

Analysis of Friction Induced Stability, Bifurcation, Chaos, Stick-slip Vibration and their Impacts on Wiping Effect of Automotive Wiper System

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ABSTRACT

A 2 DOF nonlinear dynamic model of the automotive wiper system is established. Complex eigenvalues are calculated based on the complex modal theory, and the system stability as well as its dependence on wiping velocity is analyzed. Bifurcation characteristics of frictional self-excited vibration and stick-slip vibration relative to wiping velocity are studied through numerical analysis. Research of nonlinear vibration characteristics under various wiping velocities is conducted by means of phase trajectories, Poincaré map and frequency spectrum. The pervasive stick-slip vibration during wiping is confirmed, and its temporal and spatial distributions are analyzed by way of time history and contour map. Duty ratio of stick vibration and statistics of scraping residual are introduced as quantitative indexes for wiping effect evaluation.

Results indicate that the negative slope of frictional-velocity characteristic is the root cause of system instability. As the wiping velocity decreases, the vibration state transforms from periodic to quasi-periodic and then to chaos in both high and low velocity ranges. The wiping process is accompanied with prominent stick-slip vibration except when the nominal wiping velocity exceeds 0.725m/s. To increase the wiping velocity can improve system stability, restrain stick-slip vibration and reduce adverse impacts on wiping effect caused by the uneven distribution of scrapings.

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INTRODUCTION

The automotive wiper system is designed to wipe out rain, fog, frost, snow, dust and other dirt attached to the windscreen, to provide the driver a clear vision and to guarantee the driving safety[1]. However, when the structural design, material properties or operating control parameters are inappropriate, the wiper blades demonstrate complicated nonlinear performance due to the complex features of frictional force, such as dependence on wiping velocity, nonlinearity and time-varying characteristics. Thus affects the scrape rate and further causes various adverse impacts of vibration and noise [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

Given the complexity and significance of frictional vibration of the automotive wiper system, its mechanism [2, 3, 4, 5, 6, 7, 8, 9, 14], influencing factors [9, 10, 11, 12, 13, 14, 15] and control measures [7, 11, 12] have drawn considerable attention of researchers. Previous studies have both theoretically and experimentally probed into the stability of nonlinear frictional vibration of the wiper system, and proposed relevant control methods [2, 3, 7, 11, 12] of chaotic vibration based on the analysis of fundamental characteristics of nonlinear vibration.

Meanwhile, studies on wiping effect have confirmed the wiper arm pressing force, blade features, windscreen surface topography and vehicle speed which causes variation of aerodynamic characteristics, as crucial influencing factors [15, 9, 10, 11, 12, 13, 14, 15]. Research on wiping effect improvement has been conducted through adjusting wiper arm pressing force and attack angle [5, 6, 13, 15]. Nevertheless, limitations of research so far still exist as follows: firstly, the consistency and universality of current research results remain to be verified due to different research objects and their complexities; secondly, some influential literatures [2, 3] contain certain calculation and analysis errors; thirdly, because the stick-slip vibration can cause wiping velocity fluctuation and produce scraping residues on the windscreen, the issue of wiping effect has begun to arouse the public concern, yet research into influencing mechanism and extent of nonlinear vibration on wiping effect has barely started. Therefore, it's essential to intensify the systematic study of nonlinear vibration and wiping effect of wiper system.

Under such context, this study establishes a 2 DOF nonlinear dynamic model of wiper system based on dialectic analysis of previous research [2, 9, 10, 14]. Numerical computation is adopted for analyzing system stability and bifurcation characteristics relative to wiping velocity. Additionally, changing patterns of periodic, quasi-periodic and chaotic vibrations relative to wiping velocity are investigated. Furthermore, qualitative and quantitative evaluations are proposed based on the analysis of nonlinear stick-slip vibration and its impact on wiping effect, in order to comprehensively reveal the impact mechanism. Accordingly, results in this study can provide theoretical guidance for frictional characteristics matching, wiping velocity optimization, scrape rate improvement as well as vibration control and noise reduction.

