

Vibration Analysis of Inclined Beams under Uniform Partially Distributed Moving Mass or Force Considering Coriolis and Centrifugal Forces using FEM

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ABSTRACT

This article has investigated dynamic response of simply supported beams carrying moving distributed mass or force. A one dimensional finite element based on the both the first shear deformation (FSDT) and the Classical Laminate beam (CLT) theories is assumed for beam model. A ten degree of freedom beam element for FSDT theory and six degree of freedom element for CLT theory is considered for the beam and the moving distributed mass. Combination of the element property matrices for the moving distributed mass or force and the associated overall property matrices for the beam itself determines the overall effective property matrices of the entire vibrating system. After deriving the governing equations of motion of the beam and the moving distributed mass, the corresponding equations of motion are integrated by applying the Newmark's time integration scheme to obtain the system responses in each time step. Analysis, in its general form, may well be applied to various boundary conditions. However, results from computer simulation are for simply supported beams. The numerical results of free vibration and moving force problems analysis of the beams are presented and, whenever possible, compared to the available analytical solution and other numerical results in order to demonstrate the accuracy of the present method. Effect of Coriolis and centrifugal forces induced by the moving distributed mass and also friction force between the beam and the moving distributed mass are investigated. Numerical results reveal that all above mentioned parameters have significant influence on both the vertical and the horizontal deflections of the inclined beam except the frictional force that can be neglected.

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INTRODUCTION

The dynamic behavior of a beam subjected to moving loads or masses has been studied in connection with machining process and design of railway trucks, bridges and flow of fluids inside pipes. Since the middle of the last century, when railway construction began, the problem of oscillation of bridges under travelling loads has interested engineers. Some of the initial investigations were carried out by Stokes [1] and Ayre et al. [2]. Early monograph include those by Inglis [3] and Hillerborg [4]. A more recent book by Fryba [5] includes analyses of moving masses on beams under different loading conditions. Esmaeilzadeh and Ghorashi [6, 7] have investigated the dynamic behavior of a beam traversed by uniformly and partially distributed moving masses. Mofid and shadnam [8], Cifuentes [9], Foda and Abduljabbar [10], Akin and mofid [11], Wu [12] have performed the dynamic analysis of inclined and beams due to

moving masses by means of various analytical or numerical methods. Dynamic response of eccentrically pre-stressed viscoelastic beam under moving harmonic loads using Lagrange equations has studied by Kocaturk and Simsek [13]. A mixed Ritz-DQ method is presented to study the dynamic behavior of Euler-Bernulli FGM beam under moving load by Khalili *et al.* [14]. Yas and Heshmati [15] have investigated dynamic response of functionally graded nanocomposite beam reinforced by randomly oriented carbon nanotube under action of moving load under assumptions of Timoshenko theory using finite element method. Convergence of the Galerkin method for the dynamic response of an elastic beam resting on a nonlinear foundation with viscose damping subjected to moving concentrated load has studied by Ding *et al.* [16]. Dynamic analysis of a composite laminated beam under moving concentrated load is studied by Khaya [17].

In the earlier papers the authors investigated the behavior of a beam carrying moving concentrated masses in which the inertial effect of the mass was taken into account. As an extension of that works, the present paper deal with the problem of vibration analysis of the inclined beams under the moving distributed mass with considering the Coriolis and the centrifugal forces of moving distributed mass. Some of its applications are travelling trains and flow of fluids inside pipes. Clearly, the point mass formulations can be considered as a special case of present study, if the distribution load length assumed to be small.

In the present paper the following assumptions are adopted: (1) the load moves at constant velocity and keeps in contacts with the beam at all times; (2) the computations are performed for the simply supported boundary conditions; (3) the friction force between beam and moving distributed mass is considered and it is assumed to be uniform distribution; (4) for the initial conditions, the beam is assumed to carry the whole load at the left-hand support.

MATHEMATICAL MODEL

Beam Model and its Finite Element Formulations

Beam Model

A simply supported isotropic beam of length L , width b and the thickness h is considered, with the coordinate system placed at the mid-plane of beam, as shown in Fig. 1a. $u(x, z, t)$ And $w(x, z, t)$ are the displacements in x - and z - directions, respectively, at any point. Formulations based on the FSDT theory are discussed in the next section.

