

Application of Dynamic Phasor Concept in Modeling Aircraft Electrical Power Systems

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ABSTRACT

As future commercial aircraft incorporates more EMAs, the aircraft electrical power system architecture will become a complex electrical distribution system with increased numbers of power electronic converters (PEC) and electrical loads. The overall system performance and the power management for on-board electrical loads are therefore key issues that need to be addressed. In order to understand these issues and identify high pay-off technologies that would enable a major improvement of the overall system performance, it is necessary to study the aircraft EPS at the system level. Due to the switching behaviour of power electronic devices, it is very time-consuming and even impractical to simulate a large-scale EPS with some non-linear and time-varying models. The dynamic phasor (DP) technique is one way to solve that problem. This DP technique is naturally a frequency-domain analysis method and has been successfully applied in modeling of electromagnetic machines, flexible AC transmission systems etc. our previous work has extended this technique to modeling of controlled rectifier units (CRU), auto-transformer rectifier units (ATRU) etc. This paper is to summarize the DP model library developed in the University of Nottingham for accelerated simulation studies of the aircraft EPS. The three-phase models (ABC models) are used for comparative studies. The DP modeling technique is compared with the ABC model using the simulation of a twin-generator aircraft EPS, which is based on the more open electrical technology (MOET) large aircraft EPS architecture. The switching behaviour of ATRU is included in the ABC model for the EPS studies. However, the switching behaviour of PWM converters is not considered due to computation convergence problems. The efficiency and the accuracy of the DP model are demonstrated through comparison of the simulation results under normal operation conditions.

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INTRODUCTION

The more-electric aircraft (MEA) has become a dominant trend for the next-generation aircraft [1]. The recent advance of power electronics, electrical drives and modern control theory makes it possible to replace many functions, which are conventionally managed by hydraulic, pneumatic and mechanical power, with electrical power driven devices [2], offering reduced overall system weight, as well as increased reliability, efficiency and performance of the aircraft. Compared with conventional aircraft, many new more-electric loads will emerge in the electrical power system (EPS), including electromechanical (EMA) or electrohydrostatic (EHA) actuators for flight surface actuation, wing ice protection systems, landing gears etc. The future EPS may take many forms: AC, DC, hybrid, frequency-wild, variable voltage, together with the possibility

of novel connectivity topologies. To address the stability, availability and capability issues as well as to assess the performance of the power quality and transient behavior, extensive simulation work is required to develop the EPS architectures.

Due to the switching behaviour of power electronic devices and its resulted higher harmonics, it is very time-consuming and even impractical to simulate a large-scale EPS with some non-linear and time-varying models. Considering the system dynamic frequency is normally much lower than that of higher harmonics, it is a common practice to neglect these higher harmonics when studying system dynamic behaviour. The average modelling technique, which removes the higher harmonics by averaging the variable during one fundamental period, has been widely used to model the EPS recently. In these average models, the variables are transformed from the three-phase *abc* frame to a

synchronous rotating dq frame (DQ0 models). The DQ0 model demonstrates good performance and high efficiency under balanced conditions [3]. This model, however, becomes very slow under unbalanced or faulty conditions. This is due to the second harmonic present in the DQ0 model under unbalanced conditions.

The DP technique can potentially overcome that problem and will enable fast simulations under both balanced and unbalanced conditions. The DPs are in nature some time-varying Fourier series coefficients. Therefore the DP technique is a frequency-domain analysis technique. Truncating the unimportant higher order harmonics and only considering the significant components, DP models are capable of retaining the dominant dynamic features of the EPS and suitable for transient stability studies. The slow variation of DPs allows for larger simulation time steps and results in faster simulations.

The DP method has been firstly proposed in [4] studying a series resonant converter in the 1990s. After that, it has been successfully used to analyze synchronous machines, induction machines [5], doubly-fed induction generator (DFIG) [6], flexible AC transmission system (FACTS) devices such as unified series-shunt compensator (USSC), and high-voltage direct current (HVDC) transmission system under balanced operating conditions [7,8,9,10]. When considering system unbalanced conditions, the DP modeling has been implemented with positive and negative sequences theory and the DP models for synchronous generators, DFIGs and transmission lines have been established [6, 11]. Comparison of three modeling techniques, i.e. time-domain models in the abc and the dq0 frames, DP models, has been studied in [12] where the efficiency of DP models under unbalanced conditions has been demonstrated.

This paper is to summarize the DP modeling activities in previous years in our group. A DP model library has been established. This library contains most of the essential elements in the EPS, including the synchronous generator [12], transmission lines [12], the PWM controlled rectifier [13], the transformer rectifier unit [14] etc. This paper is to use the developed DP models to study an EPS based on the More Open Electrical Technologies (MOET) large aircraft electrical system architecture from Airbus France (document WP3.11 Architecture V0 [15]). The same architecture of EPS is simulated using ABC models for comparison studies. The efficiency and accuracy of the DP model is illustrated by comparing with the corresponding ABC model.

