $c_1 = 0$. The first order shear deformation theory (FSDT) has $c_0 = 0$, $c_1 = 1$. Higher order theories can also be implemented in a similar approach.

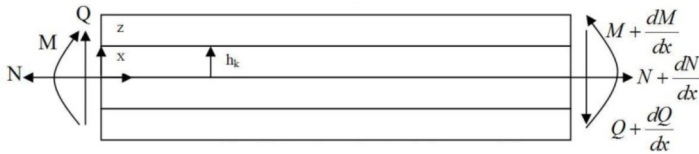


Figure 2. Parameters of a simple beam model

Tube Cross Section

Hajianmaleki and Qatu [17] used the following stiffness parameters in a FSDT formulation to study transverse vibration of generally laminated shafts.

$$A_{11} = \pi \sum_{k=1}^N E_x^k \left[(r_k^2 - r_{k-1}^2) \right]$$

$$D_{11} = \frac{\pi}{4} \sum_{k=1}^N E_x^k \left[(r_k^4 - r_{k-1}^4) \right] \quad (5a)$$

where r in the above equations is measured from middle of the laminate (Figure 3). The term B_{11} can be found in Ref 14. They used generalized differential quadrature (GDQ) for solution of the problem and the natural frequencies for two-segment shafts with different laminates were assessed.

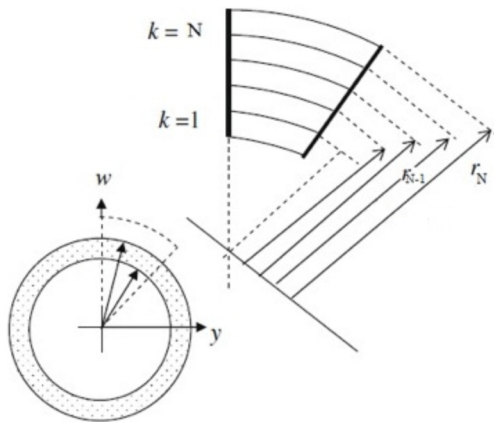


Figure 3. Geometry of a simple shaft model

Shear Stiffness

A shear stiffness parameter is proven to be needed for beam theories where we have for rectangular cross section [13]

$$A_{55} = \frac{5}{4} \sum_{k=1}^N b \bar{Q}_{55}^k \left[(h_k - h_{k-1}) - \frac{4}{3h^2} (h_k^3 - h_{k-1}^3) \right] \quad (5b)$$

and for tube cross section [17]

$$A_{55} = k \bar{Q}_{55} A$$

where A is cross sectional area and $k=0.5$ for tubular beams.

BEAM THEORIES

Classical Beam Theory

If the beam thickness is less than 1/20 of the wavelength of the deformation mode, a classical beam theory (CBT) or Euler Bernoulli (EB) beam theory where shear deformation and rotary inertia are negligible, is generally acceptable for lower frequency determination. Qatu [18] and Qatu and Elsharkawi [19] used CBT to study vibration of straight and curved laminated beams. They used Ritz method for solving the equations of motion. Qatu and Iqbal [20] used CBT to solve the vibration of a cross-ply laminated composite driveshaft with an intermediate joint.

Applying the classic beam theory for thin beams (normals to the beam midsurface remain straight and normal), strains and curvature change at the middle surface are:

$$\varepsilon_0 = \frac{\partial u}{\partial x}, \quad \kappa = \frac{\partial^2 w}{\partial x^2} \quad (6)$$

where u , and w are displacements in x and z directions, respectively (Figure 2). Normal strain at any point would be

$$\varepsilon = \varepsilon_0 + z\kappa \quad (7)$$

Force and moment resultants are calculated using

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A_{11} & B_{11} \\ B_{11} & D_{11} \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ \kappa \end{bmatrix} \quad (8)$$

The equations of motion are

$$\frac{\partial^2 M}{\partial x^2} = I_1 \frac{\partial^2 w}{\partial t^2} - p_z \quad (9)$$

$$\frac{\partial N}{\partial x} = I_1 \frac{\partial^2 u}{\partial t^2} - p_x \quad (10)$$

where I_1 , p_x , and p_z are beam mass, external loads in x and z directions per unit length, respectively.

$$I_1 = \sum_{k=1}^N b \rho^{(k)} (h_k - h_{k-1}) \quad (11)$$

Inserting the strain and curvature relations in the force and moment resultants equations and using those in the equations of motion, one can express the equations of motion in terms of displacements. Applying boundary condition and suitable