2. DOF NONLINEAR DYNAMIC MODELING OF WIPER SYSTEM

As shown in Figure 1, a 2 DOF nonlinear dynamic model of wiper system based on dialectic analysis of previous studies[2, 3] has been established. Subscripts *D* and *P* stand for driver and passenger sides, which are substituted by *i=D, P* in the following sections for specification and equation derivation. The notions L_i indicate nominal positions of wiper arms without vibration deflection. The quantities l_i are distances between the pivot centers and the wiper arm terminals. The quantities I_i are moments of inertia of each wiper arm assembly, while the K_i and K_M represent torsional angular stiffness coefficients of each wiper arm and of driver motor output terminal crankshaft. The quantities C_P and C_{DP} are torsional angular damping coefficients, M_i are frictional resistance moments of wiper blades, $\dot{\psi}_i$ are nominal input angular velocities of wiper arms and θ_i are angular deflections relative to positions L_i .

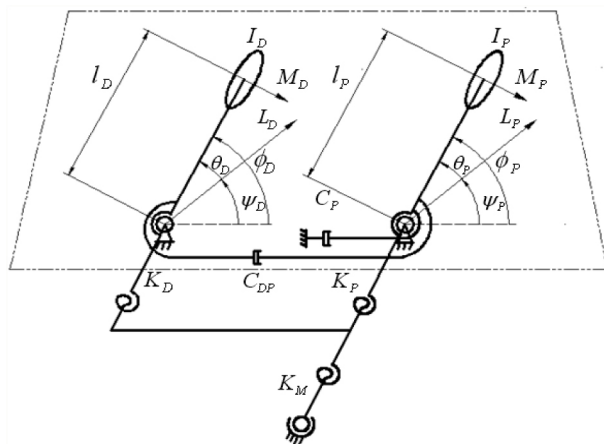


Figure 1. 2 DOF Nonlinear Dynamic Model of Wiper System

The nominal, actual and vibration deflection linear velocities at the terminals of wiper arms can be obtained through equations (1), (2), (3). Differential equations of motion of wiper arms are presented as equation (4), among which R_i and D_i represent moments produced by the elastic restoring force and the damping force, M_i are moments of frictional resistance, N_i are normal pressing forces of wiper arms, μ_0 stands for static

friction coefficient between wiper blades and windscreen, $\mu(\dot{z}_i)$ (Figure 2) are frictional characteristics derived from polynomial data fitting of experimental results[9].

$$v_i = \dot{\psi}_i l_i, (i = D, P) \quad (1)$$

$$\dot{z}_i = (\dot{\theta}_i + \dot{\psi}_i) l_i, (i = D, P) \quad (2)$$

$$u_i = \dot{\theta}_i l_i, (i = D, P) \quad (3)$$

$$\begin{cases} I_i \ddot{\theta}_i = -R_i - D_i - M_i(\dot{z}_i) & \dot{z}_i \neq 0 \\ I_i \ddot{\theta}_i = -R_i - D_i & \dot{z}_i = 0, |R_i| \geq N_i l_i \mu_0 \\ I_i \ddot{\theta}_i = 0 \quad (\dot{\theta}_i = -\dot{\psi}_i) & \dot{z}_i = 0, |R_i| \leq N_i l_i \mu_0 \end{cases} \quad (4)$$

$$\begin{bmatrix} R_D \\ R_P \end{bmatrix} = \begin{bmatrix} \frac{K_D(K_P + K_M)}{K_D + K_P + K_M} & \frac{-K_D K_P}{K_D + K_P + K_M} \\ \frac{-K_D K_P}{K_D + K_P + K_M} & \frac{K_P(K_D + K_M)}{K_D + K_P + K_M} \end{bmatrix} \begin{bmatrix} \theta_D \\ \theta_P \end{bmatrix} = K \theta \quad (5)$$

$$\begin{bmatrix} D_D \\ D_P \end{bmatrix} = \begin{bmatrix} C_{DP} & -C_{DP} \\ -C_{DP} & C_{DP} + C_P \end{bmatrix} \begin{bmatrix} \dot{\theta}_D \\ \dot{\theta}_P \end{bmatrix} = C_0 \dot{\theta} \quad (6)$$

$$M_i = N_i l_i \mu(\dot{z}_i) \quad (7)$$

$$\mu(\dot{z}_i) = \mu_0 \operatorname{sgn}(\dot{z}_i) + \mu_1 \dot{z}_i + \mu_2 \dot{z}_i^3 \quad (8)$$

