

Methods in Vehicle Mass and Road Grade Estimation

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ABSTRACT

Dynamic vehicle loads play critical roles for automotive controls including battery management, transmission shift scheduling, distance-to-empty predictions, and various active safety systems. Accurate real-time estimation of vehicle loads such as those due to vehicle mass and road grade can thus improve safety, efficiency, and performance. While several estimation methods have been proposed in literature, none have seen widespread adoption in current vehicle technologies despite their potential to significantly improve automotive controls. To understand and bridge the gap between research development and wider adoption of real-time load estimation, this paper assesses the accuracy and performance of four estimation methods that predict *vehicle mass* and/or *road grade*. These include recursive least squares (RLS) with multiple forgetting factors; extended Kalman filtering (EKF); a dynamic grade observer (DGO); and a method developed by this research: parallel mass and grade (PMG) estimation using a longitudinal accelerometer.

The estimation methods and models were constructed, numerous vehicle tests were performed, and data was evaluated off-line by the estimation approaches. It is found that RLS and EKF yield estimates within 5% of their actual values if provided initial values close to true initial states. To improve estimation when inaccurate initial values are provided, a mass selection algorithm is proposed that determines mass based on converged values from concurrently-operating EKF estimators. Its potential for accurate mass and grade estimation is demonstrated. PMG estimation provides the most reliable and accurate results, and demonstrates the greatest potential for successful real-time implementation to advance the performance, economy, and reliability of future vehicle controls.

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INTRODUCTION

Changes in inertial parameters of road vehicles have a significant effect on ride quality, performance, handling, and energy efficiency. As automotive subsystems become increasingly automated, accurate knowledge of inertial parameters is critical to ensuring passenger safety while delivering improved vehicle performance. Therefore, it is evident that vehicle mass and road grade are two parameters related to critical loads that must be estimated accurately and reliably.

For successful operation, various controllers require accurate identification of vehicle mass, but passenger and cargo loading may cause variations in mass of up to 50% for small cars from trip to trip. The difference between accurate real-time identification of mass and its assumption of constant value

throughout all operating conditions directly reflects a reduction in the accuracy and performance of automotive controls. Meanwhile, light trucks and cargo vans may vary in mass by 400% depending on payload conditions. Vehicle mass is directly related to tire normal forces, which influence lateral and longitudinal tire force generation [1]. Consequently, active safety technologies such as anti-lock braking systems, collision avoidance, and stability control can benefit from accurate knowledge of vehicle mass [2, 3]. It has been shown that for typical passenger vehicles, a 10% increase in vehicle mass corresponds to a 2.4% to 4.1% increase in energy usage [4]. Neglecting mass variations negatively affects the accuracy of distance-to-empty predictions and compromises effective battery management of hybrid electric or battery electric vehicles. The presence of road grade can introduce a

significant load to the powertrain. Transmission shift scheduling, adaptive cruise control, and hill start assist [5] are technologies that benefit from accurate knowledge of road grade.

While various controllers on-board a vehicle can independently estimate relevant parameters, it may be advantageous to have a single, accurate, on-line estimate of vehicle mass and road grade applied across controller platforms. This avoids redundant computations and conflicting parameter estimates, which is crucial when a supervisory controller is coordinating the behavior of multiple control systems [6]. While there have been numerous studies for inertial load estimation which employ various prediction methods and modeling approaches [3, 7, 8, 9, 10], many previous efforts toward on-line mass and grade estimation using existing on-board sensor systems have been based on vehicle longitudinal dynamics models due to the many common driving scenarios for which such models apply. Estimation approaches include recursive least squares (RLS) with multiple forgetting factors [11, 12, 13], extended Kalman filtering (EKF) [5], a dynamic grade observer (DGO) [14] requiring only longitudinal acceleration and an estimate of powertrain torque, and grade estimation using kinematic information provided by a longitudinal accelerometer [15]. As developed in this paper, kinematic information may also be used to estimate mass without explicit calculation of road grade: a *parallel* mass and grade estimation.

