

A Reduced Order Model for the Aeroelastic Analysis of Flexible Wings

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ABSTRACT

The aeroelastic design of highly flexible wings, made of extremely light structures yet still capable of carrying a considerable amount of non-structural weights, requires significant effort. The complexity involved in such design demands for simplified mathematical tools based on appropriate reduced order models capable of predicting the accurate aeroelastic behaviour. The model presented in this paper is based on a consistent nonlinear beam model, capable of simulating the unconventional aeroelastic behaviour of flexible composite wings. The partial differential equations describing the wing dynamics are reduced to a dimensionless form in terms of three ordinary differential equations using a discretization technique, along with Galerkin's method. Within this approach the nonlinear structural model an unsteady indicial based aerodynamic model with dynamic stall are coupled. Only three degrees of freedom in edgewise, flapwise, and torsion, are needed to describe efficiently the dynamics of the wing and to evaluate the sensitivity to system parameters, such as stiffness ratio, aspect ratio, and root angle of attack. Interesting design indicators will be highlighted. In addition to analytical results, a wind tunnel test article will be introduced to assess the validity of the proposed model.

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INTRODUCTION

Potential aeroelastic instabilities usually constrain the flight envelope of an aircraft and thus they have to be considered during the design process. Their evaluation should be done with respect to its consequence on the aircraft wing structural integrity and the airworthiness requirement by specific FAR or MIL regulations. In the design of innovative aircraft wings, the structure usually needs to be extremely lightweight and capable of carrying a considerable amount of non-structural weight. Usually, for low drag (or thrust required) the aircraft must have a high aspect ratio. All of these factors clearly imply that the resulting aircraft is highly flexible, forcing the designer to deal with specific phenomena not usually considered in classical aircraft definition and proposing different design procedures and associated indicators. Moderate static structural deflections occurring during standard flight conditions could severely modify their aeroelastic behaviour (for example reducing their flutter speed). Analyses procedures for more "conventional"

aeroelastic phenomena, such as wing flutter, also need to be revisited for very flexible aircraft. In general, the sensitivity of the flutter characteristics of the aircraft to the reference geometry can be quite important, even for relatively small deformations.

Aeroelastic stability studies need to be performed about the trimmed aircraft configuration, which will be different for each flight condition. In addition, geometrically nonlinear structural effects imply the presence of significant in- and out-of-plane wing bending displacements, the first ones are usually negligible with linear structural models. The investigation of the correct structural behaviour of such kind of configurations points out specific design indicators and important parameter to deal with. In the proximity of the critical speeds, limit cycle oscillations may also occur. Since the system incorporates structural nonlinearities, as the amplitude of oscillations grows, additional non-linear stiffness is produced. Thus, the vibrations could either grow to infinity or eventually be constrained to a bounded oscillation, into the so called Limit Cycle Oscillation mode