The displacement field for the beam based on FSDT theory can be assumed as follows.

$$\begin{aligned} u(x, z, t) &= u_0(x, t) + z \phi_x(x, t) \\ w(x, z, t) &= w_0(x, t) \end{aligned} \quad (1)$$

where w_0 and u_0 are transverse and axial displacements of a point on the mid-plane in the z - and x -directions and ϕ_x is the rotation of the normal to the mid-plane about the x - axis. The strain -displacement relationships for the theory are given by

$$\begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \gamma_{xz}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \gamma_{xz}^{(1)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial w_0}{\partial x} + \phi_x \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ 0 \end{Bmatrix} \quad (2)$$

The governing equations were derived using the dynamic form of the principle of virtual displacements same as the equation (3), where the virtual strain energy δU , virtual work done by applied δV forces and the virtual kinetic energy δK are given by

$$\delta U =$$

$$b \int_0^L \left\{ \int_{-\frac{h}{2}}^{\frac{h}{2}} [\sigma_{xx} (\delta \varepsilon_{xx}^{(0)} + z \delta \varepsilon_{xx}^{(1)}) + \sigma_{xz} (\delta \gamma_{xz}^{(0)} + z \delta \gamma_{xz}^{(1)})] dz \right\} dx \quad (3)$$

$$\delta V = -b \int_0^L (q \delta w_0) dx + b \int_0^L f \delta u_0 dx \quad (4)$$

$$\delta K = b \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho_0 [(\dot{u}_0 + z \dot{\phi}_x)(\delta \dot{u}_0 + z \delta \dot{\phi}_x) + \dot{w}_0 \delta \dot{w}_0] dz dx \quad (5)$$

where q is resultant distributed force at the surface of the laminated beam, f is the axial force, σ_{xx} is normal stress in the beam and ρ_0 is density of the beam material. The equations of motion of the beam are obtained by substituting for δU , δV and δK into the virtual work statement, integrating through the thickness of beam and then integrating by parts to relieve generalized displacements and finally, setting coefficients of δu_0 , δw_0 and $\delta \phi_x$ to zero separately,

$$\frac{\partial N_{xx}}{\partial x} = I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2} + f \quad (6a)$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial}{\partial x} \left(N_{xx} \frac{\partial w_0}{\partial x} \right) = I_0 \frac{\partial^2 w_0}{\partial t^2} + q \quad (6b)$$

$$\frac{\partial M_{xx}}{\partial x} - Q_x = I_2 \frac{\partial^2 \phi_x}{\partial t^2} + I_1 \frac{\partial^2 u_0}{\partial t^2} \quad (6c)$$

where N_{xx} and M_{xx} are the force and moment resultants per unit length, Q_x is transverse shear force, I_i is mass inertia and f and q are the moving distributed axial and traverse moving loads respectively as follows.

$$N_{xx} = A_{11} \left(\frac{\partial u_0}{\partial x} \right) + B_{11} \frac{\partial \phi_x}{\partial x} \quad (7a)$$

$$M_{xx} = B_{11} \left(\frac{\partial w_0}{\partial x} \right) + D_{11} \frac{\partial \phi_x}{\partial x} \quad (7b)$$

$$Q_x = K \left(A_{55} \left(\frac{\partial w_0}{\partial x} + \phi_x \right) \right) \quad (7c)$$

$$A_{11} = \frac{Eh}{1-\nu^2}, \quad D_{11} = \frac{Eh^3}{12(1-\nu^2)}, \quad A_{55} = Gh$$

$$(I_0, I_1, I_2) = \sum_{k=1}^N \int_{z_k}^{z_{k+1}} (1, z, z^2) \rho_0 dz \quad (8)$$

$$f = m_l g \frac{1}{\epsilon} \left[H \left(x - \xi + \frac{\epsilon}{2} \right) - H \left(x - \xi - \frac{\epsilon}{2} \right) \right] \sin \theta \quad (9)$$

$$q(x, t) = -\frac{1}{\epsilon} \left[m_l g + m_l \frac{d^2 w_0}{dt^2} \right] \left[H \left(x - \xi + \frac{\epsilon}{2} \right) - H \left(x - \xi - \frac{\epsilon}{2} \right) \right] \cos \theta \quad (10)$$

$$\frac{d^2 w_0}{dt^2} = \frac{\partial^2 w_0}{\partial t^2} + 2v \frac{\partial^2 w_0}{\partial x \partial t} + v^2 \frac{\partial^2 w_0}{\partial x^2} \quad (11)$$

$$\xi = vt + \frac{\epsilon}{2} \quad (12)$$

where v is velocity, m_l is mass, ϵ is load length, g is acceleration of gravity, θ incline angle. A_{11} and A_{55} are extensional stiffnesses and D_{11} is bending stiffness of lamina that here has one isotropic layer. $H(x)$ is Heaviside unit function that can be expressed as,