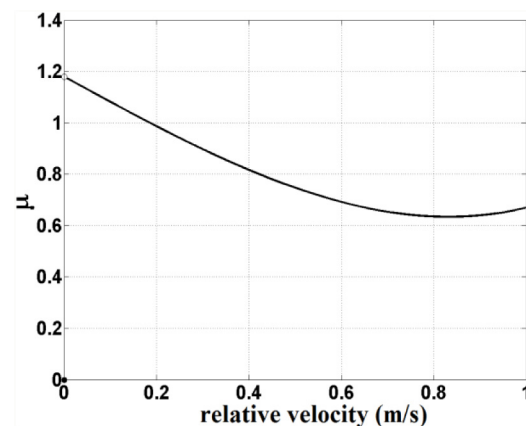


Figure 2. Frictional Characteristic

STABILITY ANALYSIS OF FRICTION INDUCED VIBRATION

Complex Modal Analysis Method

The 2 DOF nonlinear dynamic system is firstly transformed into the state space form, namely, two second-order systems are transformed into four first-order systems. Then, damping complex eigenvalues are calculated and the system stability is qualitatively analyzed based on the complex modal theory.

Frictional characteristics are simplified by removing the higher orders of $\dot{\theta}_i$ via formula (9). Thereby moments of frictional resistance M_i can be transformed by formula (10). Likewise, differential equations of motion can be converted into formula (11) and further changed into the state space form in formula (12) with the introduction of state vector Y .

System eigenvalues are calculated and unified in the form of formula (13), where the imaginary part ω represents the system natural angular frequency. When the real part σ is positive, the damping coefficient is negative and the system is unstable; otherwise, when σ is negative, the damping coefficient is positive and the system is stable.

$$\begin{aligned}\mu(\dot{z}_i) &= \mu_0 \operatorname{sgn}(\dot{z}_i) + \mu_1 \dot{z}_i + \mu_2 \dot{z}_i^3 \\ &\approx (\mu_0 \operatorname{sgn}(\dot{z}_i) + \mu_1 v_i + \mu_2 v_i^3) + (\mu_1 l_i + 3\mu_2 l_i v_i^2) \dot{\theta}_i \\ &= s_i + q_i \dot{\theta}_i\end{aligned}\quad (9)$$

Where, $s_i = \mu_0 \operatorname{sgn}(\dot{z}_i) + \mu_1 v_i + \mu_2 v_i^3$, $q_i = \mu_1 l_i + 3\mu_2 l_i v_i^2$.

$$\begin{bmatrix} M_D \\ M_P \end{bmatrix} = \begin{bmatrix} N_D l_D \mu(\dot{z}_D) \\ N_P l_P \mu(\dot{z}_P) \end{bmatrix} = \begin{bmatrix} N_D l_D q_D & 0 \\ 0 & N_P l_P q_P \end{bmatrix} \begin{bmatrix} \dot{\theta}_D \\ \dot{\theta}_P \end{bmatrix} + \begin{bmatrix} N_D l_D s_D \\ N_P l_P s_P \end{bmatrix}\quad (10)$$

$$I \ddot{\theta}_D + C \dot{\theta}_D + K \theta = T\quad (11)$$

$$\text{Where, } I = \begin{bmatrix} I_D & 0 \\ 0 & I_P \end{bmatrix}, C = C_0 + \begin{bmatrix} N_D l_D q_D & 0 \\ 0 & N_P l_P q_P \end{bmatrix},$$

$$T = \begin{bmatrix} -N_D l_D s_D \\ -N_P l_P s_P \end{bmatrix}.$$

$$A \dot{Y} + B Y = F\quad (12)$$

$$\text{Where, } Y = \begin{bmatrix} \theta_i \\ \dot{\theta}_i \end{bmatrix}, A = \begin{bmatrix} C & I \\ I & 0 \end{bmatrix}, B = \begin{bmatrix} K & 0 \\ 0 & -I \end{bmatrix}, F = \begin{bmatrix} T \\ 0 \end{bmatrix}.$$

$$\lambda = \sigma + \sqrt{-1}\omega\quad (13)$$

System Stability Analysis

System eigenvalues under the varying nominal wiping velocity are calculated through MATLAB with model parameters [2] listed in Table 1. Changes of system eigenvalues relative to the nominal wiping velocity in Figure 3 reveal that:

Firstly, the system is unstable with positive real parts of eigenvalues under relative low nominal wiping velocity V_D . As V_D increases, the positive real parts of system eigenvalues decrease gradually and begin to turn negative when V_D rises to 0.78m/s, indicating the system starts to reach the stable state. When V_D exceeds 0.795m/s, all the real parts of system eigenvalues become negative and the system eventually turns stable.

Secondly, according to Figure 2, the curve of frictional characteristic presents negative slope in the relative low wiping velocity range and the slope becomes positive in the relative high wiping velocity range. Consequently, there exists a critical turning point for this change of slope. Hence the negative slope of frictional characteristics is the root cause for system instability.

Thirdly, to increase the wiping velocity benefits the improvement of system stability.

Table 1. Model Parameters

Parameter	Value ($\times 10^{-2}$)	Unit	Parameter	Value ($\times 10^{-1}$)	Unit
I_D	4.07	kg · m ²	I_D	4.70	m
I_P	3.67	kg · m ²	I_P	4.50	m
K_D	7.20	N · m/ rad	N_D	73.5	N
K_P	7.51	N · m/ rad	N_P	59.8	N
K_M	3.53	N · m/ rad	μ_0	11.8	/
C_{DP}	1.00	N · m · s/ rad	μ_1	-9.84	/
C_P	1.00	N · m · s/ rad	μ_2	4.74	/

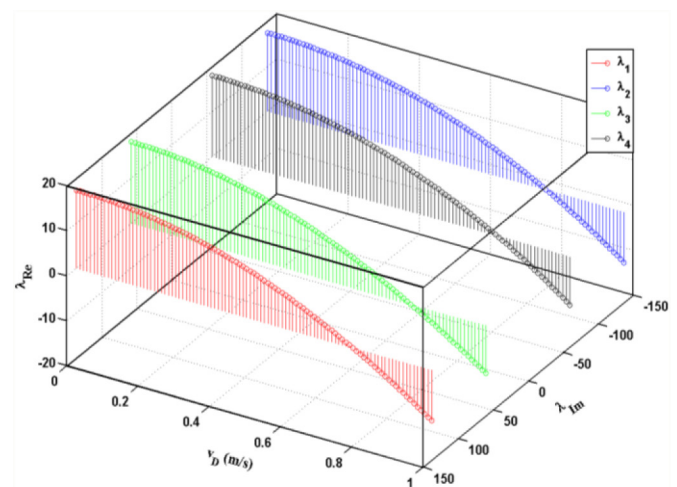


Figure 3. Complex Eigenvalues Relative to Wiping Velocity

ANALYSIS OF BIFURCATION CHARACTERISTICS OF FRICTIONAL SELF-EXCITED VIBRATION RELATIVE TO WIPING VELOCITY

Bifurcation Characteristic of Self-excited Vibration Relative to Wiping Velocity

Simulation and analysis of the wiper system dynamics is conducted with MATLAB/SIMULINK based on the system differential equation of motion and model parameters in [Table 1](#). The range of nominal wiping velocity is set as 0.1~0.8m/s in accordance with practical wiping velocity scope and the above analysis of system stability. By studying stable numerical solutions that meet the criteria of $\dot{\theta}_i = 0 (i = D, P)$, bifurcation diagram of wiper system nonlinear frictional vibration is obtained as shown in [Figure 4](#). Take the bifurcation analysis of θ_D as an example:

Firstly, the wiping velocity range can be divided into high and low ranges by V_D of 0.485m/s. As V_D decreases, vibration state changes in the same pattern during both two ranges, namely, from the periodic to the quasi-periodic and then to the chaotic. There also exists chaotic intermittency in the high wiping velocity range, indicates the alternating occurrences of chaos and quasi-periodic vibration.