(LCO). The nature of the LCO will provide important information on the behavior of the aeroelastic system in the vicinity of the flutter boundary. Patil et al. studied the effects of geometric nonlinearities arising from large deformation on the aeroelasticity of high aspect-ratio wings [1]. Due to wing deformation the dynamic aeroelastic characteristics were found to be significantly altered and dependent on the relative values of the structural frequencies. The increase in deflection resulted in a significant decrease in the flutter speed. The predominant cause of this decrease in flutter speed was shown to be steady-state wing curvature under loading, which was also found to decrease the effective lift generated by the wing in the vertical direction. This effect could be cancelled out by pre-curving the wing to give an effectively straight wing under nominal flight loads. Similarly Frulla showed that an improved flutter characteristic can be obtained by increasing the in-plane bending stiffness [2]. A follow-up paper by Patil and Hodges looked at the effects of aerodynamic and structural nonlinearities on aeroelastic behavior [16]. Static structural deflection was captured accurately by linear structural models, though the effect of the deflection on the dynamics was not accurately modeled. A model that accounts for the steady state structural deformation and uses a two-dimensional unsteady loads model, as opposed to a non-planar three-dimensional aerodynamics representation of the loads, was found to accurately predict the aeroelastic behavior of the high aspect-ratio wings. Patil et al. also developed a model for studying nonlinear aeroelasticity and LCOs and applied it to the Goland's wing [3]. The wing structure was modeled using a geometrically exact nonlinear beam theory, and the aerodynamics was modeled using an unsteady state space model augmented for compressibility and dynamic stall effects. LCOs due to stall and geometric stiffening effects appeared above speeds where linearized stability analysis predicted instabilities, and for finite perturbations from stable equilibrium points. Similar investigations on LCOs were carried out on a high-aspect ratio configuration. The dependence of post-flutter LCOs on tip displacement and pre-flutter LCOs on large perturbations were investigated. The first torsion and edge-wise bending mode were found to couple with increasing flap-wise bending deformation. The characteristics of LCOs were seen to be dependent on speed and not dependent on tip displacement. Results obtained by Tang and Dowell showed a similar dependence of flutter speed and nonlinear aeroelastic response on static aeroelastic deformation [4]. LCOs were observed due to aerodynamic effects even in the absence of structural nonlinearities. Tang and Dowell presented results from experimental and theoretical studies on the aeroelasticity of high aspect-ratio wings [4]. Structural equations, modeled using nonlinear beam theory, were combined with the ONERA stall model to generate theoretical results [5]. Experimental and theoretical results showed good agreement on the static aeroelastic response, flutter onset velocity and LCO frequency and mode shape. For the model used in the experiments, aerodynamic

nonlinearities were found to be the cause of LCOs and the hysteresis observed in the LCO amplitude vs. flow velocity graph. Harmonic balance method was used and showed good agreement with experimental data on the static aeroelastic response, flutter onset velocity and LCO frequency. Frulla et al. modeled the nonlinear aeroelastic characteristics of a high aspect ratio wing using a second-order geometrically-exact nonlinear beam formulation and an unsteady aerodynamic model based on Wagner's indicial functions [6]. Computational results were found to be similar as those presented in [3] and [5]. Wind tunnel experiments found LCOs to exhibit hysteresis and remain stable even when the wind speed was decreased below critical speed. The flutter speed measured in the wind tunnel showed good correlation with computational results obtained for the statically deformed beam.

During preliminary design simplified mathematical models, based on appropriate reduction techniques, are needed in order to describe the correct aeroelastic behaviour. The model presented in the paper is based on a nonlinear Euler-Bernoulli composite beam theory and coupled with an unsteady aerodynamic model for an incompressible flow field, based on the Wagner aerodynamic indicial function. The effect of dynamic stall and trailing edge flow separation is also considered in this study. Using a Galerkin procedure a three degrees of freedom in edgewise, flapwise, and torsion is derived and used to compare the flutter velocity computed in a non-linear equilibrium condition" (NLFS) to the classical flutter analysis based on a linear equilibrium condition (LFS). The effect of main non-dimensional aeroelastic parameters is shown and main design parameters identified by a parametric analysis. In addition to analytical results, a wind tunnel test article will be introduced to assess the validity of the proposed model.

DEFINITION OF LINEAR/NON LINEAR FLUTTER

The definition of the flutter condition is straightforward; according to [7,8], flutter can be defined as the dynamic instability of an elastic body in an air-stream. The air loads due to deflections of the elastic structure from the undeformed state are also necessary for the identification of flutter condition. The definition of stability in Liapunov sense states that a stationary point is stable if points which starts nearby stay nearby, and it is quasi-asymptotically stable if nearby points tend to it. If a point is both Liapunov stable and quasi-asymptotically stable then it is said to be asymptotically stable [9]. Consider an autonomous nonlinear dynamical system $\dot{x} = f(x(t); x(0) = x_0)$, with $x(t) \in D \subseteq \mathbb{R}^n$ the system state vector, D an open set containing the origin, and $f : D \rightarrow \mathbb{R}^n$ is continuous on D . Without loss of generality, we may assume that the origin is an equilibrium point. The following observations hold:

a. The origin of the above system is said to be *Lyapunov stable*, if, for every $\varepsilon > 0$, there exists a $\delta = \delta(\varepsilon) > 0$ such that, if $\|x(0)\| < \delta$, then $\|x(t)\| < \varepsilon$, for every $t \geq 0$.

