

Application of Laminated Composite Materials in Vehicle Design: Theories and Analyses of Composite Shells

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ABSTRACT

With rising fuel prices, light weight structures and materials (like composites) are receiving more attention. Composite materials offer high stiffness to weight ratio when compared with traditional metallic materials. Traditionally, composite materials were generally costly which made them only attractive to very limited industries (e.g., the defense industry). Advances in their manufacturing and new innovations have brought the cost of these materials down and made them reasonably competitive. They have gained more and more usage in the last 3 decades in the aerospace industry and have recently been gaining more usage in the automotive industry. In automotive design, they yield lighter structures which have positive impact on attributes like fuel economy, emission and others. Proper modeling and analyses need to be performed to make sure that other attributes (e.g. noise, vibration and harshness or NVH) are assessed properly and remain competitive. Composite materials offered for real applications in the automotive industry vary from thermo-plastics to laminated and fiber-reinforced structures. This paper will focus on composite shells and their applications in the auto industry.

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INTRODUCTION

Much of the outer body of a typical vehicle is made of shell structures ([Figure 1](#)). Some of these shell structures are flat and thus referred to in the scientific literature as plates. Most of these structures are curved in nature and are thus called shells. Because plates are a special case of shells, the focus in modeling and analyses has been on shells (e.g., finite element packages offer shell elements only). Shells are three dimensional bodies where one dimension of the structure, namely the thickness, is small in comparison to the other two dimensions. Theories that treat shell structures are based upon the three dimensional (3D) theory of elasticity. 3D (if implemented) is costly and time consuming. Thus analysts use shell theory in their work.

The equations of motion and the required boundary conditions for deep thick composite shells are shown using two formulations. One formulation is based upon what was presented initially by Rath and Das [1] and followed by others. In this formulation, plate stiffness parameters are used for thick shells, which reduced the equations to those applicable for shallow shells. The second formulation is based upon that of Qatu [2,3,4]. In this formulation, the stiffness parameters are calculated using exact integration (and/or terms truncated to a specific order) of stress resultant

equations. In addition, Qatu considered the radius of twist in his formulation. In both formulations, first order polynomials for in-plane displacements in the z-direction are utilized allowing for the inclusion of shear deformation and rotary inertia effects (first order shear deformation theory or FSDT). Exact static and free vibration solutions for isotropic and symmetric and anti-symmetric cross-ply cylindrical shells for different length-to-thickness and length-to-radius ratios are obtained using the above theories. Results of both theories are compared with those obtained using a three-dimensional (3D) analysis to test the accuracy of the shell theories presented here. Early treatment of composite thick shells [e.g. 1,5] included both shear deformation and rotary inertia rotary but failed to include accurate representation of curvature (the z/R terms in the stress resultants). We will refer to these as simply the first order shear deformation theory (FSDT). Qatu [3,4] presented equations where the term is carefully considered in the shell equations for composite deep thick shells. We will refer to his equations as the first order shear deformation shell theory by Qatu (FSDTQ). These equations will be described in detail here. Exact static and free vibration solutions for isotropic and symmetric and anti-symmetric orthotropic cross-ply cylindrical shells with shear diaphragm boundary conditions for different length-to-thickness and length-to-

radius ratios varying from thin and shallow to moderately thick and very deep shells are obtained using both FSDT and FSDTQ and compared with those of a converged 3D finite element analysis obtained using a commercial software package (ANSYS®). Dimensionless transverse displacement, moment and force resultants in a static analysis and the first five natural frequency parameters in the dynamic analysis are studied for eighteen different cylindrical shells and compared with 3D results. Also, Errors of FSDTQ and FSDT results (when comparing them to those of the 3D results) are obtained and discussed.



Figure 1. Sell structures in vehicle design

FSDTQ SHELL THEORY

A shell that is doubly curved with in-plane axes α and β , and a normal to the middle-plane axis z , principal radii R_α and R_β in α and β directions, respectively, and a twisting radius $R_{\alpha\beta}$ is considered. The boundaries of the shell coincide with the principal axes. Using the first-order shear deformation model, the displacements become [e.g. 4]

$$\begin{aligned} u(\alpha, \beta, z, t) &= u_0(\alpha, \beta, t) + z\psi'_\alpha(\alpha, \beta, t), \\ v(\alpha, \beta, z, t) &= v_0(\alpha, \beta, t) + z\psi'_\beta(\alpha, \beta, t), \\ w(\alpha, \beta, z, t) &= w_0(\alpha, \beta, t), \end{aligned} \quad (1)$$

