

The Application of Singular Value Decomposition to Determine the Sources of Far Field Diesel Engine Noise

Michael D. Hayward, J. Stuart Bolton and Patricia Davies
Purdue University

ABSTRACT

The identification of the dominant noise sources in diesel engines and the assessment of their contribution to far-field noise is a process that can involve both fired and motored testing. In the present work, the cross-spectral densities of signals from cylinder pressure transducers, accelerometers mounted on the engine surface, and microphones (in the near and far fields), were used to identify dominant noise sources and estimate the transfer paths from the various “inputs” (i.e., the cylinder pressures, the accelerometers and the near field microphones) to the far field microphones. The method is based on singular value decomposition of the input cross-spectral matrix to relate the input measurements to independent virtual sources. The frequencies at which a particular input is strongly affected by an independent source are highlighted, and with knowledge of transducer locations, inferences can be drawn as to possible noise source mechanisms. It was found that results from an analysis of fired test data were similar to those from motored test data: thus it is possible to reduce or eliminate motored testing. Also, since the transfer paths between the near- and far-field measurements can be estimated, convolutions of the input time histories and the transfer path impulse response functions can be combined to yield accurate output predictions. Furthermore, the contribution from each input measurement reveals which noise sources contribute significantly to the far field. Conclusions can then be drawn regarding source locations and spectral characteristics.

CITATION: Hayward, M., Bolton, J. and Davies, P., "The Application of Singular Value Decomposition to Determine the Sources of Far Field Diesel Engine Noise," *SAE Int. J. Engines* 6(2):2013, doi:10.4271/2013-01-1974.

INTRODUCTION

The demand for quiet diesel engines has promoted the need to understand the dominant noise source characteristics in the near and far fields. Presented here is a method to assist in the noise source identification of a diesel engine, by utilizing data measured by input cylinder pressure transducers, near field microphones, accelerometers, and output far field microphones. In previous work by the authors [1] a graphical representation of the input cross-spectral density matrix singular values was introduced to highlight the dominant noise source locations in the near field. These “contribution plots” are similar to the “signature plots” previously presented by Leclère et al. [2]. It has also been shown by Lee, Bolton and Suh [3] that the solution of a matrix problem can yield transfer path estimates from the near field measurements to the far field; a similar approach has been adopted here. Further application of singular value decomposition to the far field estimates then accentuates dominant noise sources that radiate efficiently to the far field. Implementation of this method also allows accurate transfer path estimation, thus enabling far field noise prediction based only on input measurements (i.e., nearfield transducers).

Thus, in total, the methods described here can effectively highlight dominant noise source locations in the near field, and reveal the noise sources that contribute significantly to the far field sound pressure.

SINGULAR VALUE DECOMPOSITION

Described in the following section is the method of singular value decomposition, and its application to input measurement decorrelation and noise source identification.

Multiple-Input/Multiple-Output Modeling

The relationship between the “input measurement” near field microphones and accelerometers, and far field “output measurements” can be modeled as a Multiple-Input/Multiple-Output (MIMO) system. Simplification of the model yields a Multiple-Input/Single-Output (MISO) system for each of the four far field microphone measurements (Figure 1).

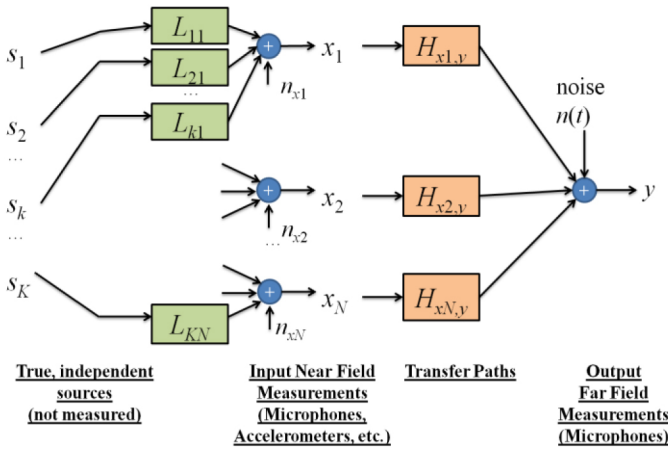


Figure 1. Schematic of Multiple-Input/Single-Output system flow of independent sources, $s_k(t)$, input measurement microphones, accelerometers and cylinder pressure transducers $x_i(t)$, and far field microphones, $y_j(t)$.