Mass and grade estimation methods reported in literature are not widely employed in production automotive controls at present. In addition, there are few reported efforts which directly compare such estimation approaches using data that represents a range of driving conditions [13], nor have they been comprehensively studied and compared using consistent experiments. A study to evaluate existing estimation schemes side-by-side may yield critical information regarding the accuracy and reliability of each method, may highlight impediments to their implementation which explain the lack of industry adoption, and may also uncover novel approaches by which the existing methods could be integrated or improved upon. Therefore, the goal of this research is to compare the existing real-time mass and grade estimation approaches, seek insights regarding suitable method implementation, and propose new solutions which advance the state-of-the-art of real-time vehicle load parameter estimation. The existing longitudinal dynamics-based RLS, EKF, and dynamic grade observer (DGO) methods are compared alongside a parallel mass and grade (PMG) estimation approach. Various driving data sets are employed to evaluate estimator reliability and accuracy. While global positioning (GPS)-based estimation approaches are reported in the literature [16, 17], comparison of such approaches is omitted because GPS information is not necessarily available or reliable as compared to vehicle sensors requiring only local access of dynamic information.

The remainder of this paper is structured as follows: The first section introduces the longitudinal dynamics model used by all the estimation methods. Then, the on-line estimation methods compared in this study are described. Experiments to evaluate

these methods are detailed and the results are presented. A mass selection algorithm for use with simultaneous mass and grade estimation methods is proposed and validated using the convergence of parameters under several different initial conditions. Then, the independent mass and grade estimation methods are assessed. Finally, the last section summarizes the conclusions and discusses future research opportunities.

LONGITUDINAL DYNAMICS MODEL

For typical driving scenarios, longitudinal motion is dominant and its measurement provides a near-continuous data set that can be used for real-time estimates of vehicle loads. Interruptions of estimation may occur during gear shifts, braking, and periods of significant vehicle yaw and roll because such events are not easily captured by conventional longitudinal dynamics modeling. The forces influencing automotive longitudinal dynamics are shown in Figure 1. The tractive force due to tire-road interactions is F_w . This study assumes no tire slip and that the tractive force may be accurately determined by $F_w = T_w/r_w$, where T_w is wheel torque and r_w is tire radius. Forces opposing longitudinal motion include aerodynamic drag F_{aero} , rolling resistance F_μ , grade forces F_{grade} and braking F_{fb} if present.

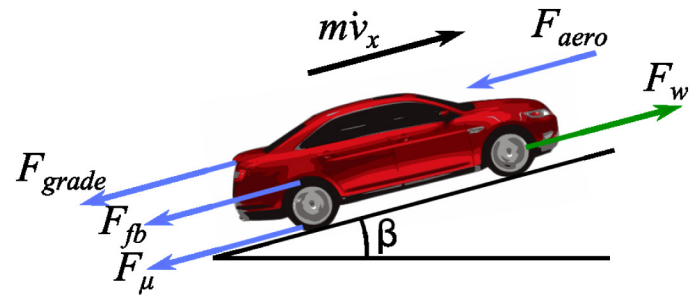


Figure 1. Illustration of forces influencing vehicle longitudinal dynamics, including tractive force F_w , aerodynamics drag force F_{aero} , grade force F_{grade} , rolling resistance force F_μ and friction brake force F_{fb} .

From these assumptions, the longitudinal dynamics of a vehicle may be expressed through Newton's 2nd law as

$$m\dot{v}_x = \frac{T_w}{r_w} - \frac{1}{2}\rho C_d A_f v_x^2 - mg(\mu \cos \beta + \sin \beta) \quad (1)$$

where m is vehicle mass; v_x is longitudinal velocity; ρ is the density of air; C_d is the vehicle's drag coefficient; A_f is the vehicle's frontal area, g is the gravitational constant, μ is the vehicle's rolling resistance, and β is road grade. The wheel torque T_w may be computed from a measurement on the driveline, combined with a driveline model that considers losses due to friction and inertial effects. Making the substitution $\beta_\mu = \tan^{-1} \mu$ allows equation (1) to be written as:

$$\dot{v}_x = \left(\frac{T_w}{r_w} - \frac{1}{2}\rho C_d A_f v_x^2 \right) \frac{1}{m} - \frac{g}{\cos(\beta_\mu)} \sin(\beta + \beta_\mu)$$

Equation (2) separates the loading effects of vehicle mass and road grade, and is used as the basic dynamic equation to which mass and grade estimation methods are applied in this study.

