

An Unusual Way to Improve TPA for Strongly-Coupled Systems

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ABSTRACT

In a vehicle's development process, Transfer Path Analysis (TPA) is commonly used for identifying sound sources and their transmission to a receiver. Forces acting on the structure are the reason for the structure-borne sound share of the vehicle interior noise. In practice it is not possible, or too extensive, to measure operational forces directly. Instead, they are often calculated indirectly from accelerations and from additionally measured inertances. As the car body is a strongly coupled system, a force acting at one position results in accelerations throughout the structure. This crosstalk must be considered by using a dense inertance matrix consisting of the ratios between each force excitation and the accelerations at every sensor position. Then a matrix inversion is performed to solve the system of equations describing the coupling of the structure. For an engine TPA, this method has the disadvantage that it does not include the mounts, although their characteristics can have a major impact on the interior noise.

Another widely-used method predicts each body acceleration from the corresponding acceleration at the engine using the mount attenuation extracted from operational data. This has the advantage that active-side accelerations are less disturbed by other sources like tire-road noise or auxiliary components than passive-side accelerations. The resulting force is calculated without a matrix inversion, only considering the local body impedance. However, ignoring the crosstalk on the structure often leads to overestimating the force.

A new approach is presented which combines the advantages of matrix inversion and active-side accelerations as input signals. Effective mount transfer functions are calculated *in situ* from operational data considering the crosstalk on the structure. Thus, active-side accelerations can be used for synthesis without overestimating the distinct sound shares.

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INTRODUCTION

Transfer Path Analysis (TPA) is a useful tool to describe the sound transmission from sound sources of a vehicle to a receiver, e. g., from the engine to the driver's ears. The transfer paths which are responsible for unwanted or annoying noise components can be identified. The primary goal is to show possibilities for improving the sound. A transfer path model combines input signals measured during operational conditions with transfer functions describing the transmission to the receiver. The transfer path synthesis can be validated by comparing it to a simultaneously measured reference signal at the receiver position. A source can emit airborne sound directly and can cause structure-borne sound if forces are induced into the structure. The resulting vibrations are then emitted as airborne sound to the receiver. In practice, it is often impossible to measure these forces directly with reasonable effort. It is very elaborate and time-consuming to place force transducers between the source and

receiver structure, and the available space, e. g., in the engine compartment, is usually limited. Another possibility is to determine forces indirectly from accelerations measured during operation.

This paper focuses on structure-borne sound caused by a source which is connected via mounts to a structure, like an engine in a car. There are two widely-used methods for modeling structure-borne transfer paths which have individual advantages and drawbacks. On the one hand, the **Indirect Force Determination (IFD)** [1, 2] method allows considering the coupling of the receiver structure but uses accelerations measured on the receiver structure which can be disturbed by other sources like rolling tires. On the other hand, there is a method based on mount attenuation functions using accelerations measured on the source, which are less disturbed due to soft mounts, but here the coupling of the structure is ignored [3]. The receiver structure is also called passive-side and the source structure is named as active-side.

In the following sections, both approaches are described in detail and a new unusual method is presented which combines the advantages of both. It is based on accelerations measured on the source structure and considers coupling of the receiver structure. After explaining the theory, examples are given.

INDIRECT FORCE DETERMINATION

For the Indirect Force Determination (IFD), three-dimensional acceleration sensors are placed at the positions where the source induces forces into the structure. In the case of a car engine, they are applied next to the engine mounts on the car body. At these positions, the structure is successively struck by an impact hammer in all spatial directions measuring the induced force and the resulting accelerations. From these measurements, the inertances, also referred to as accelerances, are calculated. The inertance of a structure is defined as the transfer function¹ from the induced force F_k at the position k to the resulting acceleration a_l at the position l :

$$I_{k,l}(f) = a_l(f) \oslash F_k(f). \quad (1)$$

To distinguish among the different directions in space at each measured position, separate indices are used not only for each position but also for each direction.