b. The origin of the above system is said to be *asymptotically stable* if it is Lyapunov stable and if there exists $\delta > 0$ such that if $\|x(0)\| < \delta$, then $\lim_{t \rightarrow \infty} x(t) = 0$.

c. The origin of the above system is said to be *exponentially stable* if it is asymptotically stable and if there exist $\alpha, \lambda, \delta > 0$ such that if $\|x(0)\| < \delta$, then $\|x(t)\| \leq \alpha \|x(0)\| e^{-\lambda t}$ for $t \geq 0$.

Perhaps a more pertinent definition of the flutter condition and the properties of an aeroelastic system can be proposed by studying the stability of infinitesimal motions about that condition. It is then sufficient to analyze the aeroelastic vibration with a complex exponential time dependence function, since all other motion can be defined by superposition. The flutter speed and flutter frequency are defined as the lowest airspeed and corresponding frequency at which a specific structure will exhibit sustained simple harmonic oscillations. Based on the small motion hypothesis, a linear aerodynamic operator can be applied, for example the Theodorsen's unsteady representation of the aerodynamic loads. The final problem led to a complex eigenvalue problem where two characteristic values have to be determined: speed and frequency. At flutter, where the approximation and the harmonic aerodynamic loads are strictly valid, the real part of at least one of the characteristic exponents transit from negative value to zero and migrate into the positive domain. The zero damping parameter identifies the flutter speed. This classical flutter calculation, known as the V-g approach [8], it is strictly referred to the small motion hypothesis superimposed to undeformed configuration. If the aeroelasto-dynamic governing equations of a generic autonomous system can be represented by the following vector relation in $R^n: \dot{q} = \Psi(q, \kappa)$, a stationary point of the system can be indicated by q^e such that $\Psi(q^e, \kappa) = 0$. The small motion hypothesis admits a representation of the governing system in the form:

$$\dot{z} = \Psi(q^e + z, \kappa) = \Psi(q^e, \kappa) + J(\Psi)_{q^e} z + \dots \cong J(\Psi)_{q^e} z \quad (1)$$

(assuming constant parameters), where z is a small perturbation and J is the Jacobian of Ψ evaluated at the equilibrium condition. If the functional Ψ is represented by the operator H such that $\Psi = [H(\kappa)]q$, then it can be shown that $J(\Psi)_{q^e} = [H]_{q^e} = [H]_{\bar{q}} = [H]$ where $\bar{q}$ is the generic reference point; for example can be assumed as the undeformed state, that is $\bar{q} = 0$. The dynamic characteristics of the perturbed system are the same whatever reference point is assumed. In this situation it is possible to investigate the flutter behaviour of the system and a flutter condition can be determined and identified as classical flutter speed or

“Linear Flutter Speed” (LFS). On the other hand as the Ψ operator is non-linear, due to high structural flexibility, the perturbed motion dynamics is influenced by the chosen equilibrium point. The reduced linear perturbation system originates a linear approximation of the behaviour of the system in the neighborhood of the static equilibrium point, implying that there is going to be a flutter speed associated to each condition identified as “Nonlinear Flutter Speed” (NLFS). Clearly, when the structure is highly flexible, as in the case of slender HALE wings, the classical approach is no longer suitable to correct identify the flutter speed. The high level of deflections under loads strongly modifies the equilibrium point. For this reason a different flutter analysis has to be defined. In this case, the condition of small motion has been applied to HALE configuration by means of perturbing the non-linear equilibrium equations; the equilibrium configuration is no longer equivalent to the undeformed one and the perturbed equations defined with respect to the actual equilibrium point is strongly influenced by the same equilibrium configuration. If the small motion axiom is no longer representative of the actual case, such as in the post-flutter regime, a non-linear form has to be studied; different phenomena will occur [3,4,10], and consequently different aerodynamic operator and solution schemes should be employed to analyze post-flutter behaviour. These very important aspects are not discussed in this paper, where only the small perturbation condition is considered.

