

UniTire Model for Tire Forces and Moments under Combined Slip Conditions with Anisotropic Tire Slip Stiffness

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ABSTRACT

The tire mechanics characteristics are essential for analysis, simulation and control of vehicle dynamics. This paper develops the UniTire model for tire forces and moments under combined slip conditions with anisotropic tire slip stiffness.

The anisotropy of tire slip stiffness, which means the difference of tire longitudinal slip stiffness and cornering stiffness, will cause that the direction of tire resultant shear stress in adhesion region is different from that in sliding region. Eventually the tire forces and moments under combined slip conditions will be influenced obviously. The author has proposed a "direction factor" before to modify the direction of resultant force in the tire-road contact patch, which can describe tire forces at cornering/braking combination accurately. However, the aligning moments which are very complicated under combined slip conditions are not considered in previous analysis.

The simplified physical tire model is introduced firstly in this paper to analyze the tire mechanics characteristics under combined cornering/braking situations, containing the tire forces and aligning moments. Then considering the influence of anisotropic tire slip stiffness and the expression of "direction factor", a semi-physical tire model for tire forces and moments is given. Finally, the effectiveness of the model for tire forces and moments under combined slip conditions with anisotropic tire slip stiffness is validated by the comparison with the experimental data.

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INTRODUCTION

Since tires are the only components of a road vehicle to directly contact with the road surface, the tire characteristics are the essential and important factor for the studies of vehicle dynamics. And the tire mechanics characteristics under combined cornering/braking situations have significant effects on the vehicle directional control [1]. There are already many existing models which can describe the tire characteristics generated at combined-slip conditions [2, 3, 4, 5, 6], however, some of them don't give the expression of aligning moment, and some are not accurate to describe the tire forces and moments when the tire has a strong anisotropy. The earlier studies of UniTire model [4,7], to some extent, consider the effects of the anisotropy of tire slip stiffness, however, the expressions of anisotropy are simple and inaccurate. The Magic Formula model [8] is an empirical tire model which is capable of describing the tire forces and moments, even if the tire stiffness is anisotropic. However, it is complicated, especially for combined slip cases, to use in vehicle dynamics simulation and control. Therefore,

developing an accurate and concise tire model is still an important task and challenge, especially for a tire with a strong anisotropy.

In this paper, firstly, a simplified physical tire model is discussed to analyze the basic relationship among tire shear forces, aligning moment and normalized slip ratio. Then, based on the physical model, considering the influence of anisotropic tire slip stiffness, a semi-physical tire model for tire forces and moments is presented. Finally, a set of test data with strong anisotropy of tire stiffness, in which the longitudinal slip stiffness is about three-times larger than cornering stiffness, are used to verify the model results. It shows that the proposed model is effective for tire forces and moments under combined conditions with anisotropic tire slip stiffness.

TIRE AXIS SYSTEM AND SLIP RATIOS

The tire axis system is shown in Fig.1. It is different from the SAE tire axis system. The X_t axis is the intersection of the



wheel plane and road plane and the positive direction is coincident with tire revolution direction, not wheel traveling direction; the Z_t axis is perpendicular to road plane and upward; the Y_t axis is satisfied with the right-hand rule. The wheel traveling speed is denoted as V , and α is the slip angle and γ is the inclination angle. The figure shows all the forces and moments associated with a wheel.

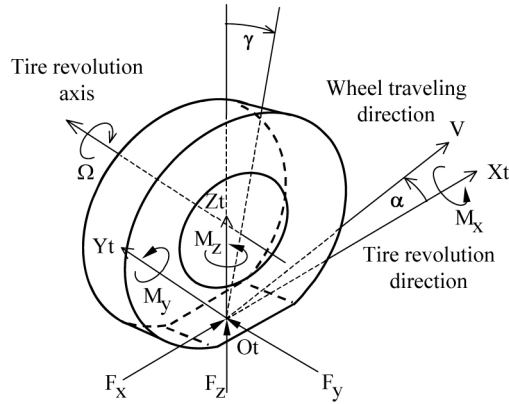


Figure 1. Tire axis system

The longitudinal and lateral slip ratios used in UniTire model are defined in the unified form [7]:

$$\begin{cases} S_x = \frac{-V_{sx}}{\Omega R_e} = -\frac{V \cos \alpha - \Omega R_e}{\Omega R_e} & S_x \in (-\infty, +\infty) \\ S_y = \frac{-V_{sy}}{\Omega R_e} = -\frac{V \sin \alpha}{\Omega R_e} & S_y \in (-\infty, +\infty) \end{cases} \quad (1)$$

where, Ω is the angular speed, R_e is the effective rolling radius. V_{sx} and V_{sy} are the longitudinal and lateral sliding speeds with respect to road surface.