Each MISO model comprises $i=1,2,\dots,N$ input measurements, x_i , and one far field measurement, y . The input measurements are related to the output measurement by an impulse response, $h_{x_iy}(t)$: i.e.,

$$y(t) = \sum_{i=1}^N \int_{-\infty}^{\infty} x_i(\tau) h_{x_iy}(t - \tau) d\tau. \quad (1)$$

The Fourier transform of the impulse response yields the frequency response function, $H_{x_iy}(f)$, which can be used to determine the cross-spectral density between any two measurements: i.e.,

$$S_{x_iy}(f) = \lim_{T \rightarrow \infty} E \left[\frac{x_{iT}^*(f) y_T(f)}{T} \right] = \sum_{j=1}^N H_{x_jy}(f) S_{x_jx_j}(f) \quad (2)$$

where T denotes windowing of the signal to a length of T seconds.

The primary goal of the present work was to determine the dominant noise sources of a diesel engine and their characteristics in both the near and far fields (i.e., at the 1 meter microphones). In order to ensure the highest possible accuracy, the number of independent sources, $s_k(t)$, should be determined, and the transfer paths between the input measurements and the far field measurements should be estimated. To achieve these goals, the previously calculated cross-spectral densities must be assembled into a cross-spectral matrix:

$$\begin{bmatrix} S_{x_1y} \\ S_{x_2y} \\ \vdots \\ S_{x_Ny} \end{bmatrix} = \begin{bmatrix} S_{x_1x_1} & S_{x_1x_2} & \dots & S_{x_1x_N} \\ S_{x_2x_1} & S_{x_2x_2} & \dots & S_{x_2x_N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{x_Nx_1} & S_{x_Nx_2} & \dots & S_{x_Nx_N} \end{bmatrix} \begin{bmatrix} H_{x_1,y} \\ H_{x_2,y} \\ \vdots \\ H_{x_N,y} \end{bmatrix} + \begin{bmatrix} S_{x_1n} \\ S_{x_2n} \\ \vdots \\ S_{x_Nn} \end{bmatrix} \quad (3)$$

The final term in Eq. 3 expresses the cross-spectral noise characteristics between each input measurement and any additive noise on the measured input. Assuming the latter noise is negligible, Eq. 3 can be expressed more compactly as;

$$\mathbf{S}_{xy} = \mathbf{S}_{xx} \mathbf{H} \quad (4)$$

Ideally, the number of independent (and unmeasurable) noise-emitting sources, s_k , would be equal to the number of input measurements, x_i , with each input directly related to only one particular noise source. This would be the case if, for example, there were no mixing between the signals sensed by the microphones and accelerometers. In that case, decorrelation of the data would be unnecessary. However, in most cases, including this diesel engine investigation, these conditions are not met.

Singular Value Decomposition

Due to the unavoidable mixing observed in the input measurements (i.e., sound radiating from a single unique source is usually detected by more than one microphone and/or accelerometer), the singular value decomposition (SVD) method was used here to decorrelate the data. Singular value decomposition is an operation that is applied to the input cross-spectral matrix, $\mathbf{S}_{xx}$, in order to connect the measurements, x_i , $i=1,2,\dots,N$, to a set of statistically independent sources, s_k , $k=1,2,\dots,K$. This procedure can be mathematically represented as

$$\mathbf{S}_{xx} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_n] \text{diag}[\lambda_1, \lambda_2, \dots, \lambda_n] [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n]^H, \quad (5)$$

where $\mathbf{U}$ and $\mathbf{V}$ are the left and right orthonormal vectors, respectively, and $\mathbf{\Sigma}$ is a diagonal matrix containing the singular values [4]. As $\mathbf{S}_{xx}$ is a function of frequency, so too are the u_{ij} , v_{ij} and λ_i terms. While the singular values, or 'virtual sources' do not necessarily represent one particular noise source, they are usually closely related to independent noise generating mechanisms within the engine. Since each singular value represents independent information collected by the input measurements, in an ideal case where there are more input measurements than there are noise generating

sources, one or more of the singular values would be zero as there would be no new independent information measured by the inputs. It is through this comparison between the number of inputs and the number of non-zero singular values that the number of noise-generating sources in the engine can be determined.

An important property of the singular value matrix is that the λ_i 's are ordered from the largest singular value to the smallest so that λ_1 is always larger than λ_2 , and so on. This observation is significant when plotting the singular values as a function of frequency; in particular frequency regions where there is a change in the dominant virtual source, there may be a 'swapping' of singular values. An example of this phenomenon can be seen in [Figure 2](#) which shows, in the top plot, the singular values derived from a set of accelerometer measurements.