(2)

where α , and β are special coordinates and t is time. The first two equations are for in-plane displacement, and the last is for out of plane displacement; and where $-h/2 \leq z \leq h/2$ and h is the shell thickness, u_0 , v_0 , and w_0 are midsurface

displacements of the shell, and ψ_α and ψ_β are midsurface rotations. Eqs. (1-2) constitute the only assumption needed to reduce a 3D elasticity equations in curvilinear coordinates to the shell theory by Qatu [2,3,4]. The strain-displacement relationships in the principal coordinates of a doubly-curved shell are [2]

$$\begin{aligned} \epsilon_\alpha &= \frac{1}{(1+z/R_\alpha)} \left(\frac{1}{A} \frac{\partial u}{\partial \alpha} + \frac{v}{AB} \frac{\partial A}{\partial \beta} + \frac{w}{R_\alpha} \right), \\ \epsilon_\beta &= \frac{1}{(1+z/R_\beta)} \left(\frac{1}{B} \frac{\partial v}{\partial \beta} + \frac{u}{AB} \frac{\partial B}{\partial \alpha} + \frac{w}{R_\beta} \right), \\ \epsilon_z &= \frac{\partial w}{\partial z}, \\ \gamma_{\alpha\beta} &= \frac{1}{(1+z/R_\alpha)} \left(\frac{1}{A} \frac{\partial v}{\partial \alpha} - \frac{u}{AB} \frac{\partial A}{\partial \beta} + \frac{w}{R_{\alpha\beta}} \right) \\ &\quad + \frac{1}{(1+z/R_\beta)} \left(\frac{1}{B} \frac{\partial u}{\partial \beta} - \frac{v}{AB} \frac{\partial B}{\partial \alpha} + \frac{w}{R_{\alpha\beta}} \right), \\ \gamma_{\alpha z} &= \frac{1}{A(1+z/R_\alpha)} \frac{\partial w}{\partial \alpha} + A(1+z/R_\alpha) \frac{\partial}{\partial z} \left(\frac{u}{A(1+z/R_\alpha)} \right) - \frac{v}{R_{\alpha\beta}(1+z/R_\alpha)}, \\ \gamma_{\beta z} &= \frac{1}{B(1+z/R_\beta)} \frac{\partial w}{\partial \beta} + B(1+z/R_\beta) \frac{\partial}{\partial z} \left(\frac{v}{B(1+z/R_\beta)} \right) - \frac{u}{R_{\alpha\beta}(1+z/R_\beta)}, \end{aligned} \quad (3)$$

By substituting Eqs (1-2) into (3), strain-displacement equations develop as

$$\begin{aligned} \epsilon_\alpha &= \frac{1}{(1+z/R_\alpha)} (\epsilon_{0\alpha} + z\kappa_\alpha), \\ \epsilon_\beta &= \frac{1}{(1+z/R_\beta)} (\epsilon_{0\beta} + z\kappa_\beta), \quad \epsilon_z = 0, \\ \epsilon_{\alpha\beta} &= \frac{1}{(1+z/R_\alpha)} (\epsilon_{0\alpha\beta} + z\kappa_{\alpha\beta}), \\ \epsilon_{\beta\alpha} &= \frac{1}{(1+z/R_\beta)} (\epsilon_{0\beta\alpha} + z\kappa_{\beta\alpha}), \\ \gamma_{\alpha z} &= \frac{1}{(1+z/R_\alpha)} [\gamma_{0\alpha z} - z(\psi'_\beta / R_{\alpha\beta})], \\ \gamma_{\beta z} &= \frac{1}{(1+z/R_\beta)} [\gamma_{0\beta z} - z(\psi'_\alpha / R_{\alpha\beta})], \end{aligned} \quad (4)$$