Usually, the longitudinal slip ratio used in tire force test is defined as:

$$\kappa = \frac{-V_{sx}}{V \cos \alpha} = \frac{\Omega R_e - V \cos \alpha}{V \cos \alpha} \quad (2)$$

The relationship between S_x and κ can be obtained easily:

$$S_x = \frac{\kappa}{1 + \kappa} \quad (3)$$

SIMPLIFIED PHYSICAL TIRE MODEL

In the simplified physical tire model, the contact patch between tire and the road is assumed to be rectangular in shape, and the contact pressure distribution is assumed to be uniform in lateral direction and of arbitrary form in circumferential direction to represent different kinds of pressure distributions. The parallel deformations of the carcass in longitudinal and lateral directions are considered.

The longitudinal and lateral deformations of the tread and carcass, under combined slip condition, are shown in Fig.2.

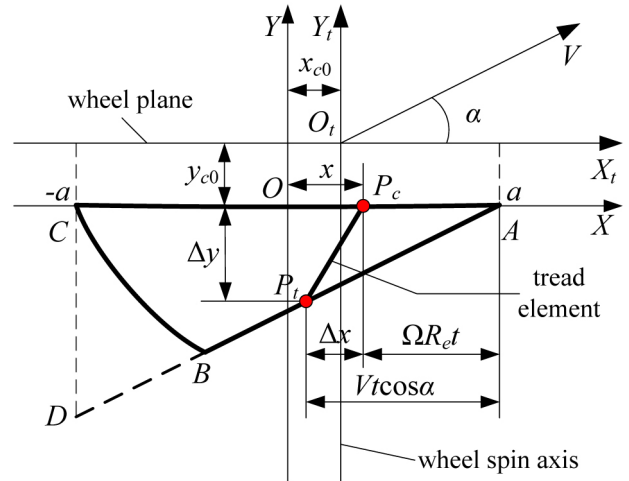


Figure 2. Deformations of tread and carcass

In this figure, $X_t O_t Y_t$ is a coordinate system which is coincident with the tire axis system shown in Fig.1, and XOY is a relative coordinate system for describing the tread deformation with respect to the carcass. The origin 'O' is set at the actual contact patch center which coincided with point 'O_t' before the tire is deformed. 'ABC' is the central line of the contact patch under combined slip condition. In general cases, the whole length of contact patch, $2a$, is divided into two parts, the adhesion region 'AB', and the sliding region 'BC', by the initial sliding point 'B'. x_{c0} and y_{c0} are the deformations of carcass along axes X_t and Y_t caused by the longitudinal and lateral forces, which are expressed as:

$$x_{c0} = \frac{F_x}{K_{cx}}, \quad y_{c0} = \frac{F_y}{K_{cy}} \quad (4)$$

where K_{cx} and K_{cy} are the longitudinal and lateral stiffness of the carcass.

As shown in Fig.2, a point in contact patch, which begins to contact at the point 'A', is now reaching the position 'P_t', after rolling for a period of time t . Meanwhile, the corresponding point on carcass moves to 'P_c'. The longitudinal and lateral deformations of the tread element can be calculated as:

$$\begin{cases} \Delta x = -(Vt \cos \alpha - \Omega R_e t) \\ \Delta y = -Vt \sin \alpha \end{cases} \quad (5)$$

Considering Eq. (1) and $a-x=\Omega R_e t$, Eq. (5) can be written as

$$\begin{cases} \Delta x = (a - x) S_x \\ \Delta y = (a - x) S_y \end{cases} \quad (6)$$

If the stiffness of tread element in X and Y directions are k_{tx} and k_{ty} , the shear stresses of point P_t in both directions can be expressed as:

$$\begin{cases} q_x = k_{tx} \cdot \Delta x = k_{tx} \cdot (a - x) S_x \\ q_y = k_{ty} \cdot \Delta y = k_{ty} \cdot (a - x) S_y \end{cases} \quad (7)$$

Then the magnitude of resultant shear stress becomes:

$$q = \sqrt{q_x^2 + q_y^2} = (a - x) \sqrt{(k_{tx} S_x)^2 + (k_{ty} S_y)^2} \quad (8)$$

In the rear of contact patch, the tread element begins to slide when the resultant shear stress equal to the maximum friction force:

$$q(u_c) = \mu q_z(u_c) \quad (9)$$

Where $u = x/a$ is the relative coordinate, and u_c means the relative coordinate of initial sliding point 'B'; μ is the directional friction coefficient, which will be described later in detail; q_z denotes the vertical load distribution along the contact patch length, which can be expressed in a unified form as[9]

$$q_z = \frac{F_z}{2a} \cdot \eta(u) \quad (10)$$

where F_z is the tire vertical load; $\eta(u)$ is the normalized pressure distribution function.