THE STUDIED EPS

This paper will focus on the study of a twin-generator aircraft EPS as shown in Figure 1. This is a large aircraft hybrid frequency-wild electrical power system (EPS) based on an Airbus-A380 EPS architecture within the European MOET project. The EPS is supplied by two 250kVA synchronous generators (SG). Each generator is controlled by its generator control unit (GCU) to maintain 230/400Vrms at

the terminals. Since the generator is directly driven by the aircraft engine, the power system is frequency-wild (360-900Hz) reflecting recent industry trends in moving away from constant frequency 400Hz power. In the EPS shown Figure 1, there are two primary electrical power distribution centers (PEPDC1 and PEPDC2) and one emergency electrical power distribution center (EEPDC). The PEPDCs are used to supply the primary electrical loads such as the wing icing protection system (WIPS), the environment condition system (ECS) and other loads. The EEPDC is assigned to supply the essential electrical loads for the flight surface control actuators. These essential electrical loads require no loss of power for supplying the vital electrical loads under emergency.

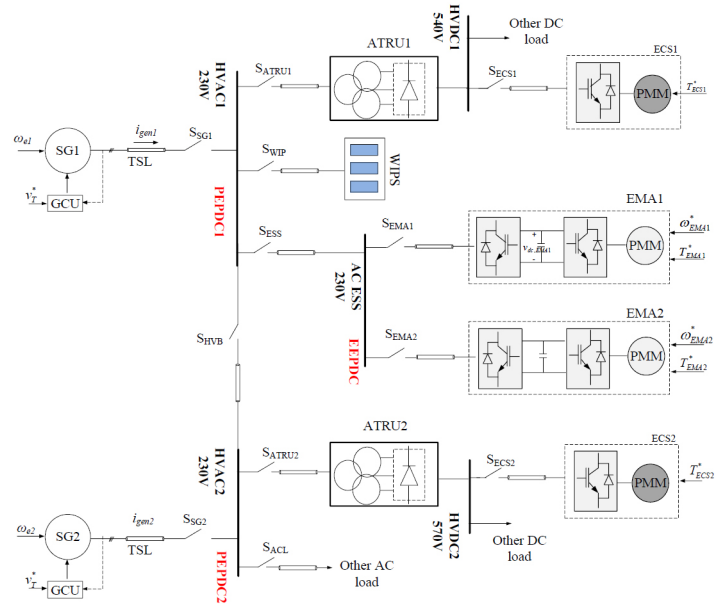


Figure 1. The EPS of the twin-generator aircraft

The PEPDC1 consists of a 230VAC bus (HVAC1) fed by the main generator and a ± 270 VDC bus (HVDC1) produced by a 150kVA auto-transformer rectifier unit (ATRU1). The HVAC1 bus supplies power to a 60kW environment control system (ECS1) through the ATRU1. The HVAC1 bus also feeds two actuators rated at 5kW. These EMA operation depends on the corresponding demands for the actuator speed ω^* and T^* . The HVDC1 also supplies some other DC loads such as DC/DC converters, battery etc. These loads are represented by some resistors in our model. Similarly, the PEPDC2 consists of a 230VAC bus (HVAC2) and a ± 270 VDC bus (HVDC2) supplied by the ATRU2. The ECS2 is fed by ATRU2.

DYNAMIC PHASOR CONCEPT

The dynamic phasors concept [4] is based on the fact that a time-domain waveform $x(\tau)$ can be approximated with arbitrary precision in the interval $\tau \in (t - T, t]$ by a following Fourier series:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k(t) e^{jk\omega_s t} \quad (1)$$

where $\omega_s = 2\pi/T$ and T the fundamental period of the waveform. $X_k(t)$ is the k^{th} Fourier coefficient in a complex form referred to as a “dynamic phasor” (DP) and determined as follows:

$$X_k(t) = \frac{1}{T} \int_{t-T}^t x(\tau) e^{-jk\omega_s \tau} d\tau = \langle x \rangle_k(t) \quad (2)$$

where k is the DP index. If only consider the term with DP index $k=0$, the DP method is the same as the state-space averaging method [16]. Also, if a state variable only has an oscillating form similar to a sine wave, the DP index can be chosen at $K=\{-1,1\}$. In addition, if a state variable has a dc coordinate as well as an oscillating form, the DP index can be set at $K=\{-1,0,1\}$. For a general waveform, the required accuracy of such approximation can be achieved by appropriate selection of a DP set.