AEROELASTIC MODEL

Structural Model

Different levels of approximation can be determined without arbitrary simplification or ad-hoc assumption, showing the limits of applicability of approximations in order to fit the physical behaviour. A nonlinear 3D composite Euler-Bernoulli beam theory [11,12] has been adopted in order to derive a structural beam model valid for moderate to large deflections. The wing is assumed to be clamped in the plane of symmetry and derived equations are also valid for beams with a mass centroid axis offset from the elastic axis, non-uniform mass and stiffness section properties. Quadratic terms have been retained in the final expression. Higher non-linear terms have been neglected assuming they would have a negligible effect on the system dynamics. Rotary inertia has been neglected according to the slender beam hypothesis. The effect of shear deformation has also been neglected.

Consider a uniform straight isotropic beam of length L and mass per unit length m . Introducing two coordinate system: 1) the Cartesian coordinate system xyz that describes the undeformed geometry, and 2) the orthogonal curvilinear coordinate system ξ, η, ζ that describes the deformed geometry.

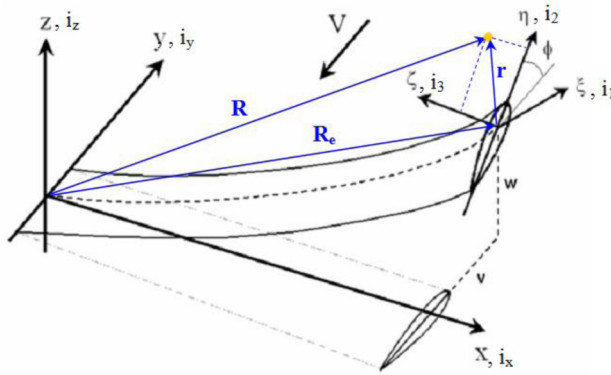


Figure 1. Reference system

Curvatures respect to axes 1, 2, and 3 are defined as $\rho_1 = i'_2 \cdot i_3$, $\rho_2 = i'_3 \cdot i_1$, $\rho_3 = i'_1 \cdot i_2$ where prime indicate first derivative respect to ξ . In the case of a second order approximation the expression of curvatures become $\rho_1 = \phi' + v''w'$, $\rho_2 = -w'' + v''\phi$, $\rho_3 = v'' + w''\phi$. Equations of motion are derived using Hamilton's principle:

$$\int_{t_1}^{t_2} \int_0^L (\delta T - \delta U + \delta W_{nc}) d\xi dt = 0 \quad (2)$$

where, neglecting the rotary inertia, the variation of kinetic energy and non-conservative terms are given by:

$$\delta T = - \int_0^L (m\ddot{w}\delta w + m\ddot{v}\delta v + j_1\ddot{\phi}\delta\phi) d\xi \quad (3a)$$

$$\delta W_{nc} = \int_0^L (-c_2\dot{v}\delta v - c_3\dot{w}\delta w - c_4\dot{\phi}\delta\phi + q_2\delta v + q_3\delta w + q_4\delta\phi) d\xi \quad (3b)$$

Structural terms are derived by the variation of the elastic energy:

$$\delta U = \int_0^L (M_1\delta\rho_1 + M_2\delta\rho_2 + M_3\delta\rho_3) d\xi \quad (4)$$

