

Longitudinal Slip Ratio Control of Electric Powertrains Using a Controller Output Observer for Disturbance Rejection

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ABSTRACT

The use of electric motors to independently control the torque of two or four wheels of a vehicle has the potential to significantly improve safety and handling. One virtue of electric motors is that their output torque can be accurately estimated. Using this known output torque, longitudinal tire force and coefficient of friction can be estimated via a controller output observer. This observer works by constructing a model of wheel dynamics, with longitudinal tire force as an unknown input quantity. A known wheel torque is input to the physical and modeled system and the resulting measured and predicted wheel speeds are compared. The error between the measured and predicted wheel speed is driven towards zero by a robust feedback controller. This controller modulates an estimate of longitudinal tire force used as an input by the wheel dynamics model. The resulting estimate of longitudinal tire force quickly converges towards the actual value with minimal computational expense. Using this estimate, a methodology for controlling tire slip ratio is presented. The estimated longitudinal force is applied as a known disturbance which is explicitly rejected by the control law. This allows for less aggressive feedback control gains and improved robustness. The resulting slip ratio control system shows promising results when simulated in the CarSim environment.

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INTRODUCTION

Electrified vehicle powertrains are increasing in prevalence in the automotive industry due to increased demand from consumers and regulators for more efficient vehicles. Implementing electric motors so that they independently drive two or all four wheels of the vehicle has the additional potential benefit of improved vehicle stability and handling. Electric motors can be packaged within the envelope of the wheel, termed a hub motor, or mounted inboard and connected to the wheel using axles. The use of independent electric motors to drive vehicle wheels has the following merits:

1. Electric motors can generate both driving and braking torque. This allows for maximum yaw moment generation [1].
2. The system can respond 10 times more quickly than an electrohydraulic brake system [2].
3. The amount of torque generated by the electric motor is known with much higher accuracy than that generated by an internal combustion engine or a conventional brake system [2].

This paper will focus on exploiting this final merit to implement a Controller Output Observer (COO) [3] to estimate immeasurable longitudinal tire forces. This observer works by comparing the output of a physical system with the output of a model of that system. Both the physical and modeled systems are subject to a known input wheel torque. The resulting wheel speed is predicted by the model and measured from the physical system. The error between the two signals is driven towards zero by a controller, the output of which is the desired estimate of longitudinal tire force. Knowledge of longitudinal tire force can be used to estimate the coefficient of friction between the road and the tires [4]. This information can be used to alert the driver of precarious driving conditions, and also to improve the response of active safety system [5].

An emerging topic of interest for independently motored vehicles is control allocation. With the redundancy of actuators, longitudinal acceleration as well as yaw moment can be

generated by an infinite combination of independent wheel torques. The decision of control allocation to the independent motors can then be refined with goals such as reduced energy consumption, or maximum stability and handling performance [6] [7]. The control allocation algorithm can directly dictate wheel motor torques, or instead can dictate desired tire slip ratio. Slip ratio is a measure of the longitudinal deformation of a tire, and under non-saturated operating conditions correlates strongly with longitudinal force generation. However, if the slip ratio exceeds some optimal value, force generation degrades in the lateral and longitudinal directions reducing vehicle stability and performance [8]. For this reason, control allocation of desired slip ratio is suggested by [7] to prevent the saturation of tires by constraining allowable slip ratios. The desired slip ratio command is then sent to lower level controllers to actuate the independent motors.

This paper proposes a slip ratio controller which quickly and accurately achieves a desired slip ratio. The longitudinal tire force estimated by the COO is represented as a disturbance, and rejected by the control law. This control architecture is similar to the disturbance observer presented in [9] [10] [11] [12]. Indeed, the proposed control structure is shown to be equivalent to the disturbance observer structure under certain circumstances. However, the proposed control structure utilizing the COO can be realized without requiring plant inversion, and can be implemented using classical feedback control techniques. Additionally, it is suggested that the near real time estimate of longitudinal tire forces provided by the COO could be used by the control allocation algorithm to adapt the control effectiveness matrix. This is a topic of future work.

An outline to this paper is as follows. A linear model of wheel and tire dynamics is presented. Next the Controller Output Observer is applied to predict longitudinal tire force. A disturbance observer is applied to the same problem, and its potential equivalence to the COO is presented. A proposed slip ratio controller utilizing the COO for disturbance rejection is proposed with some linear simulation results. Finally, the slip ratio controller is applied to a non-linear CarSim vehicle model. The proposed control structure tracked desired slip ratio quickly and accurately. Additionally, the estimate of longitudinal tire force tracked the true longitudinal force with little error.