If the vehicle is equipped with a longitudinal accelerometer, road grade is computed from equation (3) relating measured acceleration a_x to the differentiated longitudinal velocity $\dot{v}_x$. On a horizontal surface, these two quantities are identical. However, if the accelerometer is tilted due to road grade or vehicle pitch, it also captures the vector projection of gravity along the axis of sensor measurement. If the accelerometer is tilted from the horizontal plane by β and vehicle pitching is absent, then $g \sin(\beta)$ is the component of gravity projected onto its measurement axis [15, 17].

$$a_x = \dot{v}_x + g \sin(\beta) \quad (3)$$

ON-LINE ESTIMATION METHODS

This section presents mass and grade estimation methods that are evaluated and compared in this study. They include simultaneous mass and grade estimation approaches (RLS and EKF), a dynamic grade observer (DGO) that only estimates grade, and a parallel mass and grade (PMG) estimator which uses a longitudinal accelerometer. For more detailed information on the development of these estimation methods, readers are encouraged to reference the applicable sources cited in the following subsections.

Recursive Least Squares (RLS) with Multiple Forgetting Factors

Recursive least squares is a well-established method for estimating parameters in real time. It may be used with forgetting factors to account for parameters that are slowly-varying [18]. Vahidi et al. [12] presented a form of RLS that uses multiple forgetting factors for mass and grade, to reflect the fact that mass is constant while grade may slowly vary. Thus, this method *simultaneously* estimates vehicle mass and road grade. The approach is summarized below.

Equation (2) is rewritten more compactly as

$$\dot{v}_x = [\phi_1 \ \phi_2] \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad (4)$$

where

$$\phi_1 = \left(\frac{T_w}{r_w} - \frac{1}{2} \rho C_d A_f v_x^2 \right) \quad (5a)$$

$$\phi_2 = \frac{g}{\cos(\beta_\mu)} \quad (5b)$$

$$\theta_1 = \frac{1}{m} \quad (5c)$$

$$\theta_2 = \sin(\beta + \beta_\mu) \quad (5d)$$

Two different forgetting factors, $\lambda_1, \lambda_2 < 1$, are applied to the RLS update equations for mass and grade respectively. These forgetting factors influence parameter update gains as well as convergence of the covariance matrix. A forgetting factor of 1 is used when the estimated parameter is assumed to be constant, in which case all prior information is applicable for identifying the parameter's current value. Consequently, the mass forgetting factor λ_1 should be defined very close to unity because mass is assumed to be essentially constant for the duration of a trip. The forgetting factor for grade λ_2 should be less than λ_1 since grade is more likely to vary over the course of a continuous driving event as compared to vehicle mass loading. Together $\lambda_{1,2}$ represent tuning parameters for RLS mass and grade estimation and detailed evaluation of their influence may be found in [12].

Extended Kalman Filter (EKF)

Kalman filters are used extensively in state estimation, and can be implemented for *simultaneous* mass and grade estimation if these two parameters are treated as system states with low variance. Due to the effects of aerodynamic drag, equation (1) is not linear in v_x . Thus the extended Kalman filter [19] must be used to linearize the system about an operating velocity. An EKF discretized by distance to estimate constant mass and grade was presented by Winstead et al. [5] in the context of adaptive cruise control, but was not explored for varying grade as is investigated in this study. In this paper a discrete-time filter is formulated using an Euler approximation for the following state vector at time k .

$$x(k) = \begin{bmatrix} v_x(k) \\ \theta_1(k) \\ \theta_2(k) \end{bmatrix} \quad (6)$$

v_x is longitudinal vehicle velocity while θ_1 and θ_2 are defined in equation (5). The system propagates in one time step as

$$x(k+1) = x(k) + T_s f(x(k), u(k)) + w(k) \quad (7)$$

with differentiate state transition model

$$f(x(k), u(k)) = \begin{bmatrix} \phi_1(k)\theta_1(k) + \phi_2(k)\theta_2(k) \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

and process noise

$$w(k) = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \quad (9)$$

T_s is the sampling period, and process noise $w(k)$ is assumed to be zero-mean with diagonal covariance matrix Q . Velocity is the observable quantity, thus the observation model is

$$y(k+1) = Hx(k+1) + v(k+1) \quad (10)$$

The observation matrix H is $[1 \ 0 \ 0]$ and $v(k+1)$ represents observation noise with zero mean and covariance R . The model described in [equations \(7,8,9,10,11\)](#) may be implemented with the extended Kalman filter. A thorough explanation of the filter, including linearization and equations for the prediction and update step, is found in [\[13\]](#).

Dynamic Grade Observer (DGO)

If mass is known, grade is the only unknown parameter in [equation \(1\)](#). McIntyre et al. [\[14\]](#) formulate vehicle longitudinal dynamics as in the manner of [equation \(4\)](#) and propose an observer to update estimates of grade, related to θ_2 . Therefore, the DGO method *only estimates grade* and does not independently provide a vehicle mass estimate.

Consider the observation error between measured and modeled velocity

$$e = v - \hat{v} \quad (11)$$

Modeled velocity is found by integrating modeled acceleration based on estimated road grade, and the error derivative

$$\dot{e} = \dot{v} - \dot{\hat{v}} \quad (12)$$

The following observer is proposed

$$\dot{e} + e = \phi_2 \dot{\theta}_2 - (k_1 + 1)(\dot{e} + e) + k_2 \text{sgn}(e) + \dot{e} \quad (13)$$

k_1 and k_2 are observer gains. The proof and conditions for convergence, and a demonstration of the DGO in combination with an RLS estimator for mass, can be found in [\[14\]](#).

Parallel Mass and Grade (PMG) Estimator Using a Longitudinal Accelerometer

A parallel mass and grade estimation approach is devised here which *independently* estimates mass and grade when a longitudinal accelerometer is available on-board the vehicle. An increasing number of vehicles on the road today are equipped with reliable and accurate longitudinal accelerometers. These sensors capture a vehicle's longitudinal acceleration but also measure the component of gravity along the measurement axis

due to tilting caused by grade and body pitch, as described in [equation \(3\)](#). This allows direct extraction of road grade information with only two signals assuming that vehicle pitching is absent from the vehicle dynamics, as has been earlier recognized in the literature [\[15, 17\]](#).

Because the accelerometer signal is affected by noise factors including vehicle bounce and pitch motions, [equation \(3\)](#) may not always be accurate for computation of road grade. To mitigate these effects, this research utilizes a weighted moving average filter based on signal variance over a buffer length. Since grade is a slowly varying parameter, large signal variance during a short buffer period indicates significant bounce and pitch motion, and the grade estimate update during this time is treated with lower confidence. In this manner, grade is *independently estimated* using available vehicle data and the accelerometer. During periods of constant acceleration, vehicle pitching may cause the grade estimate to be biased. Although the proposed PMG approach neglects this behavior, a pitch correction factor may be determined either experimentally by measuring steady state pitch angles under a range of constant accelerations on flat ground. Alternatively, this pitch correction factor can be determined analytically as a function of longitudinal acceleration if suspension parameters are known.

Once an estimate of grade is obtained, a longitudinal accelerometer also allows for a *parallel estimation* of vehicle mass. We achieve this by rearranging [equation \(1\)](#), and assuming the rolling resistance to be unaffected by road grade ($\mu \cos(\beta) \approx \mu$). This leads to

$$m(\dot{v}_x + g(\mu + \sin \beta)) = \frac{T_w}{r_w} - \frac{1}{2} \rho C_d A_f v_x^2 \quad (14)$$

$$m(a_x + \mu g) = \frac{T_w}{r_w} - \frac{1}{2} \rho C_d A_f v_x^2 \quad (15)$$

$$m = \frac{\phi}{a_x + \mu g} \quad (16)$$

where ϕ is the right hand side of [equation \(15\)](#). Noting that situations may arise when the numerator and denominator of [equation \(16\)](#) are both zero, it is necessary to introduce a method that updates mass to account for this.

Mass estimates are updated using a single variable recursive least squares algorithm. In this study, we employ a formulation of the RLS algorithm for a single forgetting factor as described by Johnson [\[18\]](#). Here, we improve and extend the RLS algorithm to consider cases where the right hand side of [equation \(16\)](#) is nearly indeterminate. A potential approach to resolve this concern is to introduce a variable forgetting factor. However, since vehicle mass is considered to be a stationary parameter over the course of a trip, the forgetting factor should already be close to unity. A variable forgetting factor also