The functions on the right-hand side of Eqs (4) are

$$\begin{aligned}
\varepsilon_{0\alpha} &= \frac{1}{A} \frac{\partial u_0}{\partial \alpha} + \frac{v_0}{AB} \frac{\partial A}{\partial \beta} + \frac{w_0}{R_\alpha}, \\
\varepsilon_{0\beta} &= \frac{1}{B} \frac{\partial v_0}{\partial \beta} + \frac{u_0}{AB} \frac{\partial B}{\partial \alpha} + \frac{w_0}{R_\beta}, \\
\varepsilon_{0\alpha\beta} &= \frac{1}{A} \frac{\partial v_0}{\partial \alpha} - \frac{u_0}{AB} \frac{\partial A}{\partial \beta} + \frac{w_0}{R_{\alpha\beta}}, \quad \varepsilon_{0\beta\alpha} = \frac{1}{B} \frac{\partial u_0}{\partial \beta} - \frac{v_0}{AB} \frac{\partial B}{\partial \alpha} + \frac{w_0}{R_{\alpha\beta}}, \\
\gamma_{0\alpha z} &= \frac{1}{A} \frac{\partial w_0}{\partial \alpha} - \frac{u_0}{R_\alpha} - \frac{v_0}{R_{\alpha\beta}} + \psi_\alpha, \quad \gamma_{0\beta z} = \frac{1}{B} \frac{\partial w_0}{\partial \beta} - \frac{v_0}{R_\beta} - \frac{u_0}{R_{\alpha\beta}} + \psi_\beta, \\
\kappa_\alpha &= \frac{1}{A} \frac{\partial \psi_\alpha}{\partial \alpha} + \frac{\psi_\beta}{AB} \frac{\partial A}{\partial \beta}, \quad \kappa_\beta = \frac{1}{B} \frac{\partial \psi_\beta}{\partial \beta} + \frac{\psi_\alpha}{AB} \frac{\partial B}{\partial \alpha}, \\
\kappa_{\alpha\beta} &= \frac{1}{A} \frac{\partial \psi_\beta}{\partial \alpha} - \frac{\psi_\alpha}{AB} \frac{\partial A}{\partial \beta}, \quad \kappa_{\beta\alpha} = \frac{1}{B} \frac{\partial \psi_\alpha}{\partial \beta} - \frac{\psi_\beta}{AB} \frac{\partial B}{\partial \alpha}.
\end{aligned} \quad (5)$$

The stress resultants are defined as

$$\begin{aligned}
& \begin{bmatrix} N_\alpha & N_{\alpha\beta} & Q_\alpha & M_\alpha & M_{\alpha\beta} & P_\alpha \end{bmatrix}^T \\
&= \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_\alpha & \sigma_{\alpha\beta} & \sigma_{\alpha z} & z\sigma_\alpha & z\sigma_{\alpha\beta} & z\sigma_{\alpha z} \end{bmatrix}^T \left(1 + \frac{z}{R_\alpha}\right) dz, \\
& \begin{bmatrix} N_\beta & N_{\beta\alpha} & Q_\beta & M_\beta & M_{\beta\alpha} & P_\beta \end{bmatrix}^T \\
&= \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_\beta & \sigma_{\beta\alpha} & \sigma_{\beta z} & z\sigma_\beta & z\sigma_{\beta\alpha} & z\sigma_{\beta z} \end{bmatrix}^T \left(1 + \frac{z}{R_\beta}\right) dz,
\end{aligned} \quad (6)$$

The superscript T stands for the transpose of a vector. In the above equations. The applied load per unit area on the middle surface of a shell is taken as

$$\bar{q} = q_\alpha \bar{e}_\alpha + q_\beta \bar{e}_\beta + q_z \bar{e}_z, \quad (7)$$

where the unit vectors $\bar{e}_\alpha$ and $\bar{e}_\beta$ are tangent to the principal axes and $\bar{e}_z$ is perpendicular to the shell surface, respectively. Let $\sigma_{0\alpha}$, $\sigma_{0\alpha\beta}$ and $\sigma_{0\alpha z}$ be the components of applied traction on the edges $\alpha = \text{constant}$ and $\sigma_{0\beta}$, $\sigma_{0\beta\alpha}$ and $\sigma_{0\beta z}$ be the components of applied traction on the edges $\beta = \text{constant}$. Using Hamilton's principle results in following equations of motion [5,6,7]