LINEAR MODEL OF WHEEL DYNAMICS

A schematic of a modeled wheel is shown in Figure 1. The wheel has a moment of inertia J and a radius R . Torque τ can be applied to the wheel and longitudinal tire force F_x is generated at the bottom of the wheel. The wheel rotates with angular velocity ω and moves with a longitudinal velocity V . A summation of the moments about the axis of rotation of the wheel generates the dynamical equation shown in Eqn. (1).

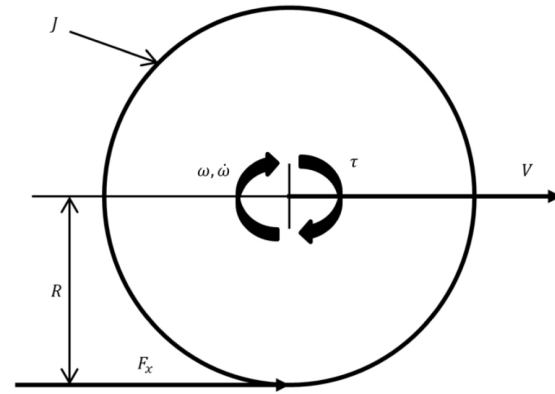


Figure 1. Wheel Schematic Diagram

$$\dot{\omega}(t) = \frac{1}{J} \tau(t) - \frac{R}{J} F_x(t)$$

(1)

To simplify analysis, only a single wheel will be studied connected to a mass m equivalent to a quarter of the vehicle's sprung mass. A summation of forces in the longitudinal direction renders a second dynamical equation, shown in Eqn. (2). Together, Eqns. (1-2) describe the dynamics of a single wheel.

$$\dot{V}(t) = \frac{1}{m} F_x(t)$$

(2)

Longitudinal force F_x is generated at the contact patch between the tire and the road surface. The generation of this force is dependent on the longitudinal deformation of the tire, termed the slip ratio λ , which is defined during braking as shown in Eqn. (3). A $\lambda - \mu$ curve for a tire operating on dry pavement is shown in Figure 2, where μ is defined as the ratio of longitudinal force to normal force N .

$$\lambda = \frac{R\omega - V}{V}$$

(3)

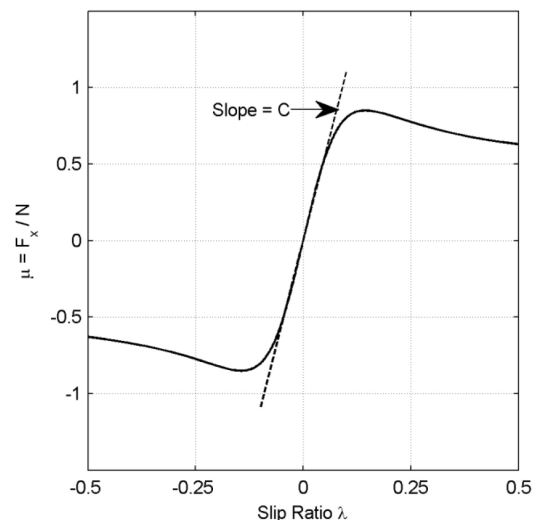


Figure 2. Linear Model of Longitudinal Tire Force Generation

As is evident in the figure, for moderate slip ratios the $\lambda - \mu$ curve can be approximated by a line with slope C . Using this approximation, the linear tire model shown in Eqn. (4) is developed. This linear tire model will be used for the analysis of the controller output observer and slip ratio controller developed in this work. C is generally not known, and varies with many factors such as road surface and tire wear.

$$F_x = CN\lambda \quad (4)$$

A variable can be linearized by representation as an initial value plus a deviation value as shown in Eqn. (5). To facilitate analysis, the non-linear slip ratio described in Eqn. (3) was linearized as is shown in Eqn. (6). Variables preceded with Δ represent linearized quantities and variables ω_o and V_o represent initial values for which the linearization centers. Combining Eqn. (1,2,4,5) the linear system of equations describing the wheel dynamics with a linear tire model were developed, as shown in Eqn. (7). Transforming this system of equations from the time domain to the Laplace domain, the transfer functions represented in Eqns. (8, 9, 10) were developed. Eqn. (8) shows the transfer function from torque input to wheel speed output, while Eqn. (9) represents the transfer function from torque input to slip ratio output. Both transfer functions are stable assuming only positive entries forming the coefficients in Eqn. (10).

$$x = x_o + \Delta x \quad (5)$$

$$\Delta\lambda = \frac{\partial\lambda}{\partial\omega}\Delta\omega + \frac{\partial\lambda}{\partial V}\Delta V = \frac{R}{V_o}\Delta\omega - \frac{R\omega_o}{V_o^2}\Delta V \quad (6)$$

$$\begin{bmatrix} \Delta\dot{\omega}(t) \\ \Delta\dot{V}(t) \end{bmatrix} = \begin{bmatrix} -\frac{R^2NC}{JV_o} & \frac{R^2NC\omega_o}{JV_o^2} \\ \frac{NCR}{mV_o} & -\frac{NCR\omega_o}{mV_o^2} \end{bmatrix} \begin{bmatrix} \Delta\omega(t) \\ \Delta V(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{J} \\ 0 \end{bmatrix} \Delta\tau(t) \quad (7)$$

$$G_{\Delta\tau\Delta\omega}(s) = \frac{\frac{1}{J}(s + C_n)}{s(s + C_d)} \quad (8)$$

$$G_{\Delta\tau\Delta\lambda}(s) = \frac{\left(\frac{R}{JV_o}\right)}{s + C_d} \quad (9)$$

$$C_n = \frac{NCR\omega_o}{mV_o^2}; C_d = \frac{RNC}{V_o} \left[\frac{R}{J} + \frac{\omega_o}{mV_o} \right] \quad (10)$$

CONTROLLER OUTPUT OBSERVER

The relationships which govern the generation of tire forces are often complicated and cannot be measured directly. Instead, an observer termed the Controller Output Observer (COO) [3] is proposed to provide an estimate of longitudinal tire force $\Delta\hat{F}_x$ which will be used by the slip ratio controller proposed later in this paper. A block diagram representation of the COO is shown in Figure 3.

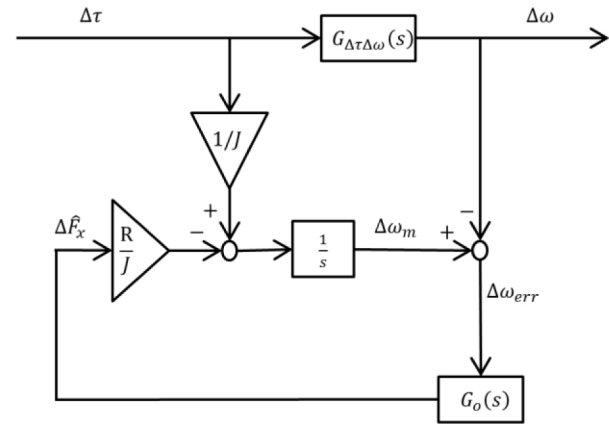


Figure 3. Controller Output Observer Block Diagram

The COO constructs a model of wheel dynamics with longitudinal tire force as an unknown input quantity, shown in Eqn. (11). The modeled system and physical system are both subjected to the same input wheel torque $\Delta\tau$. It is a merit of electric motors that this input quantity can be known accurately [2]. The resulting measured and modeled wheels speeds $\Delta\omega$ and $\Delta\omega_m$ are then compared, generating the error term $\Delta\omega_{err}$. The observer control block $G_o(s)$ is employed to minimize this error by feeding back an estimate of longitudinal tire force $\Delta\hat{F}_x$.

$$\Delta\dot{\omega}_m(t) = \frac{1}{J} \Delta\tau(t) - \frac{R}{J} \Delta\hat{F}_x(t) \quad (11)$$

Setting the input torque to zero, the transfer function from actual wheel speed to the error between modeled and actual wheel speed is shown in Eqn. (12). If G_o is defined simply as a proportional gain, the system is of type one. Therefore, the steady state error between the actual and modeled wheel speed will become zero for a step in actual wheel speed, ω . It is shown in [3] that if the modeled outputs converge toward the measured outputs, the quantities estimated by the COO will converge towards their true values. Thus the COO should provide an estimate of $\Delta\hat{F}_x$ with zero steady state error. If this level of error tracking is insufficient in practice, the observer control block $G_o(s)$ can be modified to add additional integral action to the COO structure.