

# Robust Semi-Active Ride Control under Stochastic Excitation

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## ABSTRACT

Ride control of military vehicles is challenging due to varied terrain and mission requirements such as operating weight. Achieving top speeds on rough terrain is typically considered a key performance parameter, which is always constrained by ride discomfort. Many military vehicles using passive suspensions suffer with compromised performance due to single tuning solution. To further stretch the performance domain to achieving higher speeds on rough roads, semi-active suspensions may offer a wide range of damping possibilities under varying conditions. In this paper, various semi-active control strategies are examined, and improvements have been made, particularly, to the acceleration-driven damper (ADD) strategy to make the approach more robust for varying operating conditions. A seven degrees of freedom ride model and a quarter-car model were developed that were excited by a random road process input modeled using an auto-regressive time series model. The proposed strategy shows promise as a cost-effective solution to improve the ride of a military vehicle over multiple stochastic terrains considering variation in operating weight.

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## INTRODUCTION

There have been many approaches on how to control a vehicle's suspension. The classical method, which is the passive suspension, lacks the ability to adapt to the vehicle's environment. Active suspensions were proposed to provide the ability to tune, in real time, the ride comfort, road holding or any other metric of the suspension behavior that was desired to be tuned. Active suspensions are, however, costly and consume non-trivial quantities of power.

Semi-active suspensions have been available for many years now, and a significant body of research exists on how to utilize them [1, 2, 3]. The most common method of making a suspension semi-active is to use variable value dampers, with constant spring stiffness. The controlled input of the system is the damping value.

One of the many ideas is to utilize classic control techniques such as Linear Quadratic Regulator or other dynamics based control techniques [4, 5]. The problem with employing these techniques is that the control structure, even for a simple model, is nonlinear since the control value, the damping coefficient in this case, is multiplying the velocity of the system, which is a state variable. The technique that is commonly used to bypass this is to define the system as having an active damper force, and then to clip the control signal so that only

the values that the damper can supply are used. The problem with this is that once the optimal control strategy is clipped, it may no longer be as optimal and advantageous with respect to other simpler control strategies. In addition, if nonlinear control is employed, then most likely, full state feedback is required, which is simply not a reasonable assumption. Even if it is known, it can be shown to be inferior to the logic based control systems [6]. Information that can reasonably be known is the relative displacements and velocities across the strut, and the acceleration signals at the top and bottom of the strut. Accurate knowledge of the absolute positions and velocities of the wheel and vehicle body may not be a reasonable assumption due to practical reasons.

Logic based control systems are derived from the kinematics of the vehicle suspension and require no knowledge of the vehicle's parameters. An excellent overview of these is found in [7].

The Accelerometer Driver Damper (ADD) control strategy is very simplistic in operation and requires no knowledge of the vehicle parameters [8]. The lack of knowledge about the vehicle parameters causes some actuator chatter effects in the system. The major contribution of this study is that with some knowledge about the vehicle's parameters the ADD control strategy can be significantly improved. We also show that this control strategy is invariant to the road profile.

## VEHICLE MODELING

### Seven Degree of Freedom Model

Semi-active dampers obey the passivity constraint, such that they only remove energy from the system. To show the effectiveness of the proposed control strategy, a seven degree of freedom (7DoF) vehicle model is implemented as shown in Figure 1. The equations of motion for the seven degree of freedom vehicle model are defined as:

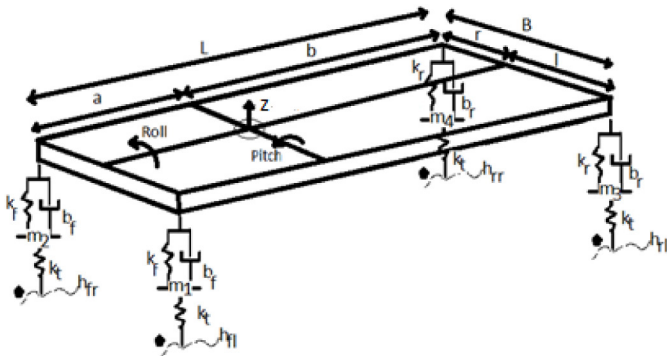


Figure 1. 7DoF Vehicle Model Diagram

$$F_{fl} = k_{fl}(z - a\theta + l\phi - z_{fl}) + c_{fl}(\dot{z} - a\dot{\theta} + l\dot{\phi} - \dot{z}_{fl}) \quad (1)$$

$$F_{fr} = k_{fr}(z - a\theta - r\phi - z_{fr}) + c_{fr}(\dot{z} - a\dot{\theta} - r\dot{\phi} - \dot{z}_{fr}) \quad (2)$$

$$F_{rl} = k_{rl}(z + b\theta + l\phi - z_{rl}) + c_{rl}(\dot{z} + b\dot{\theta} + l\dot{\phi} - \dot{z}_{rl}) \quad (3)$$

$$F_{rr} = k_{rr}(z + b\theta - r\phi - z_{rr}) + c_{rr}(\dot{z} + b\dot{\theta} - r\dot{\phi} - \dot{z}_{rr}) \quad (4)$$

where  $F_p$ ,  $k_p$ ,  $c_p$ ,  $i = fl, fr, rl, rr$ , are the front right, front left, rear left and rear right suspension force, stiffness and damping value. In relation to the vehicle body positive suspension forces pull the body downward. The geometric properties of the vehicle,  $a, b, r, l$  are the distances from the front axle to the center of gravity (CG) of the vehicle, the rear axle to the CG, the right wheels of the vehicle to the CG and the left wheels of the vehicle to the CG. The body coordinates,  $z, \theta, \phi$ , are the vertical, pitch and roll displacements of the body. The dot notation represents the derivative with respect to time.

$$\ddot{z}_{fl} = \frac{-k_{u,fl} * (z_{fl} - z_{g,fl})H(z_{g,fl} - z_{fl}) + F_{fl}}{m_{fl}} - g \quad (5)$$

$$\ddot{z}_{fr} = \frac{-k_{u,fr} * (z_{fr} - z_{g,fr})H(z_{g,fr} - z_{fr}) + F_{fr}}{m_{fr}} - g \quad (6)$$

$$\ddot{z}_{rl} = \frac{-k_{u,rl} * (z_{rl} - z_{g,rl})H(z_{g,rl} - z_{rl}) + F_{rl}}{m_{rl}} - g \quad (7)$$

$$\ddot{z}_{rr} = \frac{-k_{u,rr} * (z_{rr} - z_{g,rr})H(z_{g,rr} - z_{rr}) + F_{rr}}{m_{rr}} - g \quad (8)$$

Equations (5, 6, 7, 8) describe the motion of the wheels of the vehicle.  $z_p, z_{g,p}$ ,  $i = fl, fr, rl, rr$ , are the vertical displacements of the wheels and the ground height at the front left, front right, rear left and rear right positions.  $g$  is the acceleration due to gravity, defined as  $9.81 \frac{m}{s^2}$ .  $H$  is the Heaviside step function that enables wheel-hop. When the wheel's displacement is larger than the ground's displacement this function removes the force that would be generated by the ground, since the wheel is no longer in contact.

$$\ddot{z} = \frac{-(F_{fl} + F_{fr} + F_{rl} + F_{rr})}{mass} - g \quad (9)$$

$$\ddot{\theta} = \frac{a(F_{fl} + F_{fr}) - b(F_{rl} + F_{rr})}{J_{Pitch}} \quad (10)$$

$$\ddot{\phi} = \frac{-l(F_{fl} + F_{rl}) + r(F_{fr} + F_{rr})}{J_{Roll}} \quad (11)$$

Equations (9, 10, 11) define the vehicle body's equations of motion.  $mass, J_{Pitch}, J_{Roll}$  are the mass, pitch inertia and roll inertia of the vehicle body.

In addition to the 7DOF model, a quarter-car model was also developed to validate some of the previous work [7], and demonstrate the effectiveness of the proposed control strategy under similar assumptions.

## STOCHASTIC TERRAIN MODEL

### Time-Series Modeling

In this study we use time-series because it can model both stationary and non-stationary processes. There are three broad classes of time-series models which are of practical importance; the Auto-Regressive (AR) models, the Integrated (I) models, and the Moving Average (MA) models [12]. Combinations of these models result in autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models. Here, we use autoregressive models.

To model the stochastic road process,  $Z_g$ , consider a sample function  $u(t) = Z_g$  of a weakly ergodic process which is discretized using a constant time step  $\Delta t$ , such that  $u_1, u_2, \dots, u_i$  represent the values at discrete times. An autoregressive model, AR( $p$ ), of order  $p$  is represented as

$$u_i - \bar{u} = \phi_1(u_{i-1} - \bar{u}) + \phi_2(u_{i-2} - \bar{u}) + \dots + \phi_p(u_{i-p} - \bar{u}) + \epsilon_i \quad (12)$$

where  $\bar{u}$  is the mean of the random process  $\epsilon_i = N(0, \sigma_\epsilon^2)$ , is a Gaussian white noise and  $\phi_1, \phi_2, \phi_p$ , are feedback parameters to be estimated depending upon the order  $p$  of the AR model. Different order AR models are usually created to determine the best fit. For an AR(2) model,  $\sigma_\epsilon^2$  is determined from

$$\gamma(0) = \text{Var}(u_i) = \frac{\sigma_\epsilon^2}{1 - \phi_1 \rho_1 - \phi_2 \rho_2 - \dots - \phi_p \rho_p} \quad (13)$$

Where  $\gamma(0)$  is the variance of the random process, and  $\rho_p$  is the autocorrelation at the time lag corresponding to  $p$ . More details are provided in [12].

After the feedback parameters of the desired model are estimated, the residual series must be tested for uncorrelated, normally distributed white-noise using a series of statistical tests [9,12]. The residual series is the difference between the actual and the estimated processes.

The model type can be identified by visually inspecting the plots of the autocorrelation and the partial sample autocorrelation functions for different lags (multiples of  $\Delta t$ ). The autocorrelation provides information on the correlation between random variables  $u(t_i)$  and  $u(t_i + h)$  where  $h$  denotes the lag. For a stationary random process, the autocorrelation depends only on  $h$  and not on  $t_i$ . For autoregressive models, the autocorrelation function dies out quickly with increasing  $h$ . The sample autocorrelation function  $\hat{\rho}(h)$  is defined as

$$\hat{\rho}(h) = \frac{n^{-1} \sum_{i=1}^{n-h} (u_{i+h} - \hat{u})(u_i - \hat{u})}{\hat{\sigma}^2} \quad (14)$$

where  $\hat{\sigma}$  is the estimated standard deviation of the random process.

The partial autocorrelation represents the autocorrelation between  $u_i$  and  $u_{i+h}$  without considering the contribution from lags 1 to  $h-1$ , inclusive. It is useful in identifying the order of an autoregressive model. The partial autocorrelation of an AR( $p$ ) process is zero for lags greater or equal to  $p+1$ .

After the order  $p$  of the model is identified, the  $\phi$ 's and  $\bar{u}$  are estimated either by using the Yule-Walker equations [9, 12] or equivalently, by minimizing

$$\sum_{i=p+1}^n \left( (u_i - \bar{u}) + \phi_1(u_{i-1} - \bar{u}) + \dots + \phi_p(u_{i-p} - \bar{u}) \right)^2 \quad (15)$$

In the current example, a third order Auto-Regressive (AR) model is used to represent the road profile. The equation for the AR(3) model is defined in [16]

$$u_i = \phi_1 u_{i-1} + \phi_2 u_{i-2} + \phi_3 u_{i-3} + \epsilon_i \quad (16)$$

where  $u_i$  is the height of the road profile at each discrete time step,  $z_i$ ;  $\phi_1 = 1.2456$ ,  $\phi_2 = -0.2976$ , and  $\phi_3 = -0.1954$  are feedback parameters estimated from road profile experimental data; and  $\epsilon_i$  is Gaussian white noise with mean,  $\mu = 0$ , and variance,  $\sigma^2 = 0.2634$ . These coefficients were generated based on number of statistical tests [9,12]. The effect of road variance was studied by varying the variance of the white noise signal,  $\epsilon_i$ .

Equation (16) generates a stochastic road profile similar to Figure 2. The x-axis is the longitudinal distance in units of feet and the y-axis is the road height in units of inches. Since the quarter-car model parameters are given in SI units, it is necessary to implement unit conversion in the model. For a vehicle traveling at 20 mph on the stochastic terrain, the sampling rate of the x-axis is 0.0341 to convert the longitudinal distance into time for the simulation. The road input after the unit conversion is shown in Figure 3.

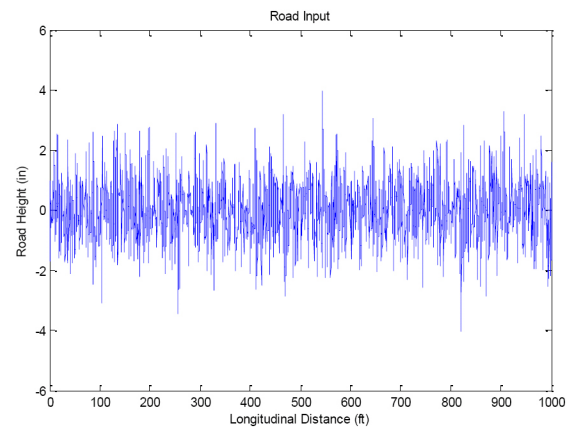


Figure 2. Initial stochastic road profile generated by AR(3) model

The left and right front wheels of the vehicle model were excited by two independent road samples generated from the stochastic model, from equation (12). The rear wheels were excited following front wheels by the time delay determined by the vehicle speed.