$$\begin{aligned}
& \frac{\partial(BN_\alpha)}{\partial \alpha} + \frac{\partial(AN_{\beta\alpha})}{\partial \beta} + N_{\alpha\beta} \frac{\partial A}{\partial \beta} - N_\beta \frac{\partial B}{\partial \alpha} + \frac{AB}{R_\alpha} Q_\alpha + \frac{AB}{R_{\alpha\beta}} Q_\beta + ABq_\alpha \\
&= AB\bar{I}_1 \ddot{u}_0 + AB\bar{I}_2 \ddot{\psi}_\alpha, \\
& \frac{\partial(AN_\beta)}{\partial \beta} + \frac{\partial(BN_{\alpha\beta})}{\partial \alpha} + N_{\beta\alpha} \frac{\partial B}{\partial \alpha} - N_\alpha \frac{\partial A}{\partial \beta} + \frac{AB}{R_\beta} Q_\beta + \frac{AB}{R_{\alpha\beta}} Q_\alpha + ABq_\beta \\
&= AB\bar{I}_1 \ddot{v}_0 + AB\bar{I}_2 \ddot{\psi}_\beta,
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial(BQ_\alpha)}{\partial \alpha} + \frac{\partial(AQ_\beta)}{\partial \beta} - AB \left(\frac{N_\alpha}{R_\alpha} + \frac{N_{\alpha\beta}}{R_{\alpha\beta}} + \frac{N_{\beta\alpha}}{R_{\beta\alpha}} + \frac{N_\beta}{R_\beta} \right) + ABq_z = AB\bar{I}_1 \ddot{w}_0, \\
& \frac{\partial(BM_\alpha)}{\partial \alpha} + \frac{\partial(AM_{\beta\alpha})}{\partial \beta} + M_{\alpha\beta} \frac{\partial A}{\partial \beta} - M_\beta \frac{\partial B}{\partial \alpha} - ABQ_\alpha + \frac{AB}{R_{\alpha\beta}} P_\beta \\
&= AB\bar{I}_2 \ddot{u}_0 + AB\bar{I}_3 \ddot{\psi}_\alpha, \\
& \frac{\partial(AM_\beta)}{\partial \beta} + \frac{\partial(BM_{\alpha\beta})}{\partial \alpha} + M_{\beta\alpha} \frac{\partial B}{\partial \alpha} - M_\alpha \frac{\partial A}{\partial \beta} - ABQ_\beta + \frac{AB}{R_{\alpha\beta}} P_\alpha \\
&= AB\bar{I}_2 \ddot{v}_0 + AB\bar{I}_3 \ddot{\psi}_\beta,
\end{aligned} \quad (8)$$

where in Eq. (8)

$$\bar{I}_i = \left(I_i + I_{i+1} \left(\frac{1}{R_\alpha} + \frac{1}{R_\beta} \right) + \frac{I_{i+2}}{R_\alpha R_\beta} \right), \quad i = 1, 2, 3, \quad (9)$$

where $[I_1, I_2, I_3, I_4, I_5] = \sum_{k=1}^N \int_{h_{k-1}}^{h_k} \rho^{(k)} [1, z, z^2, z^3, z^4] dz$. The stress resultant terms are shown in Figure. 2.

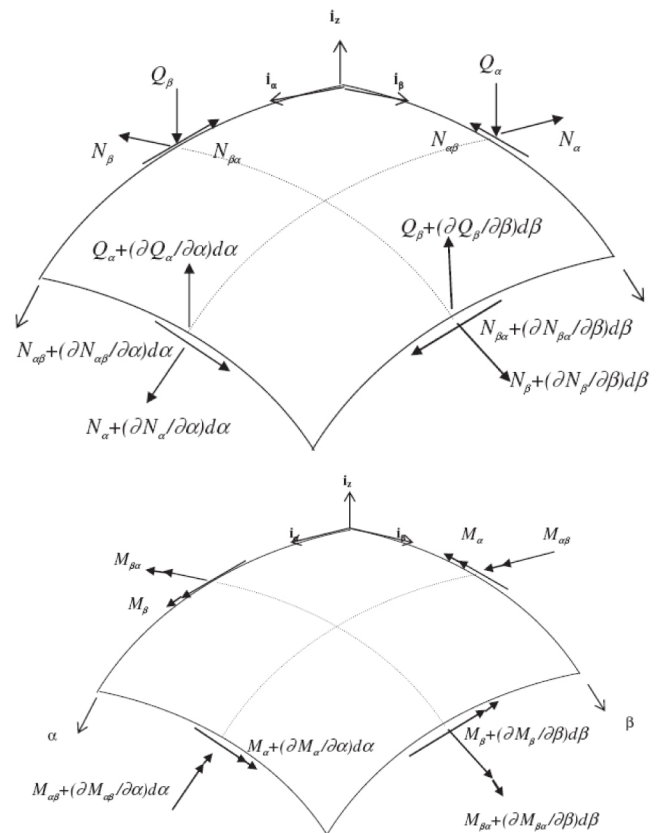


Figure 2. Force and moment resultants in shell coordinates

Also, the remaining terms in the Hamilton's principle functional lead to the natural and geometric boundary conditions for the shell. The boundary data on an edge $\alpha = \text{constant}$ are