At first, it is assumed that there is no coupling between the different positions or directions. That means, a force at position k causes only an acceleration at $l = k$ and no acceleration at the other positions with $l \neq k$. In other words, each measured acceleration signal is only a result of a force acting at the same position and in the same direction. Vice versa, only the acceleration signal with the index k which is recorded during operational conditions of the car or machine, needs to be considered to calculate the operational force F_k . The inertance is simply inverted to yield the apparent mass $H_k^{AMF}(f)$, which will be used to calculate the force indirectly from the acceleration:

$$H_k^{AMF}(f) = (a_k(f) \oslash F_k(f))^{-1} = \frac{1}{I_{k,k}(f)}. \quad (2)$$

In the time domain TPA, the operational force can be calculated by filtering the operational acceleration with the impulse response $h_k^{AMF}(t)$ of the apparent mass transfer function $H_k^{AMF}(f)$:

$$\tilde{F}_k^{AMF}(t) = a_k(t) * h_k^{AMF}(t). \quad (3)$$

However, as the car body is a strongly coupled system, a force acting at one position causes vibrations throughout the

structure. Then, it is not sufficient to consider only the inertance for $l = k$ to calculate the force with the index k but instead all combinations of impact and acceleration sensor positions must be evaluated. A frequency dependent inertance matrix $\mathbf{I}(f)$ with $k = 1 \dots K$ and $l = 1 \dots L$ can be set up:

$$\mathbf{I}(f) = \begin{bmatrix} I_{1,1}(f) & I_{1,2}(f) & \dots & I_{1,K}(f) \\ I_{2,1}(f) & I_{2,2}(f) & \dots & I_{2,K}(f) \\ \vdots & \vdots & \ddots & \vdots \\ I_{L,1}(f) & I_{L,2}(f) & \dots & I_{L,K}(f) \end{bmatrix}. \quad (4)$$

In the frequency domain, the operational accelerations are equal to the product of the inertance matrix and the operational force vector:

$$\begin{bmatrix} a_1(f) \\ a_2(f) \\ \vdots \\ a_L(f) \end{bmatrix} = \begin{bmatrix} I_{1,1}(f) & I_{1,2}(f) & \dots & I_{1,K}(f) \\ I_{2,1}(f) & I_{2,2}(f) & \dots & I_{2,K}(f) \\ \vdots & \vdots & \ddots & \vdots \\ I_{L,1}(f) & I_{L,2}(f) & \dots & I_{L,K}(f) \end{bmatrix} \cdot \begin{bmatrix} F_1(f) \\ F_2(f) \\ \vdots \\ F_K(f) \end{bmatrix}. \quad (5)$$

The coupling or crosstalk between spatial directions or different positions is described by the non-diagonal elements of the inertance matrix. All force signals can contribute to any acceleration signal. The above mentioned case without coupling is a special case leading to a diagonal matrix. Equation (5) can be expressed in a vector-matrix notation:

$$\mathbf{a}(f) = \mathbf{I}(f) \cdot \mathbf{F}(f). \quad (6)$$

The operational forces are calculated by multiplying the apparent mass matrix $\mathbf{M}(f)$ with the acceleration vector:

$$\mathbf{F}(f) = \mathbf{M}(f) \cdot \mathbf{a}(f). \quad (7)$$

This is a short notation for

$$\begin{bmatrix} F_1(f) \\ F_2(f) \\ \vdots \\ F_K(f) \end{bmatrix} = \begin{bmatrix} M_{1,1}(f) & M_{1,2}(f) & \dots & M_{1,L}(f) \\ M_{2,1}(f) & M_{2,2}(f) & \dots & M_{2,L}(f) \\ \vdots & \vdots & \ddots & \vdots \\ M_{K,1}(f) & M_{K,2}(f) & \dots & M_{K,L}(f) \end{bmatrix} \cdot \begin{bmatrix} a_1(f) \\ a_2(f) \\ \vdots \\ a_L(f) \end{bmatrix}. \quad (8)$$

If the number of acceleration signals L is equal to the number of force excitation positions K , the apparent mass matrix is the inverted inertance matrix. The inversion is performed for every frequency f :

$$\mathbf{M}(f) = \mathbf{I}^{-1}(f) \text{ with } L = K. \quad (9)$$

The accuracy and numerical stability can be improved by using additional acceleration sensors applied somewhere on the structure. At these indicator positions, impact measurements are not performed but the related inertances

¹The operator $\oslash$ is used to indicate that the transfer function is calculated based on correlation techniques.

are calculated. This leads to a matrix $\mathbf{I}(f)$ with $L > K$ describing an over-determined system of equations which is solved using the pseudo-inverse (pinv) [4]:

$$\mathbf{M}(f) = \text{pinv}(\mathbf{I}(f)) \text{ with } L > K. \quad (10)$$

Generally, the apparent mass transfer functions $H_k^{AMF}(f)$ (equation (2)) are not equal to the diagonal elements $M_{k,k}(f)$ of the apparent mass matrix $\mathbf{M}(f)$. The former are the multiplicative inverse of the inertances $I_{k,k}(f)$ (equation (1)) considering no coupling. The latter are the result of a matrix inversion of a dense inertance matrix. Both are equal if a diagonal inertance matrix is used.