A key factor in developing dynamic models based on DP is the relation between the derivatives of the variable $x(\tau)$ and the derivatives of the k^{th} Fourier coefficient given as:

$$\frac{dX_k(t)}{dt} = \left\langle \frac{dx}{dt} \right\rangle_k(t) - jk\omega_s X_k(t) \quad (3)$$

This can be verified using (1), (2), and may be used in evaluating the k^{th} phasor of time-domain model. Another important property of DP is that the k^{th} phasor of a product of two time-domain variables can be obtained via the convolution of corresponding dynamic phasors:

$$\langle xy \rangle_k = \sum_i \langle x \rangle_{k-i} \langle y \rangle_i \quad (4)$$

The properties (3) and (4) play a key role when transforming the time-domain models into DP domain. Algebraic manipulations in this paper will also exploit the following property of real functions $x(\tau)$:

$$X_{-k}(t) = X_k^*(t) \quad (5)$$

where the notation $*$ denotes a complex conjugate. This property allows us to only use the positive DPs in modeling time-domain systems.

DEVELOPMENT OF DYNAMIC PHASOR MODEL LIBRARY

In this section, we will briefly describe the development of some essential EPS components in our DP model library. These DP models are focused on the functional modeling level. In this level, the model covers the dynamics up to $1/3^{rd}$

of the base EPS frequency and for aircraft EPS modeling, the bandwidth is nominally about 150Hz [17].

Transmission Lines

In the EPS, the transmission line is represented with an RLC circuit as shown in Figure 2. The application of DP concept in modeling this RLC circuit will be demonstrated in this section. This modeling concept will be extended to modeling other EPS elements introduced in this paper. The mathematic model of the RLC circuit is written as:

$$C \frac{dv_c}{dt} = i_L - i_{out} \quad (6)$$

$$L \frac{di_L}{dt} = v_{in} - Ri_L - v_c \quad (7)$$

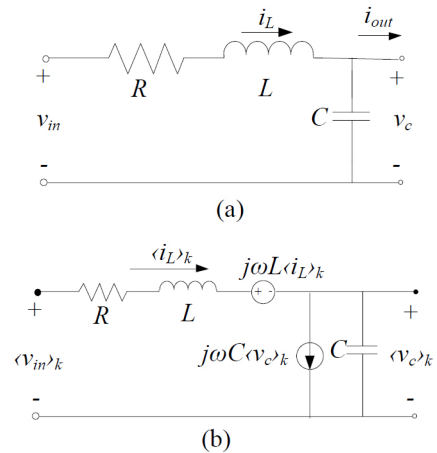


Figure 2. Transformation of RLC circuits from time domain to DP domain. (a) time-domain model; (b) DP model

Using the properties of DPs (3), (4), (5) yields the DP model of the RLC circuit:

$$L \frac{d\langle i_L \rangle_0}{dt} = \langle v_{in} \rangle_0 - R\langle i_L \rangle_0 - \langle v_c \rangle_0 \quad (8)$$

$$L \frac{d\langle i_L \rangle_k^R}{dt} = \langle v_{in} \rangle_k^R - R\langle i_L \rangle_k^R - \langle v_c \rangle_k^R + k\omega L \langle i_L \rangle_k^I \quad (9)$$

$$L \frac{d\langle i_L \rangle_k^I}{dt} = \langle v_{in} \rangle_k^I - R\langle i_L \rangle_k^I - \langle v_c \rangle_k^I - k\omega L \langle i_L \rangle_k^R \quad (10)$$

$$C \frac{d\langle v_c \rangle_0}{dt} = \langle i_L \rangle_0 - \langle i_{out} \rangle_0$$

$$C \frac{d\langle v_c \rangle_k^R}{dt} = \langle i_L \rangle_k^R - \langle i_{out} \rangle_k^R - k\omega C \langle v_c \rangle_k^I \quad (12)$$

$$C \frac{d\langle v_c \rangle_k^I}{dt} = \langle i_L \rangle_k^I - \langle i_{out} \rangle_k^I + k\omega C \langle v_c \rangle_k^R \quad (13)$$

It can be seen that the order of DP model, (8), (9), (10), (11), (12), (13), is higher than its corresponding time-domain model (6)-(7). However, the variables in the DP model vary much slower than those in the time-domain model. This will be illustrated in the simulation result later.

