

Application of a New One-Equation Turbulence Model to Computation of Separated Flows

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ABSTRACT

Accurate turbulence modeling remains a critical problem in the prediction capability of computational fluid dynamics. One particular area lacking accurate simulations is separated turbulent flows. In this paper, the recently developed one - equation Rahman-Agarwal-Siikonen (RAS) turbulence model is used to simulate the flow of several canonical turbulent flow cases. The commercially available software ANSYS Fluent and the open source software OpenFOAM are used for the flow field calculations. It is shown that the RAS model improves the accuracy of flow simulations compared to the commonly used one - equation Spalart-Allmaras (SA) and two-equation SST $k-\omega$ turbulence models.

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INTRODUCTION

Computational Fluid Dynamics (CFD) is routinely used in the design and performance prediction of aircraft, turbomachinery, automobiles, and in many other industrial applications where fluid flow is present. Despite CFD's wide range of use, deficiencies in its prediction capability still exist. One critical weakness is the accurate simulation of turbulent flows. The Reynolds-Averaged Navier-Stokes (RANS) equations based approach, employing one- and two-equation eddy viscosity type turbulence models, is now the industry standard for the analysis and design of aerospace vehicles. Large Eddy Simulation (LES), LES-RANS Hybrid methods, and Direct Numerical Simulation (DNS) methods of modeling turbulent flows have so far found limited use due to their computational intensity and high grid resolution requirements. It appears that in the foreseeable future, eddy viscosity based one- and two-equation models will remain the

main workhorse for prediction of turbulent flows in industrial applications.

This paper compares the recently developed one - equation Rahman-Agarwal-Siikonen (RAS) [1] model to the industry standard one-equation Spalart-Allmaras (SA) [2] and two-equation SST $k-\omega$ [3] models. The simulation results for three flows are compared. The first flow is that of a turbulent flat plate. This flow demonstrates the low Reynolds number accuracy of the models and their capability of predicting skin friction coefficient. The next case considered is the flow over a 2D hump in which a separation bubble occurs at the back of the hump. Accurate prediction of the separation and reattachment points and the pressure distribution across the hump are the main points of interest. The third case studied is the flow over a NACA4412 airfoil at high angle of attack. In this case, a trailing edge separation occurs. The goal of this case is the accurate prediction of the surface pressure

coefficient which can only be accomplished with accurate prediction of the separation point.

RAS TURBULENCE MODEL

The newly developed RAS turbulence model is a one equation eddy viscosity turbulence model developed using the k - ε closure. It was derived in a manner similar to the Baldwin-Barth (BB) model. The RAS model, however, avoids the shortcomings of the BB model. In the BB derivation, the assumptions made severely weakened its link with the k - ε model. This renders the BB model ill-conditioned in the shear layer regions. Additionally the eddy viscosity remains constant in a uniform stream rather than decaying. This error is caused by the destruction term equaling zero and can produce non-physical solutions for some simulations. The RAS model corrects the destruction term and revives the link to the k - ε model by modifying the coefficients with additional limits and relaxation functions. This improves the accuracy for equilibrium flows and improves the ability to account for the non-equilibrium effects. The addition of an elliptic relaxation equation to account for kinematic blocking by the wall allows for accuracy up to the wall. The equations of the RAS turbulence model are presented below.

The RAS turbulence model determines $R = k^2/\varepsilon$ by the following transport equation.

$$\frac{\partial \rho R}{\partial t} + \frac{\partial \rho u_j R}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_T}{\sigma} \right) \frac{\partial R}{\partial x_j} \right] + C_1 \sqrt{P \tilde{R}} - C_2 \left(\frac{\partial \tilde{R}}{\partial x_j} \right)^2 \quad (1)$$

The turbulent eddy viscosity is given by the equation:

$$\mu_T = C_\mu f_\mu \rho \tilde{R} \quad (2)$$

The definitions of the constants C_μ , C_1 , and C_2 all depend nonlinearly on both the rotational and irrotational strains. This extends the ability of the model to account for non-equilibrium and anisotropic effects. Commonly used isotropic turbulent viscosity models employ a constant $C_\mu = 0.09$. This becomes unrealizable in the case of large mean strain rate parameter. To ensure realizability, the model coefficient C_μ is related to the mean flow deformation rate. The formulation for C_μ as suggested by Gatski and Speziale [4, 5] is adopted as follows:

$$C_\mu = \frac{3(1 + \eta^2)\alpha_1}{3 + \eta^2 + 6\eta^2\xi^2 + 6\xi^2}, \quad \eta = \alpha_2 T_t S, \quad \xi = \alpha_3 T_t \quad (3)$$

Here S and W are the mean strain rate and the mean vorticity respectively described below.

$$S = \sqrt{2S_{ij}S_{ij}}, \quad S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \\ W = \sqrt{2W_{ij}W_{ij}}, \quad W_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \quad (4)$$

The dynamic time scale k/ε is replaced by a realizable time scale T_t . This prevents the time scale from falling below the Kolmogorov time scale, which is dominant in the neighborhood of solid walls. T_t is defined as

$$T_t = \sqrt{\frac{k^2}{\varepsilon^2} + C_T^2 \frac{\nu}{\varepsilon}} \quad (5)$$

where k , ε , and ν have the usual definitions of turbulent kinetic energy, turbulent dissipation, and kinematic viscosity respectively. The empirical constant $C_T = \sqrt{2}$ is estimated from the behavior of k in the viscous sub-layer [6].

The other coefficients associated with C_μ are given below with additional coefficient definitions available in Ref. [5].

$$\alpha_1 = g \left(\frac{1}{4} + \frac{2}{3} \Pi_b^{1/2} \right), \quad \alpha_2 = \frac{3}{8\sqrt{2}} g \\ \alpha_3 = \frac{3}{\sqrt{2}} \alpha_2, \quad g = \left(1 + 2 \frac{P}{\varepsilon} \right)^{-1} \quad (6)$$

In order to account for the anisotropy of turbulence and to reinforce the link of the present model and the k - ε model, the coefficients C_1 and C_2 are defined as a function of mean shear and vorticity parameters. These definitions assist in predicting flows with separation and reattachment.

$$C_1 = 2\sqrt{\Pi_b^*} (1 - \sqrt{\Pi_b^*}), \\ C_2 = \min \left[2C_\mu, \tilde{C}_\mu \sqrt{1 + \left(\frac{C_1}{6\tilde{C}_\mu} \right)^2} \right] \quad (7)$$

Here $\tilde{C}_\mu = 0.09$ and $\sqrt{\Pi_b^*} = C_\mu \zeta$. By examining Eq. (3) along with $\zeta = T_t \text{Smax}(1, |W/S|)$, it can be seen that shear and vorticity effects are accounted for in both C_1 and C_2 .

The role of turbulent diffusion in the near-wall region is substantial. For this reason the Prandtl number is modeled, rather than the common practice of assigning a constant value. The definition of the Prandtl number is as follows.

$$\sigma = C_\mu + f_\mu / C_T \quad (8)$$