

Modelling and Analyzing Hybrid Electric Vehicles with Single and Dual Epicyclic Power Split Transmissions

Ahmed Elmarakbi, Derek Dixon, Rob Trimble, and Qinglian Ren
University of Sunderland

ABSTRACT

The main aim of this paper is to investigate the performance of hybrid electrical vehicles (HEVs) using single and dual epicyclic power split transmissions (PST). The development of PST has been a crucial feature in the technological success of hybrid driveline vehicles. In this paper, single and twin epicyclic power split transmissions models are analyzed and the relationships of speed and torque of the engine, motor and generator are developed. The models are further extended to include all subsystems and a controller designer to be fitted in a typical hybrid electrical vehicle (HEV). Computer programmes for the analysis of epicyclic transmission based on a matrix method are developed and used. In addition, the performance and control strategy of the new system is presented. Three vehicle models are built-up; namely: traditional ICE vehicle, HEV with single epicyclic gearbox, and HEV with twin epicyclic gearbox. The vehicles are simulated over typical driving cycles. The results focus on fuel consumption comparisons but it is also shown how the single and twin epicyclic gearboxes use the engine and motor generator units differently. The simulations show that the twin epicyclic offers substantial improvements of reduction in energy consumption, though the benefits are sensitive to the driving cycle used.

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INTRODUCTION

Hybrid electric vehicles (HEVs) are considered to be an intermediate step towards purely electric drive (fuel cells or batteries) [1]. Commercial interest in hybrid vehicle technology has grown at a much more dramatic rate than was predicted a decade ago. Around that time, many industry observers were substantially more optimistic about a major leap from current petroleum based technology straight to hydrogen, fuel cells and bio fuel systems. However, it is now widely accepted that hybrid vehicles will have a significant role to play over the next couple of decades as these other technologies continue to be developed.

Hybrid electric vehicle technology has made a massive impact over the past decade on the automotive engineering industry [2, 3, 4]. The growth in interest has been fuelled by increasing concerns about the environment and fuel efficiency savings. But also, the market uptake of hybrid vehicles - led mainly by the Toyota Prius - has been much greater than most observers originally predicted; this in turn has led most of the other original equipment manufacturers

(OEMs) and Tier One suppliers to develop their own systems, often in collaborative partnerships.

Although many versions of hybrid vehicle have been tried, by far the most common layout is the 'series/parallel' hybrid, in which an IC engine and electric motor can either work independently or together. This means that the transmission system must incorporate (a) a power combining device and (b) a regeneration scheme so that the battery can be recharged either by the engine or by the kinetic energy of the vehicle during braking. It is perhaps not widely recognised, but the transmission design has been a crucial issue in the success of hybrid vehicles. These transmissions are also often referred to as power split devices (PSD) - and the control strategy to manage all the engine, motor generator (MG) and transmission elements is also crucial to the goal of achieving improved fuel efficiency from the hybrid vehicle compared with that available from conventional vehicles.

This paper is focused on transmissions for hybrid electric vehicles-NexxtDrive system [5] which is marketed as 'DualDrive' for automotive and off-highway applications. The transmission provides a continuously variable gearbox

(CVT) based on two epicyclic gear sets plus two electric motor/generator units. This unique, patented scheme offers potential efficiency benefits over its competitors. It has a crucial benefit of providing two ratios at which the electrical path transmits no power (and therefore no losses). A key design issue is to design the transmission to match the vehicle application such that these two points occur at common operating speeds. The DualDrive system is covered by patent protection and potentially has applications in other industrial drive situations, bicycle drive units, engine ancillary units etc. It is similar in principle to the successful Toyota Prius unit, but differs in particular in that the Toyota scheme only has one epicyclic gear set.

Continuously Variable Transmissions (CVTs) have been around for many years and the cost-benefit issues relating to CVTs are well understood. The potential advantages are improved performance, economy and emissions or more importantly an improved compromise between them. Their disadvantages have been cost, complexity, noise and driving refinement. Only over the past five years or so has the development of CVTs reached a stage at which they are beginning to be genuinely competitive with the alternatives, e.g. conventional, torque converter automatics and automated manual gearboxes, such as the twin clutch VW DSG system.

