

Yawing Angular Misalignment Provides Positive Damping to Suppress Frictional Vibration: Basic Applicability to Disc Brake Systems

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ABSTRACT

A novel method for suppressing frictional vibration in sliding systems without using additional devices (e.g., dampers or actuators) is theoretically proposed. This method is based on the principle that yawing angular misalignment provides positive damping to sliding systems. Even if frictional vibration occurs due to a negative slope of the friction versus velocity characteristic, it can be suppressed by applying a misalignment angle ϕ that is larger than the critical misalignment angle ϕ_{cr} . To examine the basic applicability of this method to disc brake systems, a braking test at a constant normal load was simulated numerically. It was found that yawing angular misalignment can not only suppress frictional vibration but also shorten the braking time as a secondary effect of suppressing frictional vibration.

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INTRODUCTION

In passenger cars, there are many mechanical parts employing solid surfaces that contact and move each other under normal loading, such as bearings, slide guides, windshield wipers, clutches, transmissions, and brakes. Since they are designed under the assumption that they run smoothly, vibrations induced by friction (i.e., frictional vibrations) are always problems to be solved.

For examining the essence of frictional vibration, a single degree-of-freedom (SDOF) sliding system is commonly used [1,2,3,4,5,6,7,8,9,10,11,12]; an object, supported by a spring attached to a stationary wall, is subjected to a frictional force from a floor moving at a constant speed. By applying various friction models to this system, many aspects of frictional vibration have been discussed and clarified. However, in conventional SDOF models, three types of velocities (i.e., the "driving velocity" of the floor, "moving velocity" of the object, and "relative velocity" between the object and floor) are implicitly limited to be in the same straight line, although they are inherently vectors.

In this study, a novel extension is added to the simplest SDOF sliding system. A yawing angular misalignment is applied intentionally between the driving velocity and moving velocity. An analysis reveals that a non-zero misalignment angle always provides positive damping to the system, meaning that even if frictional vibration occurs due to a negative slope of the friction versus velocity characteristic, it can be suppressed by applying yawing angular misalignment.

THEORY

Theoretical Model

Figure 1 shows a theoretical model: a SDOF sliding system having a yawing angular misalignment. A ball (mass: m) is supported by a spring (stiffness: k) attached to a stationary wall. The motion of the ball is limited in the x direction. The ball is in contact with a plate moving at a constant driving velocity V under a constant normal load W . As the top view shows, V makes an angle ϕ with the x -axis, where ϕ is referred to as the misalignment angle.

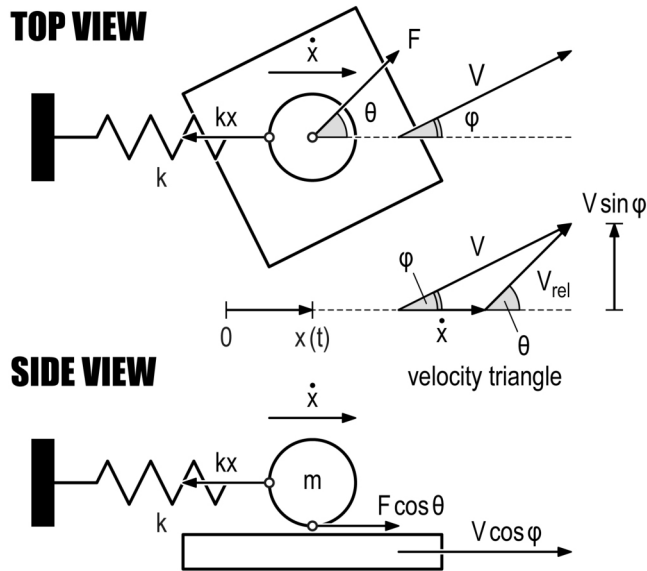


Figure 1. Theoretical model: SDOF sliding system having yawing angular misalignment

Let $x(t)$ be the position of the ball (or elongation of the spring) at time t , where the origin $x = 0$ is the position of the ball when the spring has its natural length. The moving velocity of the ball is denoted by $\dot{x}$, where $(\cdot)$ denotes the time derivative. The relative velocity between the ball and plate is denoted by V_{rel} and defined as follows:

$$V_{\text{rel}} = V - \dot{x} \quad (1)$$

The angle that V_{rel} makes with the x -axis is denoted by θ . Since frictional force generally acts so as to diminish the magnitude of relative velocity, the frictional force $\mathbf{F}$ acting on the ball is in the same direction as V_{rel} . Therefore, the angle that $\mathbf{F}$ makes with the x -axis is also θ . It should be noted that when $\varphi \neq 0$, $\mathbf{F}$ is not static friction but always kinetic friction since V_{rel} always has a non-zero lateral slip component $V \sin \varphi$. It should be also noted that when $\varphi \neq 0$, the direction of $\mathbf{F}$ is changed by change in ball motion.

Governing Equations

Let the magnitude of $\mathbf{F}$ be a function of the magnitude of V_{rel} (i.e., $F = F(V_{\text{rel}})$). Then, the equation of motion of the ball in the x direction is

$$m\ddot{x} + kx = F(V_{\text{rel}})\cos\theta \quad (2)$$

where, from the velocity triangle in Figure 1,

$$V_{\text{rel}} = \sqrt{V^2 - 2V\dot{x}\cos\varphi + \dot{x}^2} \quad (3)$$

$$\cos\theta = (V\cos\varphi - \dot{x})/V_{\text{rel}} \quad (4)$$

Stability of Equilibrium Point

From the balance between the spring force and frictional force in the x direction, the equilibrium point x_{eq} is determined by

$$x = x_{\text{eq}} \equiv F(V)\cos\varphi/k \quad (5)$$

To examine the stability of the equilibrium point, we introduce a disturbance $\tilde{x}$ from x_{eq}

$$x = x_{\text{eq}} + \tilde{x} \quad (6)$$

Using the equations above and linearizing the equation of motion under the condition $\dot{\tilde{x}}/V \ll 1$, we obtain

$$m\ddot{\tilde{x}} + (c_1 + c_2)\dot{\tilde{x}} + k\tilde{x} = 0 \quad (7)$$

The coefficients in the left-hand side of Equation (7) are

$$c_1 = F'(V)\cos^2\varphi \quad (8)$$

$$c_2 = (F(V)/V)\sin^2\varphi \quad (9)$$

where $(\cdot)$ denotes the derivative with respect to V_{rel} . The coefficient c_1 is the viscous damping generated due to the friction versus velocity characteristic $F(V_{\text{rel}})$. The sign of c_1 depends on the slope of $F(V_{\text{rel}})$. The coefficient c_2 is the viscous damping generated due to the angular misalignment φ .

The sign of c_2 is positive if $\varphi \neq 0$, meaning that angular misalignment provides positive damping to the sliding system.

Considering that the equilibrium point is stable when $c_1 + c_2 > 0$, we obtain the stability condition:

$$\tan^2\varphi > -VF'(V)/F(V) \quad (10)$$

When $F'(V) > 0$, Equation (10) is always true. However, when $F'(V) < 0$, Equation (10) is true if

$$\varphi > \varphi_{\text{cr}} \equiv \tan^{-1}\sqrt{-VF'(V)/F(V)} \quad (11)$$

where φ_{cr} denotes the critical misalignment angle. This means that even if the contact has a negative slope of its friction versus velocity characteristic, and therefore frictional vibration occurs when $\varphi = 0$, it can be suppressed by applying a sufficiently large misalignment angle $\varphi > \varphi_{\text{cr}}$.