Secondly, the wiper system becomes stable with no more self-excited vibration when V_D exceeds 0.795m/s.

Obviously, same rules apply equally to the bifurcation characteristic of θ_P . Division of different wiping velocity ranges and corresponding vibration states are summarized in [Table 2](#).

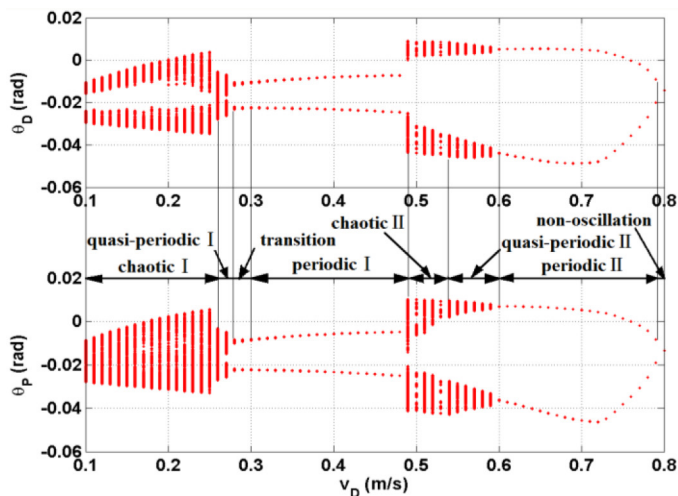


Figure 4. Bifurcation Diagram of Wiper Arm Angular Deflection

Table 2. Wiping Velocity Ranges and Vibration States

Low Velocity Range (m/s)	Vibration State	High Velocity Range (m/s)	Vibration State
[0.100, 0.258]	chaotic	[0.485, 0.535]	chaotic (alternates with quasi-periodic)
[0.258, 0.275]	quasi-periodic	[0.535, 0.600]	quasi-periodic
[0.275, 0.300]	transition	[0.600, 0.795]	periodic
[0.300, 0.485]	periodic	[0.795, 0.800]	non-oscillation

Self-excited Vibration under Various Wiping Velocities

Typical wiping velocities are selected from various velocity ranges in [Table 2](#) for detailed analysis of system self-excited vibration. Analytical methods including time histories, 2D phase trajectories, Poincaré map and frequency spectrum are adopted. The Poincaré map is obtained from the spatial 3D phase trajectories $(\theta_D, \dot{\theta}_D, \dot{\theta}_P)$ intercepted by the hyperplane of $\dot{\theta}_P = 0$ [20]. System self-excited vibration characteristic under various wiping velocities are demonstrated in [Appendix 1](#) and summarized in [Appendix 2](#), among which f_{d1} and f_{d2} stand for system second-order damping natural frequencies. Based on nonlinear bifurcation theory [15, 16, 17], joint analysis of [Table 2](#), [Appendix 1](#) and [2](#) manifests that:

Firstly, as wiping velocity decreases, the vibration state changes in the same pattern during both high and low velocity ranges, namely, from periodic to quasi-periodic and then to chaotic. There also accompanies prominent stick-slip vibration which seriously affects the scrape rate.

Secondly, chaotic intermittence happens in the high velocity range, indicates the alternating occurrences of chaos and quasi-periodic vibration, thus fully illustrates the sensitivity of system complex vibration toward wiping velocity.

Thirdly, there is no more stick-slip vibration in the high velocity range when the system is in the state of periodic vibration or non-oscillation. Hence the ideal wiping effect can be achieved.

Nonlinear Stick-slip Vibration of Wiper System

As previously discussed, the stick-slip vibration during wiping can lead to the uneven distribution of scraping residues on the windscreen which severely affects the wiping effect. Therefore, it is essential to investigate into the stick-slip vibration characteristics of the wiper system. This section starts with the bifurcation diagram of wiper arm angular deflection to study the prevalence and divergence of stick-slip vibration under various wiping velocities. Then the vibration phase trajectories of $(\theta_D, \dot{\theta}_D, \dot{\theta}_P)$ and $(\theta_P, \dot{\theta}_P, \dot{\theta}_D)$ are obtained to further analyze the spatial and temporal distributions of stick-slip vibration under various wiping velocities.