

Dynamics Analysis of Car-Trailer Systems with Active Trailer Differential Braking Strategies

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ABSTRACT

To date, various control strategies based on linear vehicle models have been researched and developed for improving lateral stability of car-trailer (CT) systems. Is a linear-model-based controller applicable to active safety systems for CT systems under emergency operating conditions, such as an evasive maneuver at high lateral accelerations? In order to answer the question, the applicability of an active trailer differential braking (ATDB) controller designed using a linear CT model is tested and evaluated, while the controller being applied to a CT system represented by a linear and a nonlinear CT model. The current research leads to the following insightful findings: the ATDB controller designed using the linear model can effectively improve the lateral stability of CT systems under regular evasive maneuvers at low lateral accelerations, but the controller is not applicable to CT active safety systems under emergency evasive maneuvers at high lateral accelerations. The insightful findings resulted from the paper will provide valuable design guidelines for the development of active safety systems for CT systems.

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INTRODUCTION

With respect to single unit vehicles, car-trailer (CT) systems have different unstable motion modes, including jack-knifing, trailer sway and roll-over [1]. Jack-knifing is one of the most common causes for severe traffic accidents, in which the car and trailer are involved. It is mainly attributed to tire/ground friction force saturation that may occur in curved path negotiations or during braking processes. If the articulation angle between the leading and trailing units exceeds a critical limit, the driver is unable to control the motion of the vehicle by steering or braking. Similar to the jack-knifing, trailer sway is also an unstable yaw motion mode in terms of divergent trail yaw response. This unstable motion mode usually occurs when side forces cause the trailer to move side to side behind the towing unit [2]. The last type of the unstable motion modes is CT roll-over. Three major factors contributing to CT roll-over are: 1) high-speed curved path negotiations, such as a CT system operating at a high merging speed on highway ramps; 2) sudden course deviation from high initial speed, e.g., lane-change maneuvers; and 3) load shift. The roll-over stability is related to the vehicle's static roll-over threshold expressed as the lateral acceleration in gravitational units (g). Roll-over thresholds for single unit cars are higher than 1g, while the threshold of a CT system may be as low as 0.6 g [3]. The aforementioned unstable motion modes may lead to fatal accidents [4, 5].

In order to address the safety issues of CT systems, various active safety systems (ASSs), including active trailer steering (ATS) and active trailer differential braking (ATDB), have been proposed and investigated [6]. In controller designs for these ASSs, different vehicle models with various degrees of freedom (DOF) have been used [6, 7, 8]. A comparative study of CT system models was conducted in terms of examining the numerical simulation results of four linear and nonlinear CT system models [9]; insightful findings regarding fidelity, complexity, and applicability for lateral dynamics analysis of CT systems have been achieved. However, to the authors' knowledge, the applicability of ASS controllers designed using linear CT system models has not been discussed and reported in the literature.

In order to examine the applicability of ASS controllers designed using linear CT system models, an ATDB controller is designed using a 3-DOF yaw-plane linear model. The objective of designing the ATDB controller is to improve the lateral stability of the CT system. To investigate the performance measures of the ATDB controller based on the linear model, the controller is 'installed' on the virtual CT system represented by a nonlinear 4-DOF, a nonlinear 6-DOF and a nonlinear 21-DOF CT system models, respectively; the corresponding numerical results derived from a simulated single lane-change maneuver have been compared and evaluated. The research based on the numerical simulations discloses the limitations of the ATDB controller derived from the linear CT system model.

3-DOF Linear Model for ATDB Controller Design

The 3-DOF yaw-plane linear model reported in Ref. [9] is used to design the ATDB controller. To this end, the linear model is augmented by introducing an additional external yaw moment on the trailer to improve the lateral stability of CT systems. Hence, adding the yaw moment M_z , the equation of the trailer yaw motion can be shown as Eq. (1)

$$I_t \cdot \ddot{\psi}_t = c \cdot Y_f - d_2 \cdot (X_{\psi_2} \cdot \sin \psi_2 - Y_{\psi_2} \cdot \cos \psi_2) + M_z \quad (1)$$

The governing equations of motion in state space form for the linear model with the ATDB controller is shown as Eq. (2)

$$M \cdot \dot{X} + D \cdot X + C_B \cdot u + F \cdot \delta_f = 0 \quad (2)$$

where the state variable vector X is shown in Eq. (3),

$$X = [V \ \dot{\psi}_1 \ \dot{\psi}_2 \ \psi_2]^T \quad (3)$$

where δ_f is the car front wheel steering angle, matrices M , D , F , and C_B are system and control matrices, which are given in Appendix B. The control variable vector u is defined as Eq. (4),

$$u = M_z \quad (4)$$

To design the ATDB controller using the linear quadratic regulator (LQR) technique, the governing equation of motion of Eq. (2) is re-organized as

$$\dot{X} = (A - B \cdot K) \cdot X \quad (5)$$

where,

$$A = -M^{-1} \cdot D \quad (6)$$

$$B = -M^{-1} \cdot C_B \quad (7)$$

Note that the notations are offered in Appendix A.

ATDB CONTROLLER DESIGN

The proposed ATDB controller is designed using the LQR technique, and the strategy is shown in Figure 1. The essential concept of this control strategy is to use the trailer yaw moment resulting from the trailer differential braking to enhance the stability of the CT system. As shown in Figure 1, four sensors are employed to collect the vehicle state variables, including the car lateral speed, yaw rate, articulation angle, and the time rate of the articulation angle. The sensor information will be sent to the controller for manipulating the trailer braking system to produce an external trailer yaw moment through differential

braking control. Then, this yaw moment will be applied on the CT system. The resulting yaw moment will prevent the unstable motion modes, such as the jack-knifing, trailer swing, and roll-over.

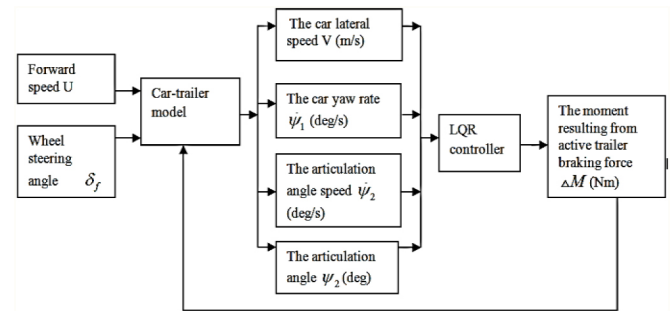


Figure.1. The proposed ATDB strategy

With the ATDB control strategy, the vehicle state variables from the seniors are analyzed and the performance measures are to be calculated. If there is a potential for unstable motion models, such as the jack-knifing, the ATDB controller will manipulate the trailer braking system. Through the trailer differential braking, the resulting yaw moment of the trailer will align the trailer with the towing unit. Thus, the articulation angle between the towing and trailing units can be reduced and the jack-knifing can be prevented. In this process, the external trailer yaw moment M_z is produced through intelligently adjusting differential braking forces on the left and right-side wheels of the trailer, which can be defined as in Eq.(8)

$$M_z = \frac{T}{2} \cdot \Delta F_t \quad (8)$$

Where ΔF_t is the longitudinal force difference between the left and the right wheels of the trailer and T is the track between the left and right wheels of the trailer.

The LQR controller design can be described as an optimization problem, and the goal is to minimize the performance index:

$$J = \int_0^\infty [Q_1 \cdot (\dot{V} + U \cdot \dot{\psi}_1)^2 + Q_2 \cdot (\dot{\psi}_t + U_t \cdot \dot{\psi}_t)^2 + Q_3 \cdot M_z^2] dt \quad (9)$$

subject to Eq. (5). By solving the algebraic Riccati equation, the solution of the optimization problem is the control vector of the form:

$$u = -K \cdot X = -(K_1 \cdot V + K_2 \cdot \dot{\psi}_1 + K_3 \cdot \dot{\psi}_2 + K_4 \cdot \psi_2) \quad (10)$$

where K is the control gain matrix, which can be determined by the LQR algorithm, X and u are the state and control variable vectors defined by Eqs. (3) and (4), respectively. In Eq. (9), Q_1 , Q_2 , and Q_3 are the weighting factors that impose penalties upon the magnitude and duration of the lateral acceleration at the car's Center of Gravity (CG), the lateral acceleration at the

trailer CG, and the active trailer yaw moment, M_z . Note that the third term on the right hand side of Eq. (9) represents the energy consumption of the ATDB control system.

As shown in Eq. (9), by coordinating the relationship between the lateral accelerations at the car CG and the trailer CG, the articulation angle between the car and trailer and the roll angles of car and trailer bodies may be controlled. Thus, the dangerous jack-knifing and trailer swing may be prevented. Moreover, both the magnitudes of the lateral accelerations at the car CG and the trailer CG are to be minimized. Thus, the roll-over stability will be enhanced. The above design considerations will be justified for other models.

ATDB Controller Modified for the 4-DOF and 6-DOF Nonlinear CT Models

In order to implement the ATDB control strategy, the nonlinear equations of the CT models can be expressed as nonlinear functions of both the state variable vector and time as,

$$M(\int X dt, t) \cdot \dot{X} = F(\int X dt, X, t) \quad (11)$$

where $M(\int X dt, t)$ is the generalized mass matrix, $F(\int X dt, X, t)$ is the generalized force, and they are nonlinear functions of both system's the state variable vector X and the simulation time t .

In the case of the nonlinear 4-DOF CT model reported in Ref. [9], the essential concept of the ATDB control strategy is to introduce a stabilizing external yaw moment on the trailer in order to improve the lateral stability of the CT system. Hence, with the ATDB control system involved, the state space form expressed in Eq. (11) becomes the Eq. (12)

$$M(\int X dt, t) \cdot \dot{X} = F(\int X dt, X, t) + C_B \cdot u \quad (12)$$

where X is shown as Eq. (3), u is shown as Eq. (10) by using the set control gains K which are based on the 3-DOF linear model and the LQR technique. C_B is the same as that for the 3-DOF model. The roll-over stability will also be enhanced.

In the case of 6-DOF nonlinear model reported in Ref. [9], all considered motions are the same as those used for the 4-DOF model with the addition of roll motions. Therefore, all dynamics analysis must include lateral motions and roll motions of CT system. In the following detailed design of the ATDB controller, these characteristics will be considered.

Based on the Lagrange equations and assuming that the 6-DOF model adapts the ATDB control strategy, the nonlinear simulation equations must be collated as nonlinear functions of both state variable and time as shown as Eq. (11) by the above rules, where X is the state variable vector of the 6-DOF model and is defined as,

$$X = [V \ \dot{\psi}_1 \ \dot{\psi}_2 \ \psi_2 \ \phi_1 \ \phi_2]^T \quad (13)$$

However, not like the 4-DOF model, all six tires are considered individually in the 6-DOF model, including the effects of individual tire force. According to these dynamics features of the 6-DOF model, tire forces of each tire affects not only lateral motion but also roll motion. Hence, the essential concept of this case is that through intelligently adjusted differential braking forces on the left or right-side wheels of trailer, we can improve the lateral stability of CT systems. The object of the ATDB strategy is to manipulate the differential braking forces ΔF_t on left and the right-side wheels of the trailer, which can be derived from Eq. (8),

$$\Delta F_t = 2 \cdot \frac{M_z}{T} \quad (14)$$

With the ATDB control system involved, the state space form of the governing equation becomes,

$$M(\int X dt, t) \cdot \dot{X} = F(\int X dt, X, t) + F_B(\int X dt, X, \Delta F_t, t) \quad (15)$$

where $F_B(\int X dt, X, t)$ is defined as the generalized control force, which are nonlinear functions of the differential braking force ΔF_t on the left and right-side wheels of the trailer, the state variable vector X and the simulation time t . The nonlinear function can take the following form,

$$F_B(\int X dt, X, \Delta F_t, t) = 2 \cdot \Delta F_t \cdot C_{BF} = 4 \cdot \frac{M_z}{T} \cdot C_{BF} = \frac{4}{T} \cdot C_{BF} \cdot u \quad (16)$$

where M_z is the yaw moment and can be produced by using the set control gains K which are based on the 3-DOF linear model and the LQR technique. C_{BF} is the generalized coefficient matrix of the ATDB control system and it is defined as:

$$C_{BF} = \begin{bmatrix} \psi_2 \\ -\psi_2 \cdot P_k \cdot \phi_1 - \psi_2 \cdot d_1 \\ \psi_2^2 \cdot P_i - \psi_2 \cdot P_k \cdot \phi_2 + \psi_2 \cdot P_i + \psi_2^2 \cdot P_k \cdot \phi_2 \\ 0 \\ \psi_2 \cdot (H_H + H_{TC}) \\ -H_{TC} \cdot \psi_2 \end{bmatrix} \quad (17)$$

where H_H is the vertical distance from the car C.G to hitch, H_{TC} is the vertical distances of the roll center of the trailer suspension, respectively. P_i , P_j and P_k are the longitudinal, lateral and vertical position of the center of trailer wheels, which are measured with respect to the hitch point of the trailer.

ATDB Controller Modified for the 21-DOF Nonlinear Model

To implement the ATDB control strategy, the ATDB controller is constructed in Matlab/Simulink and the 21-DOF nonlinear model is generated in the CarSim software; then the ATDB

controller in combined with the CarSim model through the interface of the two software packages. The simulation environment is shown in Appendix C. With the CarSim model, the yaw moment is generated through the manipulation of individual trailer tire braking force. Figure 2 shows the working principle of the brake system of the CarSim model.

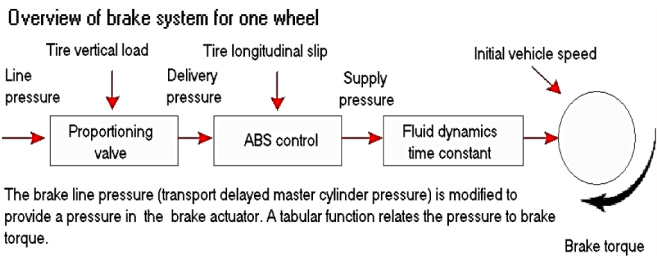


Figure.2. The working principle of the brake system of the CarSim model with the ATDB

For this case, in order to achieve this ATDB control strategy, the master cylinder of the brake system is controlled by the controller to provide different brake line pressure on the brake actuator of the corresponding wheel, then through a tabular function based on the principle of friction to produce different brake torques. Eventually, those brake torque are converted into different brake force on the corresponding wheel to control the lateral and roll motions. Hence, there are two differential braking control strategies: one for the control of the trailer (ATDB controller) yaw moment, which is shown in Figure 3, and the other for the control of both the car (Active Car Differential Braking, ACDB, controller) yaw moment and the trailer (ATDB controller) yaw moment, which is shown in Figure 4. The ACDB controller is based on the yaw moment control strategy of the car body. Note that the ABS controller is disabled in this case.

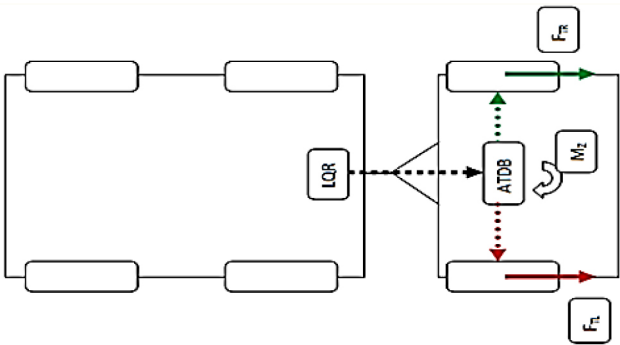


Figure.3. The CarSim model with the ATDB

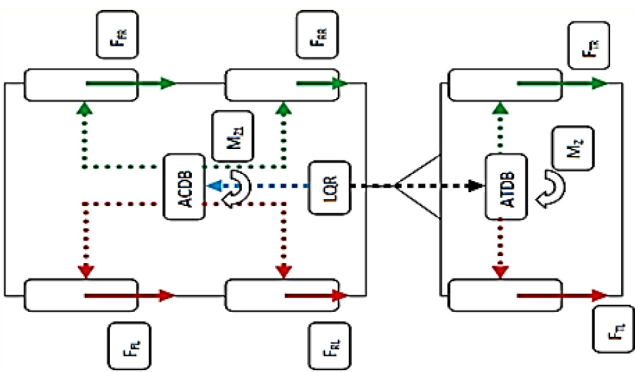


Figure.4. The CarSim model with both the ATDB and ACDB

NUMERICAL SIMULATION

The vehicle system models reported in Ref. [9] and the corresponding ATDB controllers described above are jointly constructed and integrated in Matlab/ Simulink [10]. To perform numerical simulations, the vehicle system parameters take the values listed in Table 1, other parameters take the values provided by the “E-Class, Sedan w/1A Trailer” model generated in the CarSim software [11]. The numerical simulations are conducted under the emulated single lane-change maneuver. For the purpose of comparison, the numerical simulations based on each CT model both with and without the ATDB controller have been performed. The respective numerical simulation results are presented and discussed in the following subsection.

Table.1. The primary parameters of the car-trailer system

Car mass	m_c	2000 kg
Car yaw inertia	I_c	3000 kgm ²
Car dimension	a	1.5 m
Car dimension	b	1.7 m
Car dimension	d_1	2.8m
Trailer mass	m_t	2300 kg
Trailer yaw inertia	I_t	3900 kgm ²
Trailer dimension	c	5 m
Trailer dimension	d_2	0 m
Front tire cornering stiffness (combined)	C_f	-80000 Nm/rad
Rear tire cornering stiffness (combined)	C_r	-80000 Nm/rad
Trailer tire cornering stiffness (combined)	C_t	-60000 Nm/rad

SIMULATION RESULTS AND DISCUSSION

To examine the proposed ATDB controller design, which will be applied to the design of the CT system represented by the 3-DOF (linear model), 4-DOF (nonlinear model), 6-DOF (nonlinear model) and 21-DOF (CarSim) models as introduced in previous section, the numerical simulations based on the CT models with and without the ATDB controller will be performed. At the same time, to make the simulation results more concise, all simulation data's names are simplified as follows: the case of the 3-DOF model with the controller is denoted as “I3”, the 4-DOF model with the controller is represented as “I4”, the 6-DOF model with the controller is named “I6”, the CarSim