$$H(x) = \begin{cases} 0, & x < 0 \\ 1, & x > 0 \end{cases} \quad (13)$$

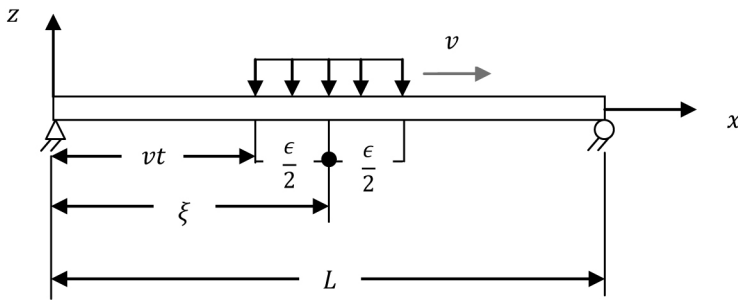


Figure 1. The mathematical model of the problem with showing parameters

Finite Element Formulations of the Beam

The weak forms of the equations (6a), (6b) and (6c) are obtained by multiplying them with δu_0 , δw_0 , and $\delta \phi_x$ and integrating over the length of element. Weak forms are brought as follow.

$$\int_0^\ell \left[\left(A_{11} \frac{\partial u_0}{\partial x} + B_{11} \frac{\partial \phi_x}{\partial x} \right) \frac{\partial \delta u_0}{\partial x} + \left(I_0 \frac{\partial^2 u_0}{\partial t^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2} + f \right) \delta u_0 \right] dx \quad (14a)$$

$$\int_0^\ell \left[K A_{55} \left(\frac{\partial w_0}{\partial x} + \phi_x \right) \frac{\partial \delta w_0}{\partial x} + \left(I_0 \frac{\partial^2 w_0}{\partial t^2} + q \right) \delta w_0 \right] dx \quad (14b)$$

$$\int_0^\ell \left[\left(B_{11} \frac{\partial u_0}{\partial x} + D_{11} \frac{\partial \phi_x}{\partial x} \right) \frac{\partial \delta \phi_x}{\partial x} + \left(K A_{55} \left(\frac{\partial w_0}{\partial x} + \phi_x \right) + I_2 \frac{\partial^2 \phi_x}{\partial t^2} + I_1 \frac{\partial^2 u_0}{\partial t^2} \right) \delta \phi_x \right] dx \quad (14c)$$

For linear element dependent variables u_0 , w_0 and ϕ_x can be approximated as follow.

$$u_0 = \sum_{j=1}^2 \psi_j^{(1)}(x) u_j^e(t) \quad (15a)$$

$$w_0 = \sum_{j=1}^4 \psi_j^{(2)}(x) w_j^e(t) \quad (15b)$$

$$\phi_x = \sum_{j=1}^4 \psi_j^{(3)}(x) \phi_j^e(t) \quad (15c)$$

where $\psi_j^{(1)}$, is linear Lagrangian interpolation function, $\psi_j^{(2)}$ and $\psi_j^{(3)}$ are cubic Hermitian interpolation function, the element used in this study for the FSDT theory has ten degree of freedom.

Moving Distributed Mass Model and its Finite Element Formulations

Moving Distributed Mass Model

Figure 2 shows a moving distributed mass on the inclined beam. If incline angle be zero, it changes to a straight beam (see Figure 1). At any time, moving distributed mass is on one or more beam element that depends on the length of the beam element. Therefore mass, stiffness, damping and force matrices of the moving distributed mass are function of time. The interaction forces between the moving distributed mass and the beam in x and y directions are respectively given by,

$$F_x = m_l \ddot{u}_x \quad (16a)$$

$$F_y = m_l (\ddot{u}_y + 2v \dot{u}_y' + v^2 u_y'') \quad (16b)$$

$$u_{\bar{x}} = u_x \cos \theta - u_y \sin \theta \quad (16c)$$

$$u_{\bar{y}} = u_x \sin \theta + u_y \cos \theta \quad (16d)$$

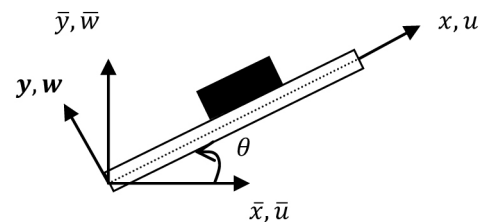


Figure 2. shows the geometry of the problem and local and global relationship of displacements.