

The Proper Use of Plane Wave Models for Muffler Design

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ABSTRACT

In many industries, muffler and silencer design is primarily accomplished via trial and error. Prototypes are developed and tested, or numerical simulation (finite or boundary element analysis) is used to assess the performance. While these approaches reliably determine the transmission loss, designers often do not understand why their changes improve or degrade the muffler performance. Analyses are time consuming and models cannot be changed without some effort. The intent of the current work is to demonstrate how plane wave muffler models can be used in industry. It is first demonstrated that plane wave models can reliably determine the transmission loss for complicated mufflers below the cutoff frequency. Some tips for developing dependable plane wave models are summarized. Moreover, it is shown that plane wave models used correctly help designers develop intuition and a better understanding of the effect of their design changes.

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INTRODUCTION

Plane wave muffler modeling techniques are familiar to most acoustic engineers. At frequencies below what is known as the plane wave cut-on (or cutoff), the sound pressure across the cross-section of the muffler can be assumed constant. In that case, transfer matrix theory can be used to model most common acoustic elements like ducts, resonators, quarter wave tubes, and other components.

Though a little dated, the text by Munjal [1] provides an excellent overview of transfer matrix theory and develops the transfer matrices for most of the more common elements. Munjal et al. give a more concise summary in Reference [2], and Munjal and Mechel include the most complete set of transfer matrices for muffler and silencer elements to our knowledge in Reference [3].

The transfer matrix theory described in the aforementioned references is applicable to the case where muffler elements are cascaded in series with respect to one another. In that case, the successive transfer matrices can be multiplied together and an overall 2×2 transfer matrix can be calculated. Glav and Åbom [4] developed a scattering matrix approach in which elements can be connected arbitrarily and multiple paths can be considered. Other authors have followed suit. Panigrahi and Munjal [5] used a global $N \times N$ transfer matrix, and Mimani and Munjal used an impedance matrix approach [6].

Though scattering, global transfer, and impedance matrices are different, the plane wave assumption is still used in each case. More significantly, the modeling strategy employed will be the same. Commercial codes including SIDLAB [7] and ADEM [8] are available for the analysis of complex muffler systems.

In spite of the availability of suitable tools, there seems to be an averseness to using plane wave muffler software in many industries. In our experience, mufflers are primarily designed in 3 ways.

1. Designers adapt a muffler that has worked on a prior application to the task at hand. However, the frequency content of the source is often entirely different. Moreover, the muffler used is frequently more costly than necessary.
2. Designers use deterministic methods like boundary and finite element analysis to design the muffler. While deterministic methods are generally accurate, design changes require an adjustment to the CAD model, remeshing, and re-solving. Moreover, the transfer impedance properties of perforates, which are common in mufflers, must be input into the models whereas properties are already included in most commercial plane wave software. Accordingly, deterministic methods are best used for virtual prototyping, but turnaround times are too long for effective design usage.

3. Designers design, fabricate, and test for several iterations. This approach has some advantages. Many companies have machine shops that can fabricate mufflers quickly. Additionally, small mufflers can be mounted on the actual product and tested. However, plane wave simulation is preferred due to shorter turnaround times.

The chief drawback to using plane wave modeling is that some expertise is required to divide up a complicated muffler into appropriate components. Many engineers assume that plane wave assumptions are not appropriate for their muffler because of the geometric complexity. However, plane wave models can be used regardless of the complexity of the muffler provided the cutoff frequency is not exceeded.

The current work provides guidelines and tips for dividing a muffler into plane wave components. Moreover, it is shown that plane wave models are sometimes preferred. Two example mufflers are shown and results are compared with measurement with good agreement.

TRANSFER MATRIX THEORY

What follows is a very brief introduction to transfer matrix theory. The objective of this discussion is to provide the reader with just enough information to understand the modeling strategies that follow. Readers are encouraged to look at References [1, 2] for a more thorough overview of transfer matrix theory.

In general, there are three primary types of transfer matrix elements: ducts, transfer impedances, or parallel impedances. The transfer matrix for a duct (Figure 1) can be expressed as

$$\begin{Bmatrix} p_1 \\ Su_1 \end{Bmatrix} = \begin{bmatrix} \cos(kl) & \frac{j\rho c}{S} \sin(kl) \\ \frac{jS}{\rho c} \sin(kl) & \cos(kl) \end{bmatrix} \begin{Bmatrix} p_2 \\ Su_2 \end{Bmatrix} \quad (1)$$

where k is the acoustic wavenumber, l is the length, and S is the cross-sectional area of the duct. Note that a duct filled with fiber can be modeled using a complex wavenumber [3]. Additionally, specialized elements for catalysts (CC) and diesel particulate filters (DPF) are modeled as duct-like elements. The equations take into account cell size, wall thickness, and the flow resistivity through the membrane in the case of a DPF [9,10].

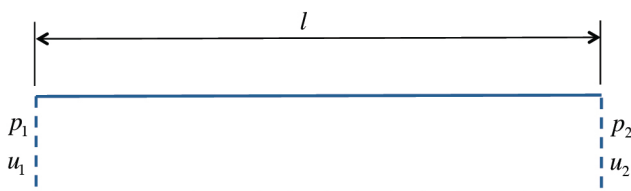


Figure 1. Schematic showing a duct and variables for the transfer matrix.

An acoustic transfer or series impedance is routinely used to model perforates (Figure 2). It is assumed that the particle velocity is continuous on each side and that there is an acoustic pressure drop. Accordingly, $u_1 = u_2$ and the transfer impedance can be expressed as

$$Z_{tr} = \frac{p_1 - p_2}{Su} \quad (2)$$

It follows that the transfer matrix can be expressed as

$$\begin{Bmatrix} p_1 \\ Su_1 \end{Bmatrix} = \begin{bmatrix} 1 & Z_{tr} \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} p_2 \\ Su_2 \end{Bmatrix} \quad (3)$$

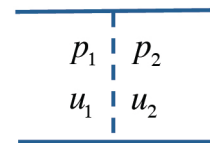


Figure 2. Schematic showing a transfer impedance.

Several equations are available in the literature for the modeling of perforates. References [3] and [11] provide suitable formulas which also include the effect of flow. In general, the effect of flow should not be neglected in the case of perforates. If there are no perforates, flow can often be ignored.

A parallel or branch impedance is representative of side branches like quarter wave tubes or Helmholtz resonators (Figure 3). Continuity of sound pressure is assumed across the opening of the branch (i.e., $p_1 = p_2 = p_B$) and conservation of mass is enforced (i.e., $Su_1 = Su_2 + S_B u_B$). The transfer matrix for a parallel element is

$$\begin{Bmatrix} p_1 \\ Su_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{Z_B} & 1 \end{bmatrix} \begin{Bmatrix} p_2 \\ Su_2 \end{Bmatrix} \quad (3)$$

The branch impedance for elements like Helmholtz resonators and quarter wave tubes is well known.

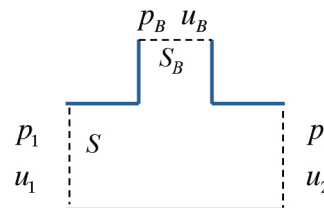


Figure 3. Schematic showing a branch or parallel impedance.

It is especially important to recognize that quarter wave tubes can be oriented in a number of different ways. Some examples are shown below in Figure 4. Though quarter wave tubes are occasionally used as a side branch (Figure 4a), they are more commonly part of a bend (Figure 4b) or integrated into an expansion chamber as an extended inlet or outlet (Figure 4c).