An important type of CVT is the E-CVT (Electronically-Controlled Continuously Variable Transmission), a good example of which is Toyota Hybrid System (THS). It combines the characteristics of an electric drive and a continuously variable transmission, using motor generator units in addition to toothed gears [6, 7]. In a THS system (see Fig. 1), one of the motor generators (MG2) is mounted on the driveshaft, and thus couples torque into or out of the driveshaft. The second motor generator (MG1) is connected with the sun gear and used to change the sun gear speed. Because MG2 is connected with the driveshaft, it cannot change speed and torque freely. Hence there are three power input/output branches in the system: the engine, MG1, the output, MG2. Because the speed of the output shaft is decided by the speed of the vehicle, there is some limitation on the control strategy to achieve optimum performance.

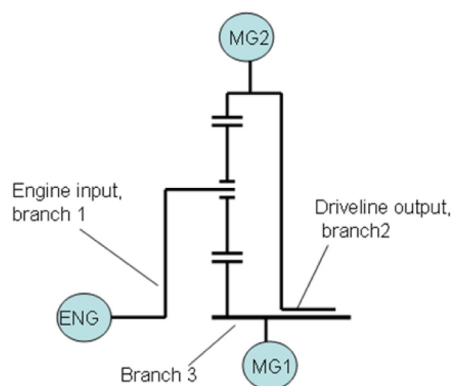


Figure 1. Single epicyclic transmission - 3 branch system.

On the other hand, in a four branch transmission system, neither of the motor generator units is mounted on the driveshaft or on the engine input shaft, which gives more freedom and benefits to the system (see Fig. 2).

One motor/generator is connected to the sun gear, and the other motor/generator is connected with a ring gear. So there are four branches of power input/output: the engine, the output shaft, and two motor generator units, MG1 and MG2.

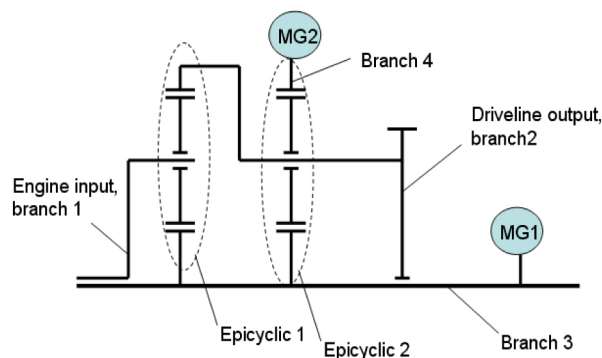


Figure 1. Twin epicyclic transmission- 4 branch system.

This type of four branch transmission system has been described recently by Moeller [4] who proposed that it offers advantages in many automotive applications, including hybrid and non-hybrid vehicles, such as light electric vehicles or off-road vehicles. However, its usage in a hybrid electric vehicle driveline will be analyzed here for the first time.

In previous work on hybrid vehicle modelling, a software tool, Transmissions System Analysis Tool (TSAT) was described [8]. This has been developed specifically to analyze these types of system which comprise collections of epicyclic gears, brakes, clutches and motor generator units interconnected in a wide variety of possible ways.

In this paper, the two PSD arrangements based on the single and twin epicyclic designs are modeled in a medium sized hybrid passenger vehicle using a modeling approach based on a combination of TSAT, QSS and Matlab components. The resulting vehicle performance is predicted over a standard driving cycle and relative merits of the two PSD arrangements are compared.

ANALYSIS OF POWER SPLIT SYSTEMS

A matrix method is used here to analyze the planetary transmission system, as introduced by Tian and Lu [9]. The key point of this method is to generate matrices to represent all the planetary train elements and other auxiliary components of the transmission. In this approach, the planetary transmissions are broken into finite functioning units, and then matrices are created representing both the configuration relationships between those units and the transmission manipulating characteristics. Once these matrices are generated, the kinematic and dynamic problems

of the transmissions can be solved by means of standard matrix operations.

Single Epicyclic Transmission

To use the matrix method to analyze the transmission, each basic part is numbered, from 1 to 10. There are 4 connected parts: I to IV. The connected part I includes basic part 1, 5 and 8; the connected part II include basic part 2, 7 and 9; the connected part III includes basic part 4 and 6; the connected part IV includes basic parts 3 and 10. There are two gear units X1 and X2.

The basic parts, the connected parts and the gear units are as shown in Fig. 3. In the matrix method, all the basic parts are numbered as 1, 2, 3, etc. and nearly all the basic parts are repeatedly named. For example, numbers 1, 5 and 8 point to the same part: 1 stands for input shaft, 5 stands for the carrier of the gear mesh X1 and 8 stands for the carrier of the gear mesh X2.

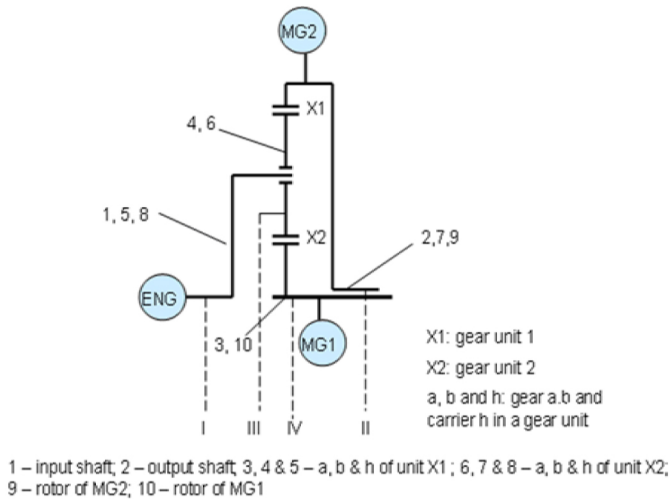


Figure 1. Encode of basic parts and connected parts.

The gears turn in a ratio determined by the number of teeth in each gear, where α is the gear ratio and N is the number of teeth. $\alpha = N_b / N_a$ (inside engaged) and $\alpha = -N_b / N_a$ (outside engaged). Let α_1 to α_2 represent the gear ratios of units X1 and X2 respectively. The tooth number for the sun gear, the pinion gear and the ring gear is 30, 24, and 78 respectively. So the gear ratios for X1 and X2 are

$$\alpha_1 = -N_4 / N_3 \quad (1)$$

$$\alpha_2 = N_7 / N_6 = N_7 / N_4 = (N_3 + 2N_4) / N_4 = 2 - (1/\alpha_1) \quad (2)$$

where N_n represents the tooth number for gear defined by n in Fig. 3. In equation, α_2 is expressed in α_1 , so there is actually only one parameter for the single epicyclic gearbox (α_1). To analyze the speeds of the system, first, the coefficient matrix is derived for the speed terms, which are made up by all the speed constraints for each part.

The following describes the rotation equation group for the single epicyclic transmission:

$$\text{Connected part 1: } \omega_1 - \omega_5 = 0, \quad \omega_1 - \omega_8 = 0 \quad (3)$$

$$\text{Connected part 2: } \omega_2 - \omega_7 = 0, \quad \omega_2 - \omega_9 = 0 \quad (4)$$

$$\text{Connected part 3: } \omega_3 - \omega_{10} = 0 \quad (5)$$

$$\text{Connected part 4: } \omega_4 - \omega_6 = 0 \quad (6)$$

$$\text{Gear system unit X1: } \omega_3 - \alpha_1 \omega_4 + (\alpha_1 - 1) \omega_5 = 0 \quad (7)$$

$$\text{Gear unit X2: } \omega_6 - \alpha_2 \omega_7 + (\alpha_2 - 1) \omega_8 = 0 \quad (8)$$

The above linear equation group (3, 4, 5, 6, 7, 8) can be written in the following vector form:

$$A_n \vec{\omega} = 0 \quad (9)$$

where $\vec{\omega}$ equals $[\omega_1 \ \omega_2 \ \dots \ \omega_{10}]^T$; A_n is the speed coefficient matrix for the single epicyclic gearbox. A Matlab program is developed to obtain the speed coefficient matrix A_n and to derive the relationship of rotation speeds for each component. From running the program, the speed coefficient matrix A_n is calculated and the relationship of rotation speeds for the single epicyclic gearbox is shown in Appendix A.1. The speed relationships for MG1 and MG2 with the engine and the driveline are derived as:

$$n_3 - \frac{18}{5} n_1 + \frac{13}{5} n_2 = 0; \quad n_9 - n_2 = 0 \quad (10)$$

The engine speed is set to be 1 as a reference point so that the speed of part 1, namely $n_1 = 1$. The speed of each other part is actually the speed of that part relative to the speed of the engine. The transmission ratio of the system is defined as

$$i = \frac{\text{engine_speed}}{\text{output_speed}} = \frac{n_1}{n_2} = \frac{1}{n_2} \quad (11)$$

The Matlab program to plot MG1 and MG2 speed vs. transmission ratio for the single epicyclic gearbox is also developed. From running the program, the result is shown in Fig. 4. When the transmission ratio is 0.72, the speed of MG1 is 0. This means that at this point, if MG2 does not add torque to the wheels, zero power is transmitted via the electrical path. The point is called a node point. At the node point, the transmission efficiency is higher. The design of the powertrain should therefore make use of this node point. For example, this node point could be arranged to coincide with a

vehicle speed around 70 mile/h, which is a typical cruising speed on the highway, with a transmission ratio of 0.72.

A similar approach is used to derive the coefficient matrix for torques from the torque equation groups, which are made up by all the torque constraints for each part. The full torque and speed matrix is shown in the [Appendix A.2](#).

The torque relationships for MG1 and MG2 with the engine and the driveline are:

$$T_9 + \frac{13}{18}T_1 + T_2 = 0; T_{10} + \frac{5}{18}T_1 = 0 \quad (12)$$

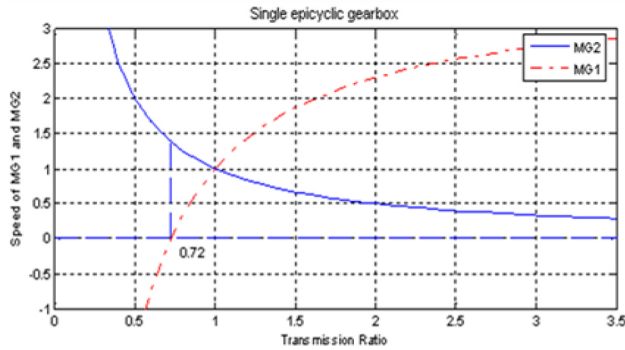


Figure 2. Relationship of transmission ratio and motor generator speeds for the single epicyclic gearbox.

Twin Epicyclic Transmission

Similarly, the simplified structure and encode of the 4 branch system is shown in [Fig. 5](#). There are 16 basic parts which are numbered from 1 to 16 respectively. There are 6 connected parts which are numbered from I to VI respectively. There are 4 gear units which are numbered from X1 to X4 respectively.

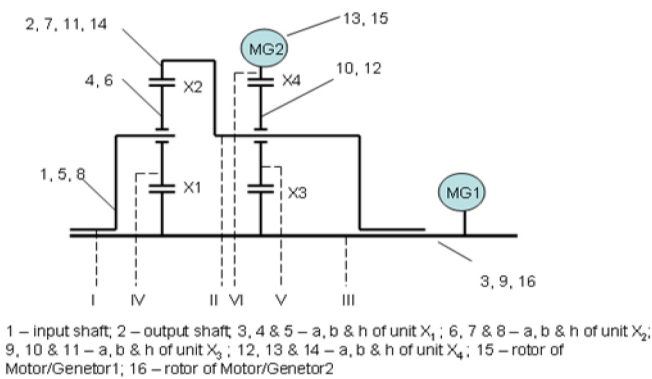


Figure 5. Encode of basic parts and connected part of the twin epicyclic transmission.