The beam is assumed to be inextensional lengthwise. This is a valid approximation if the extensional stiffness of the wing is large. Applying the Hamilton's principle to the energy expressions, the governing differential equations of motion are obtained and a nondimensional form of equations of motion for in-plane bending (v), out-of-plane (w), and torsion (ϕ) can be written as:

$$\begin{cases} \ddot{\chi}_v + \frac{\Gamma\Theta r_\alpha^2}{\Lambda^2} \chi_v'''' + N_v = L_v \\ \ddot{\chi}_w + x_\alpha \ddot{\chi}_\phi + \frac{\Theta r_\alpha^2}{\Lambda^2} \chi_w'''' - \frac{\bar{K}r_\alpha^2}{\Lambda} \chi_\phi'''' + N_w = L_w \\ r_\alpha^2 \ddot{\chi}_\phi + x_\alpha \ddot{\chi}_w - r_\alpha^2 \chi_\phi'' + \frac{\bar{K}r_\alpha^2}{\Lambda} \chi_w'' + N_\phi = M_\phi \end{cases} \quad (5)$$

Herein $\chi_v = \frac{v}{b}$, $\chi_w = \frac{w}{b}$, $\chi_\phi = \phi$ are the in-plane, out of plane and torsion dimensionless displacements. In addition, $\Theta = \frac{EI_\eta}{GJ}$, $\Gamma = \frac{EI_\xi}{EI_\eta}$, $\bar{K} = \frac{K}{GJ}$, $x_\alpha = \frac{e}{b}$, $r_\alpha = \frac{r}{b}$, $\Lambda = \frac{L}{b}$ are system dynamic parameters, while L_v , L_w , M_ϕ are the dimensionless aerodynamic force and moments function of reference reduced frequency and mass parameter $k^* = \frac{b\omega_\alpha}{U}$, $\mu = \frac{m}{\pi\rho b^2}$. N_v , N_w , N_ϕ are the second order nonlinear dimensionless terms expressed by:

$$\begin{aligned} N_v &= (\Gamma - 1) \frac{\Theta r_\alpha^2}{\Lambda^2} [\chi_\phi \chi_w'''] + \frac{r_\alpha^2}{\Lambda^2} [\chi_\phi' \chi_w''] + \\ &+ \frac{\bar{K}r_\alpha^2}{\Lambda} \left[(\chi_\phi'' \chi_\phi)' + (\chi_\phi' \chi_\phi')' \right] - \frac{\bar{K}r_\alpha^2}{\Lambda^3} (\chi_w'' \chi_w')' \\ N_w &= \chi_\phi''' (\Gamma - 1) \frac{\Theta r_\alpha^2}{\Lambda^2} [\chi_\phi \chi_w'''] - \frac{r_\alpha^2}{\Lambda^2} [\chi_\phi' \chi_w''] - \frac{\bar{K}r_\alpha^2}{\Lambda^3} (\chi_w'' \chi_w')' \\ N_\phi &= (\Gamma - 1) \frac{\Theta r_\alpha^2}{\Lambda^2} \chi_v'' \chi_w'' - \frac{r_\alpha^2}{\Lambda^2} [\chi_v'' \chi_w''] - \frac{\bar{K}r_\alpha^2}{\Lambda} (\chi_v'' \chi_\phi') \end{aligned} \quad (6)$$

Aerodynamic Model

The aerodynamic model is composed by a Linear Unsteady Model based on Wagner function and a stall model based on a modified Beddoes-Leishman (B-L) formulation.

Linear unsteady aerodynamic model

When the flight speed is low enough so that the incompressible aerodynamics assumption is acceptable, and the aspect ratio large enough to justify the use of a 2D strip theory, the unsteady aerodynamic forces can be obtained using Wagner's indicial function in Duhamel integral form [7,13]:

$$L = -\frac{1}{\mu} C_L(\hat{t}) ; \quad M = \frac{2}{\mu} C_M(\hat{t}) \quad (7)$$

Where lift and moment coefficients can be expressed through the Wagner function: