

Vehicle Sideslip Angle EKF Estimator based on Nonlinear Vehicle Dynamics Model and Stochastic Tire Forces Modeling

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ABSTRACT

This paper presents the extended Kalman filter-based sideslip angle estimator design using a nonlinear 5DoF single-track vehicle dynamics model with stochastic modeling of tire forces. Lumped front and rear tire forces have been modeled as first-order random walk state variables. The proposed estimator is primarily designed for vehicle sideslip angle estimation; however it can also be used for estimation of tire forces and cornering stiffness. This estimator design does not rely on linearization of the tire force characteristics, it is robust against the variations of the tire parameters, and does not require the information on coefficient of friction. The estimator performance has been first analyzed by means of computer simulations using the 10DoF two-track vehicle dynamics model and underlying magic formula tire model, and then experimentally validated by using data sets recorded on a test vehicle.

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INTRODUCTION

The sideslip angle information is essential for implementation of advanced active safety systems in order to increase the robustness and performance of vehicle stability control. However, direct measurement of the sideslip angle is difficult and it requires usage of sophisticated speed-over-ground optical sensors. These sensors are too expensive to be installed on the mass production vehicles and therefore the sideslip angle is typically estimated. Numerous estimation approaches from the literature typically use available measurements of vehicle dynamics state variables, and appropriate kinematic or dynamic vehicle models. The commonly found techniques used for the kinematic sideslip angle estimation include the use of direct integration of standard Electronic Stability Control (ESC) system measurements for design of open-loop [1, 2, 3] and closed-loop [4, 5] observers, or application of Inertial Measurement Unit (IMU) and appropriate, more complex 6DoF kinematic models [6]. The dynamic model-based estimation concepts include: (i) application of vehicle dynamic model with linear [7, 8, 9] or piecewise linear [2, 3] tire characteristics and various types of observers, (ii) approaches based on utilization of nonlinear vehicle dynamics models and nonlinear observers and known tire road friction model [10,11], and (iii) application

of nonlinear vehicle dynamics model with stochastic description of tire forces (e.g. random-walk state variables [12, 13, 14] or Gauss-Markov tire models [15]).

In general, it can be concluded that the kinematic model-based sideslip angle estimators are sensitive to sensor offsets and road bank disturbances, which results in potentially large drift-like estimation errors. On the other hand, estimators using vehicle dynamics models with analytical or semi-empirical tire models are sensitive to model parameters uncertainties and unmodeled dynamics. More precisely, according to the analysis given in [16] the sideslip angle observers based on vehicle dynamics model and the analytical, deterministic tire models are primarily sensitive to the uncertainties in cornering stiffness and steering input amplitudes in linear region, and to road surface coefficient of friction in the nonlinear region of the lateral tire static characteristic. The sideslip angle estimators based on the stochastic tire modeling are consequently robust against the uncertainties of tire model parameters and do not require the information on the tire-road interface coefficient of friction.

The extended Kalman filter-based sideslip angle estimator, proposed in this paper, is based on a nonlinear single-track vehicle dynamics model with stochastic description of lateral and longitudinal tire forces. In its foundation, the estimator

design utilizes the concept similar to the one proposed in [12, 13], which was used therein primarily for tire-road coefficient of friction estimation. In contrast, the reduced-order estimator proposed in this paper deals with vehicle sideslip angle estimation, and uses the vehicle longitudinal velocity as an additional measurement in order to increase the estimator robustness against the inaccuracies of braking torques measurements and uncertainties of model parameters. The proposed estimator concept has been verified off-line by using the experimental data recorded on a test vehicle, equipped with a high-performance inertial measurement unit (IMU), during various driving maneuvers performed on low- μ surfaces (e.g. snow and ice). The obtained results (used for proof-of-concept analysis) show good potential of the proposed estimator concept for on-line implementation and its potential for application in estimation of other vehicle dynamics variables, as well (e.g. tire forces, tire slip angles, cornering stiffness, coefficient of friction).

VEHICLE DYNAMICS MODELING

The single-track nonlinear vehicle dynamics model with five degrees of freedom, illustrated in Fig. 1 has been utilized as a basis for design of the proposed sideslip angle estimator. This vehicle dynamics model is defined by following expressions:

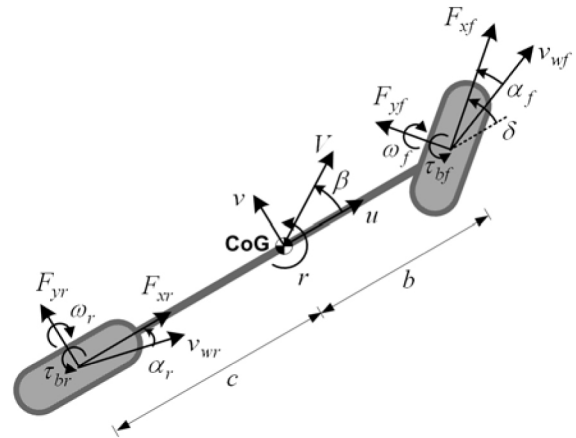


Figure 1. Illustration of a single-track nonlinear vehicle dynamics model with five degrees of freedom.

The state vector $\mathbf{x}$ of the continuous-time state-space model representation (Eq. (6)) of the utilized 5DoF vehicle dynamic model is augmented with four additional state variables corresponding to the front and rear axis lumped tire forces, the input vector $\mathbf{u}$ comprises the front and rear wheels braking torque, and the output vector $\mathbf{y}$ comprises the measurements of yaw rate, front and rear wheel speeds, lateral and longitudinal accelerations and vehicle longitudinal velocity:

$$\dot{u} = \frac{1}{m} [F_{xf} \cos \delta - F_{yf} \sin \delta + F_{xr}] + vr \quad (1)$$

$$\dot{v} = \frac{1}{m} [F_{yf} \cos \delta + F_{xf} \sin \delta + F_{yr}] - ur \quad (2)$$

$$\dot{r} = \frac{1}{I_z} \left[b(F_{yf} \cos \delta + F_{xf} \sin \delta) - cF_{yr} \right] \quad (3)$$

$$\dot{\omega}_f = \frac{1}{I_w} [-F_{xf} r_w - \tau_{bf}] \quad (4)$$

$$\dot{\omega}_r = \frac{1}{I_w} [-F_{xr} r_w - \tau_{br}] \quad (5)$$

where u and v are longitudinal and lateral velocities, respectively, r is the yaw rate, ω_f and ω_r are the front and rear rotational wheel speeds, δ is the front wheel steering angle, b and c denote distances of the front and rear axes from the vehicle CoG, respectively, r_w and I_w are the wheel radius and wheel moment of inertia, τ_{br} and τ_{bf} are the front and rear braking torques. The tire forces F_{xf} , F_{xr} , F_{yf} , and F_{yr} are modeled as first-order random walk type stochastic variables.

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{\Phi}(\mathbf{x}) + \mathbf{B}\mathbf{u} + \mathbf{W}\mathbf{v} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{e}\end{aligned}\tag{6}$$

where: $\mathbf{x} = [u \ v \ r \ \omega_f \ \omega_r \ F_{xf} \ F_{xr} \ F_{yf} \ F_{yr}]^T$, $\mathbf{u} = [T_{bf} \ T_{br}]^T$, $\mathbf{y} = [r \ \omega_f \ \omega_r \ a_x \ a_y \ u]^T$ $\mathbf{W} = \mathbf{I}_{9 \times 9}$, and front wheel steering angle δ has been considered as a time variant model parameter in state transition and output matrices, updated to a instantaneous value of the steering angle measurement, at each time step t_k .

Note that the model nonlinearities reside in state equation, more precisely in Eqs. (1) and (2), while the output equation is linear. Therefore, in order to implement Kalman filter, the state-space model nonlinear state transition function $\Phi(\mathbf{x})$ needs to be linearized around the operating point (r_{0k}, δ_{0k}) at each time step t_k in order to derive the state transition matrix $\mathbf{A}$:

$$\mathbf{A}_k = \left. \frac{\partial \Phi(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_0, t=t_k} . \quad (7)$$

The observability of the linearized state-space model (6) has been tested by means of algebraic analysis of the observability matrix $\mathbf{O}_b = [\mathbf{C} \ \mathbf{C} \mathbf{A} \ \mathbf{C} \mathbf{A}^2 \ \dots \ \mathbf{C} \mathbf{A}^{n-1}]^T$ properties. This analysis has shown that the considered model becomes unobservable in case of near zero yaw rate and lateral acceleration (i.e. in straight driving conditions). However, in these conditions the sideslip angle also equals zero, so the estimator can be paused or switched off in order to prevent the estimation error build-up.

ESTIMATOR DESIGN

The discrete-time vehicle dynamics model in Eq. (8), needed for implementation of EKF-based estimator, is obtained from continuous-time model given by Eq. (6) after applying Z-transform based upon the Zero-Order-Hold (ZOH) method:

$$\begin{aligned}\mathbf{x}(k) &= \mathbf{F}(k-1)\mathbf{x}(k-1) + \mathbf{G}(k-1)\mathbf{u}(k-1) \\ &\quad + \mathbf{\Omega}(k-1)\mathbf{v}(k-1) \\ \mathbf{y}(k) &= \mathbf{H}(k)\mathbf{x}(k) + \mathbf{e}(k)\end{aligned}\quad (8)$$

where the discrete-time model matrices have been derived from the following approximate expressions:

$$\mathbf{F} = e^{\mathbf{A}T_s} \approx \mathbf{I} + \mathbf{A}T_s, \mathbf{G} \approx \mathbf{B}T_s, \mathbf{\Omega} \approx \mathbf{W}T_s, \text{ and } \mathbf{H} \text{ equals } \mathbf{C}.$$

Kalman filter equations read:

$$\hat{\mathbf{x}}(k|k-1) = \mathbf{F}(k-1)\hat{\mathbf{x}}(k-1|k-1) + \mathbf{G}(k-1)\mathbf{u}(k-1) \quad (9)$$

$$\mathbf{P}(k|k-1) = \mathbf{F}(k-1)\mathbf{P}(k-1|k-1)\mathbf{F}^T(k-1) + \mathbf{Q}(k-1) \quad (10)$$

$$\hat{\mathbf{e}}(k|k-1) = \mathbf{y}(k) - \mathbf{H}(k)\hat{\mathbf{x}}(k|k-1) \quad (11)$$

$$\mathbf{K}(k) = \frac{\mathbf{P}(k|k-1)\mathbf{H}^T(k)}{\mathbf{H}^T(k)\mathbf{P}(k|k-1)\mathbf{H}^T(k) + \mathbf{R}(k)} \quad (12)$$

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{K}(k)\hat{\mathbf{e}}(k|k-1) \quad (13)$$

$$\mathbf{P}(k|k) = \mathbf{P}(k|k-1) - \mathbf{K}(k)\mathbf{H}(k)\mathbf{P}(k|k-1) \quad (14)$$

where: $\hat{\mathbf{x}}(k|k-1)$ and $\hat{\mathbf{x}}(k|k)$ are a-priori and a-posteriori state vector updates, respectively; $\hat{\mathbf{e}}(k|k-1)$ is a-priori prediction error; $\mathbf{P}(k|k-1)$, $\mathbf{P}(k|k)$ are a-priori and a-posteriori state estimation error covariance matrices, respectively; $\mathbf{K}(k)$ is the Kalman filter gain matrix; $\mathbf{Q}(k)$, $\mathbf{R}(k)$ are state and measurement noise covariance matrices, respectively. Equations (9) and (10) constitute the prediction phase, while Eqs. (11, 12, 13, 14) relate to measurement update phase of the Kalman filter algorithm.

Assuming that the stochastic state perturbations and the measurement errors are mutually independent, the state covariance ($\mathbf{Q}$) and the measurement noise covariance ($\mathbf{R}$) matrices are defined as:

$$\mathbf{Q} = \text{diag}\left(\begin{bmatrix} q_u & q_v & q_r & q_{\omega f} & q_{\omega r} & q_{Fxf} & q_{Fxr} & q_{Fyf} & q_{Fyr} \end{bmatrix}\right)$$

$$\mathbf{R} = \text{diag}\left(\begin{bmatrix} r_r & r_{\omega f} & r_{\omega r} & r_{ax} & r_{ay} & r_u \end{bmatrix}\right)$$

where the components of matrix $\mathbf{Q}$ corresponds to the variance of the state perturbations and the components of matrix $\mathbf{R}$ correspond to the variance of the measurement noise.

The sideslip angle estimate has been derived from Eq. (15) by using the estimated vehicle lateral and longitudinal velocities (i.e. the elements of the a-posteriori updates of the state vector estimate).

$$\hat{\beta}(k) = \text{atan}\left(\frac{\hat{v}(k|k)}{\hat{u}(k|k)}\right) \quad (15)$$

SIMULATION RESULTS

The performance of the proposed EKF-based sideslip angle estimator is first analyzed by means of computer simulation. For that purpose, the simulation environment outlined in Fig. 2 is applied, utilizing a 10DoF vehicle dynamics model [17], implemented in Matlab Simulink™. This model enables selection of different driving maneuvers and setting of relevant maneuver parameters in order to generate the reference vehicle dynamics signals and inputs for the single-track 5DoF model used in estimation (i.e. front wheel steering δ angle and braking torques $\mathbf{T}_b = [\tau_{bf} \tau_{br}]^T$ for front and rear axis. Initially, the 5DoF model outputs have been used as measurements signals $\mathbf{y}$ for the EKF estimator, as illustrated in Fig. 2, in order to test its nominal performance and emphasize the difference between the underlying 5DoF and referent 10DoF vehicle dynamics model. The presented simulation results correspond to the high- μ conditions ($\mu = 1$).

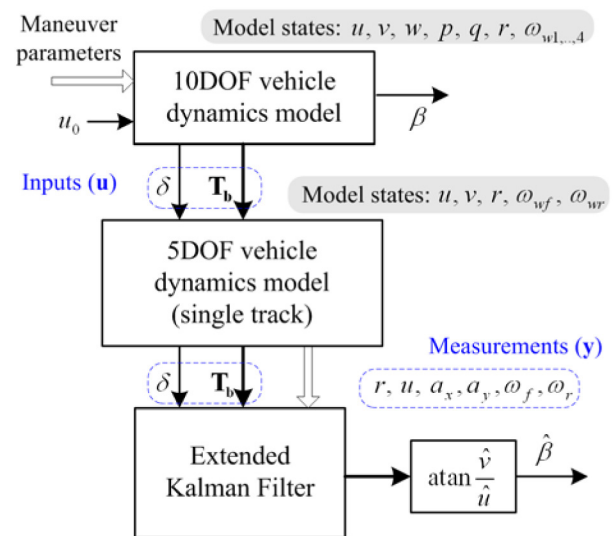


Figure 2. Illustration of EKF estimator simulation environment.