Similarly, Let α_1 to α_4 represent the gear ratios of units X1 to X4 respectively. Hence,

$$\alpha_1 = -N_4 / N_3 \quad (13)$$

$$\alpha_2 = N_7 / N_6 = N_7 / N_4 = (N_3 + 2N_4) / N_4 = 2 - 1/\alpha_1 \quad (14)$$

$$\alpha_3 = -N_{10} / N_9 \quad (15)$$

$$\alpha_4 = N_{13} / N_{12} = N_7 / N_{10} = (N_9 + 2N_{10}) / N_{10} = 2 - 1/\alpha_3 \quad (16)$$

So actually there are 2 variables, α_1 and α_3 in the system. The values of α_1 and α_3 are both set to be 0.8. To analyze the speeds of the system, first, the coefficient matrix is derived for the speed terms, which are made up by all the speed constraints for each part. For example, as a connected part 4, a simple constraint equation is $\omega_4 = \omega_6$, and as a gear system unit X1, $\omega_3 - \alpha_1\omega_4 + (\alpha_1 - 1)\omega_5 = 0$. Similar equations for all other parts lead to the complete speed matrix. The matrix is then solved here using the Gauss-Jordan elimination method in Matlab and the relationship of rotation speeds for the twin epicyclic gearbox is presented in [Appendix A.3](#). The relationships MG1 and MG2 with the engine and the output shaft are:

$$n_{15} + \frac{18}{13}n_1 - \frac{31}{13}n_2 = 0; n_{16} - \frac{18}{5}n_1 + \frac{13}{5}n_2 = 0 \quad (17)$$

A Matlab program is developed to plot the speeds of MG1 and MG2 relative to the transmission ratio. From running the program, the result is shown in [Fig. 6](#). There are two node points for the twin epicyclic gearbox, when the transmission ratios are 0.72 and 1.72 respectively. In contrast, for a single epicyclic gearbox, there is only one node point, when the transmission ratio is 0.72 ([Fig. 4](#)).

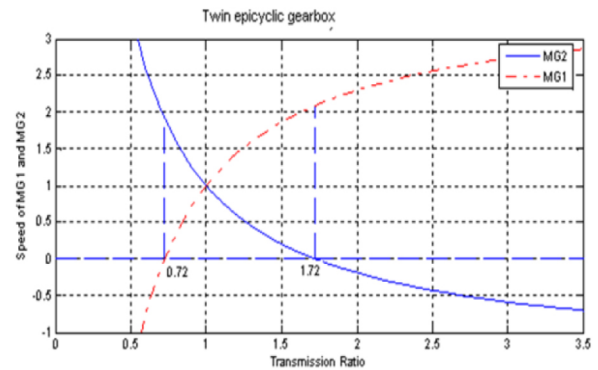


Figure 6. Relationship of transmission ratio and motor generator speeds for the 4 branch systems.

Similarly, a Matlab program is developed to obtain the torque coefficient matrix A_T and to obtain the relationship of torque for each component. From running the program, the torque coefficient matrix A_T and the relationship of torque for the twin epicyclic gearbox is presented in [Appendix A.4](#). The relations of the torque of the electrical machines with the

torque of the engine and the torque of the output shaft are obtained as the following equations.

$$T_{15} + \frac{169}{324}T_1 + \frac{13}{18}T_2 = 0; T_{16} + \frac{155}{324}T_1 + \frac{5}{18}T_2 = 0 \quad (18)$$

MODELING HYBRID ELECTRIC VEHICLE (HEV) PERFORMANCE

In this section, the modeling is further developed to investigate in more detail the effect of the single and twin epicyclic gearbox when fitted in a typical HEV. The modeling is therefore extended to include all the subsystems and the controller design of a typical HEV, and the vehicle is simulated over more typical driving cycles.

Three vehicle models are built-up: traditional ICE vehicle, and HEV with single and twin epicyclic gearboxes, respectively. Firstly, the subsystem models, such as the engine, transmission, and motor/generator are introduced. Then, the control strategy for an HEV is studied and the single and dual epicyclic systems are described in detail. The vehicle models are built based on the quasi-static approach [10]. This approach allows the fuel consumptions of each vehicle under different driving cycles to be calculated. The QSS toolbox [11] is used for some of the system elements such as driving cycles, engine, and battery model.