Controlled Synchronous Generators

The primary electrical power source in the EPS shown in Figure 1 is the synchronous generator (SG) with its excitation voltage being controlled by a generator control unit (GCU). The DP modeling of the synchronous generator is based on its time-domain model introduced in [18]. Using the DP transformation on the time-domain voltage and flux equations gives:

$$\langle \mathbf{v}_{abcs} \rangle_k = -\mathbf{r}_s \langle \mathbf{i}_{abcs} \rangle_k + \frac{d\langle \lambda_{abcs} \rangle_k}{dt} + j\omega \langle \lambda_{abcs} \rangle_k \quad (14)$$

$$\langle \mathbf{v}_{dqr} \rangle_k = \mathbf{r}_r \langle \mathbf{i}_{dqr} \rangle_k + \frac{d\langle \lambda_{dqr} \rangle_k}{dt} + j\omega \langle \lambda_{dqr} \rangle_k \quad (15)$$

$$\langle \lambda_{abcs} \rangle_k = -\sum_m \langle \mathbf{L}_{ss} \rangle_m \langle \mathbf{i}_{abcs} \rangle_{k-m} + \sum_n \langle \mathbf{L}_{sr} \rangle_n \langle \mathbf{i}_{dqr} \rangle_{k-n} \quad (16)$$

$$\langle \lambda_{dqr} \rangle_k = \sum_m \langle \mathbf{L}_{rs} \rangle_m \langle \mathbf{i}_{abcs} \rangle_{k-m} + \sum_n \langle \mathbf{L}_{rr} \rangle_n \langle \mathbf{i}_{dqr} \rangle_{k-n} \quad (17)$$

where the general vector

$$\mathbf{x}_{abcs} = [x_{as} \ x_{bs} \ x_{cs}]^T \quad (18)$$

$$\mathbf{x}_{dqr} = [x_{fd} \ x_{kd} \ x_{kq}]^T \quad (19)$$

The s and r subscripts denote variables associated with the stator and rotor windings respectively. The symbol $\mathbf{L}_{ss}$ denotes the stator self-inductance matrix; $\mathbf{L}_{rr}$ is the rotor self-inductance; $\mathbf{L}_{rs}$ and $\mathbf{L}_{rs}$ are the mutual-inductance matrix. These matrices are given in the Appendix.

The DP model of GCU controlled SG is shown in Figure 3. The DP model of the GCU raises two issues: the PI controller and the DP expression of the magnitude of the voltage vector $|\vec{v}|$. The state-space equation for the PI controller in the time domain is:

$$\dot{x} = k_i u \quad (20a)$$

$$y = k_p u + x \quad (20b)$$

where u is the input, x is the state variable, and k_p and k_i are the proportional and integral gains correspondingly. This equation can be converted into dynamic phasors as:

$$\frac{d\langle x \rangle_k}{dt} = k_i \langle u \rangle_k - jk\omega \langle x \rangle_k \quad (21a)$$

$$\langle y \rangle_k = k_p \langle u \rangle_k + \langle x \rangle_k \quad (21b)$$

The DP form of $|\vec{v}|$ has been discussed in our previous publications [19] and is briefly introduced here. The main idea is to express the voltage vector $|\vec{v}|$ using DPs $\langle v_{abc} \rangle_1$ with the help of $\langle v_{dq} \rangle_{0,2}$.

The voltage vector is defined as:

$$\vec{v} = 2/3 (v_a + v_b e^{j2\pi/3} + v_c e^{-j2\pi/3}) \quad (22)$$

The current vector is defined in the same way. In the rotating dq frame, the voltage vector in (4-1) can be expressed as:

$$\vec{v} e^{-j\omega t} = v_d + jv_q = \{V_{d0} + jV_{q0}\} + e^{-2j\omega t} \{V_{d2} + jV_{q2}\} \quad (23)$$

The first term on the right side of (23) arises from the positive sequence with no harmonics and the real and imaginary part represents the DC components of v_d and v_q respectively. The second term arises from the negative sequence and represents the second harmonic of v_d and v_q . Thus the DP index set for $\langle v_d \rangle_k$ and $\langle v_q \rangle_k$ should be chosen at $K=\{0,2\}$. The variables V_{d0} , V_{q0} , V_{d2} and V_{q2} can be calculated as follows:

$$V_{d0} = \frac{2}{3} \text{Re}[\langle v_a \rangle_1 + \langle v_b \rangle_1 e^{j2\pi/3} + \langle v_c \rangle_1 e^{-j2\pi/3}] \quad (24a)$$

$$V_{q0} = \frac{2}{3} \text{Im}[\langle v_a \rangle_1 + \langle v_b \rangle_1 e^{j2\pi/3} + \langle v_c \rangle_1 e^{-j2\pi/3}] \quad (24b)$$

$$V_{d2} = \frac{2}{3} \text{Re}[\langle v_a \rangle_1^* + \langle v_b \rangle_1^* e^{j2\pi/3} + \langle v_c \rangle_1^* e^{-j2\pi/3}] \quad (24c)$$

$$V_{q2} = \frac{2}{3} \text{Im}[\langle v_a \rangle_1^* + \langle v_b \rangle_1^* e^{j2\pi/3} + \langle v_c \rangle_1^* e^{-j2\pi/3}] \quad (24d)$$