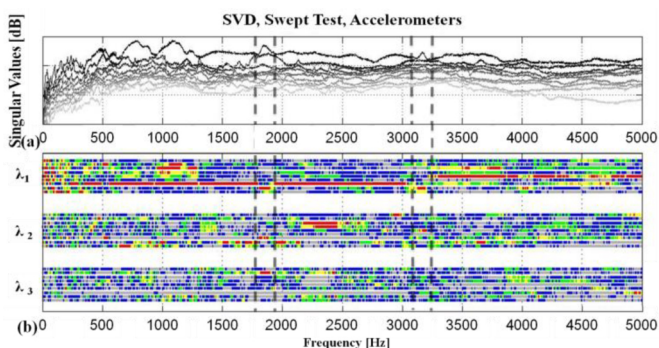


Figure 2. (a): Example of singular value decomposition applied to input measurement accelerometer cross-spectral matrix for a swept fired test; (b) graphical representation of singular value contributions to input measurement power spectra for first three singular values.

These measurements were recorded at Roush Industries on a 15L Cummins diesel engine. The test included 9 accelerometer transducers placed on the engine's surface. An unloaded sweep test was performed in which the rpm was constantly increased from low to high rpm. At frequencies below 1800 Hz there is one particular virtual source dominating the input measurements; however, at frequencies near 1800 Hz, the second singular value approaches the first, and then "swaps" to become the dominant virtual source for a range of approximately 100 Hz, before dropping back to become the second most dominant singular value. Similar behavior can be seen at 3100 Hz to 3200 Hz: this behavior indicates a change in the most significant physical source mechanism.

Singular Value Contributions

In order to properly interpret the singular values, the contributions from each virtual source to the individual input measurements must be investigated. By expanding the matrix multiplication term in [Eq. 5](#), the power spectral density of any

particular input measurement can be represented as the addition of N singular value terms: i.e.,

$$S_{x_i x_i} = \sum_{j=1}^N |u_{ij}|^2 \lambda_j \quad (6)$$

Each of the N terms in this summation represents the contribution of the singular value, λ_i , $i=1,2,\dots,N$ to the power spectral density $S_{x_i x_i}$. By using this method, the percentage contribution of each virtual source to a particular input can be calculated as a function of frequency. This percentage contribution is an indication of the strength of a particular singular value within the input measurement. By examining the relative contribution of the strongest singular values to each input measurement and by combining that information with a knowledge of each measurement's physical location, inferences can be drawn regarding noise source dominance and characteristics. Additionally, the frequencies of dominant noise source changes can be made clear.

Graphical Representation

Note, however, that the SVD contribution method yields a percentage contribution value for each input measurement, singular value and frequency analyzed, thus resulting in a very large amount of data to be interpreted. To make this task easier, a method to graphically represent the data was developed, and an example is shown in [Figure 2\(b\)](#). The labels beside each thick bar, λ_1 , λ_2 , etc., represent each singular value, while each of the smaller lines within the thick bar represent the measurements, x_1 , x_2 , etc., ordered from top to bottom. The colors plotted represent the contribution of the j^{th} singular value to the i^{th} input measurement, where red indicates a contribution of greater than 75 percent, yellow greater than 50 percent, green greater than 25 percent, and blue greater than 5 percent.

It can be seen in [Figure 2\(b\)](#), for example, that between the frequencies 500 Hz and 1800 Hz, over 75 percent of input measurement 7's power spectra is contributed by the first (and most dominant) singular value. Similarly, the first singular value dominates input 7 between 2000 Hz and 3000 Hz. By combining a knowledge of the location of input measurement 7 on the engine, with a knowledge of likely significant noise sources located near that measurement point, inferences can be readily drawn regarding the dominant noise sources in these frequency ranges in the near field.

Another important feature of the contribution plots can be seen between 1800 Hz and 2000 Hz. As previously mentioned, [Figure 2\(a\)](#) demonstrates a swapping of dominant singular values in this frequency range. Further interpretation of the contribution plot [Figure 2\(b\)](#) not only confirms this to be the case, but also provides information regarding the location of the dominant source. It is clear that at 1800 Hz, the input measurement location being dominated by the most dominant singular value changes from input measurement 7 to measurement 8, before changing back at 2000 Hz. Similar behavior can be seen at 3200 Hz. Additionally, the