Vehicle Model

The modeling of the hybrid electric vehicle performance is done using the QSS Toolkit [11]. This is a quasi-static simulation package based on a collection of Simulink blocks and the appropriate parameter files that can be run in any Matlab/Simulink environment. The vehicle model is shown in Fig. 7.

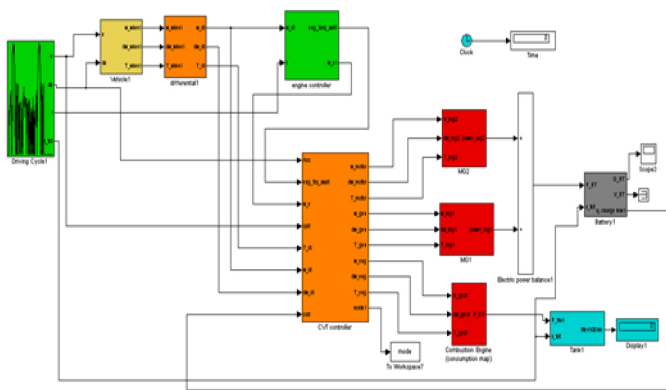


Figure 7. Overview of the hybrid vehicle model.

The data for the engine, motor generator and battery are taken from generic data in the QSS package and the other vehicle data is taken as follows: vehicle curb weight=1257 kg; drag coefficient, $C_d = 0.29$; frontal area = 2.23 m²; Tire radius = 0.292 m; and final drive = 3.95:1. It is not intended to represent any specific vehicle but rather to act as a generic

vehicle platform to focus attention on the differences obtainable from the two different PST arrangements. The traditional ICE vehicle model itself is straightforward.

There are 5 sub-systems: the driving cycle subsystem, vehicle subsystem, the gearbox subsystem, the combustion engine subsystem, and the fuel tank subsystem. The data for the engine and gearbox are taken from generic data in the QSS package as well.

The input to the model is one of the standard driving cycles - the NEDC and USA FTP-75 cycles are used extensively in this work. The solution procedure is based on stepping through the driving cycle at typically one second steps, calculating the equilibrium condition and then collecting all the data for plotting at the end of the cycle. The modeling assumptions are kept very simple in this initial work, so that no account is included for example of losses in the gear sets or differential. Thus, the focus of attention is on the overall efficiency of the engine and motor generator units and the major issue of whether it is possible to improve overall energy usage by operating the whole system at or near to the best efficiency points.

Engine Model

The engine model from QSS Toolbox is used in this research. The function of the engine model is to compute the fuel consumption from a consumption map. Inputs for the model include engine speed, engine acceleration and engine torque. The output of the model is the fuel consumption of the engine at each sampling point. The function of overload and over speed detection is built in the engine model. As soon as the engine torque or speed is over the limit, the simulation is stopped. The similar detection function is built in the motor/generator models. To finish the simulation with the whole driving cycle, once any overload or over speed is detected, the controller will reselect the related speed and/or torque, to make sure every component, including the engine and the motors, work within these the speed-torque limit.

The data for the fuel consumption map represents a small engine with maximum speed 500 rad/s and maximum torque 118 Nm. There are 3 parameters for the map: a vector ($1 \times n$) containing the rotational speed, a vector ($m \times 1$) containing the torque and an efficiency map ($n \times m$) containing the fuel efficiency point (kg/s) at each combination of speed and torque.

Transmission Models

Single epicyclic gearbox model

The layout of a single epicyclic gearbox is shown in Fig. 1 and the power flow of the single epicyclic gearbox is shown in Fig. 8. MG1, which is mainly used as a generator and MG2, which is mainly used as a motor, are connected to the battery, taking or saving electricity from or to the battery. The power of the engine is split into two ways: to the wheel via the ring gear, and to the MG1. The vehicle can be driven on