Then, the coordinate of initial sliding point will satisfy the following equation:

$$q(u_c) = (a - au_c) \sqrt{(k_{tx} S_x)^2 + (k_{ty} S_y)^2} = \frac{\mu F_z}{2a} \eta(u_c) \quad (11)$$

The normalized longitudinal slip ratio ϕ_x , lateral slip ratio ϕ_y , and combined slip ratio ϕ are defined as:

$$\begin{cases} \phi_x = \frac{2a^2 k_{tx} S_x}{\mu_x F_z} = \frac{K_x S_x}{\mu_x F_z} \\ \phi_y = \frac{2a^2 k_{ty} S_y}{\mu_y F_z} = \frac{K_y S_y}{\mu_y F_z} \\ \phi = \sqrt{\phi_x^2 + \phi_y^2} \end{cases} \quad (12)$$

where $K_x = 2a^2 k_{tx}$, $K_y = 2a^2 k_{ty}$ represent the longitudinal slip stiffness and cornering stiffness.

In Eq. (11) and Eq. (12), μ_x and μ_y are the longitudinal and lateral friction coefficients. μ is the directional friction coefficient along the resultant shear stress direction [10]. According to the friction ellipse concept, it is defined as

$$\mu = \sqrt{\left(\frac{\phi_x}{\phi} \cdot \mu_x\right)^2 + \left(\frac{\phi_y}{\phi} \cdot \mu_y\right)^2} \quad (13)$$

Substituting the given expressions into Eq. (11), it yields

$$\frac{\eta(u_c)}{1 - u_c} = \phi \quad (-1 \leq u_c \leq 1) \quad (14)$$

If the ϕ_x and ϕ_y are given, the relative coordinate of the initial sliding point, u_c , could be solved. Then the resultant shear force can be expressed as:

$$\begin{aligned} F &= \int_{x_c}^a q dx + \int_{-a}^{x_c} \mu \cdot \frac{F_z}{2a} \cdot \eta(u) dx \\ &= \frac{(1 - u_c)^2}{4} \cdot 2a^2 \sqrt{(k_{tx} S_x)^2 + (k_{ty} S_y)^2} + \mu F_z \cdot \frac{m_0(u_c)}{2} \end{aligned} \quad (15)$$

or in a normalized form as follow:

$$\bar{F} = \frac{F}{\mu F_z} = \frac{(1 - u_c)^2}{4} \cdot \phi + \frac{m_0(u_c)}{2} \quad (16)$$

where m_0 is defined as $m_0(u_c) = \int_{-1}^{u_c} \eta(u) du$.

Considering the assumption that the direction of the resultant shear stress in the sliding region will be the same as that in the adhesion region, the normalized longitudinal and lateral force can be obtained as follow:

$$\begin{cases} \bar{F}_x = F_x / (\mu_x F_z) = \bar{F} \cdot \phi_x / \phi \\ \bar{F}_y = F_y / (\mu_y F_z) = \bar{F} \cdot \phi_y / \phi \end{cases} \quad (17)$$

Considering the deformations of carcass, the aligning moment can be calculated as follow:

$$\begin{aligned} M_z &= \int_{x_c}^a q_y x dx + \int_{-1}^{u_c} \frac{q_y}{q} \cdot \mu \frac{F_z}{2a} \eta(u) \cdot x dx - F_x y_{c0} + F_y x_{c0} \\ &= \frac{1 - 3u_c^2 + 2u_c^3}{4} \cdot K_m S_y + \frac{q_y}{q} \cdot \frac{m_1(u_c)}{2} \cdot a \mu F_z - F_x y_{c0} + F_y x_{c0} \end{aligned} \quad (18)$$

where $K_m = 2a^3 k_{ty}/3$ represent the aligning stiffness; m_1 is defined as $m_1(u_c) = \int_{-1}^{u_c} u \eta(u) du$.

Since M_z could also be written as