For each element $M_{k,l}(f)$ of the apparent mass matrix, a transfer function is built by combining the corresponding values of all frequencies. Impulse responses $h_{k,l}^M(t)$ are calculated from the resulting transfer functions by means of the inverse Fourier transform. Afterwards, the acceleration signals are filtered with the related impulse responses and summed. The result is the force signal at position k :

$$\tilde{F}_k^{IFD}(t) = \sum_{l=1}^L a_l(t) * h_{k,l}^M(t). \quad (11)$$

Finally, the vibro-acoustic transfer function (ATF) is needed to perform an auralization at a receiver position, e. g., the ears of the driver. The ATF describes the relation between the force at position k and the resulting sound pressure at receiver position i :

$$H_{k,i}^{ATF}(f) = p_i(f) \oslash F_k(f). \quad (12)$$

The ATFs can be determined together with the inertances by recording the sound pressure at the receiver positions simultaneously during the impact hammer measurements. Another possibility is a reciprocal measurement with a suitable volume velocity source at the receiver position [5, 6]. The structure-borne sound can be calculated by filtering the indirectly determined forces with the impulse responses related to the ATFs (Figure 1). The total structure-borne sound synthesis is equal to the sum of all paths:

$$\tilde{p}_i(t) = \sum_{k=1}^K \tilde{F}_k^{IFD}(t) * h_{k,i}^{ATF}(t). \quad (13)$$

In general, the source structure must be separated from the receiving structure during impact measurements in order to calculate the operational forces at their interface. Otherwise, if the force determination is based on the inertances of the coupled system, the result of equation (11) is the so-called blocked force [7] which, in general, is not equal to the operational force. The reason is that an unknown

portion of the force induced by the impact hammer is acting on the source structure although only the inertance of the receiver structure is wanted. This leads to different inertances compared to the case with separated structures, and therewith to different indirectly-determined forces. However, if the inertances of the source structure are sufficiently high compared to the inertances of the receiver structure, the difference between the two forces is negligibly small [7]. In this case, the force of the impact hammer acts almost exclusively on the receiver structure and not on the source structure. The blocked forces must be combined with the (vibro-) acoustic transfer functions of the coupled system. Despite the difference between blocked and operational forces, the structure-borne sound share synthesized using blocked forces and acoustic transfer functions of the coupled system is equal to the structure-borne sound share based on operational forces and acoustic transfer functions of the uncoupled system [7].

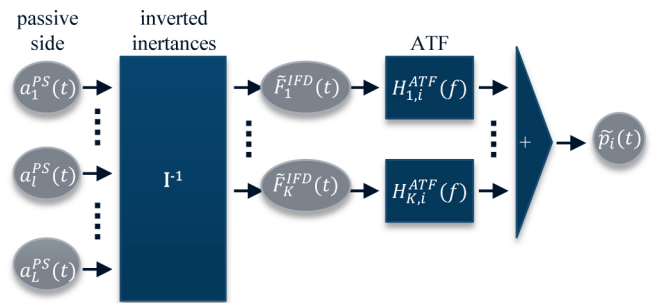


Figure 1. Structure-borne sound synthesis using indirectly-determined forces.

For an engine TPA, this method has the disadvantage that it gives no information about the mounts although their characteristics can have a major impact on the interior noise. For that reason, they could be of special interest regarding possible improvements of the interior noise by changing the mounts. A further disadvantage is that accelerations on the car body, which are used for Indirect Force Determination, can be disturbed by additional sources, which are not considered in the inertance matrix, e. g., tire-road noise or auxiliary components. The accelerations on the active-side, the engine, are usually not affected by these components due to the high engine mass and soft engine mounts. A determination of the induced forces based on active-side accelerations would eliminate these disadvantages.

MOUNT ATTENUATION FUNCTION

Another widely-used method for structure-borne sound synthesis predicts each acceleration on the passive-side (car body) from the corresponding acceleration on the active-side (engine) using a mount attenuation function [3]. For each position and direction of space, it is defined as transfer function between active-side (AS) and passive-side (PS) accelerations measured during operational conditions: