

Torque Vectoring Control for Distributed Drive Electric Vehicle Based on State Variable Feedback

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ABSTRACT

Torque Vectoring Control for distributed drive electric vehicle is studied. A handling improvement algorithm for normal cornering maneuvers is proposed based on state variable feedback control: Yaw rate feedback together with steer angle feedforward is employed to improve transient response and steady gain of the yaw rate, respectively. According to the feedback coefficient's influence on the transient response, an optimization function is proposed to obtain optimum feedback coefficients under different speeds. After maximum feedforward coefficients under different speeds are obtained from the constraint of the motor exterior characteristic, final feedforward coefficients are calculated according to an optimal steering characteristic. A torque distribution algorithm is presented to help the driver to speed up during the direct yaw moment control. Simulations performing multiple maneuvers are carried out: The transient response and the steady gain of the yaw rate are improved; the direct yaw moment control can rectify the understeering caused by accelerating, balance the utilization of the road adhesion between two axles, and increase the lateral stability margin of the vehicle.

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INTRODUCTION

For internal combustion engine vehicles, an extra hydraulic control unit or limited slip differential is necessary to achieve direct yaw moment control (DYC). In a distributed drive electric vehicle (DDEV), electric motors are connected to each wheel separately, so the drive/brake torque of each wheel can be independently and precisely controlled. This will introduce a new form of DYC.

So far, DYC algorithms[1, 2, 3, 4, 5, 6] of DDEV are mostly designed for critical cornering (i.e. low road friction or high speed), with a hierarchical control structure which consists of the model following control and the torque allocation. Researchers recently on the adaptive model following control algorithm[5] and control allocation considering tire longitudinal-lateral force coupling[6]. However, for the normal cornering maneuver, the torque vectoring control (TVC) of DDEV which can improve the handling is seldom studied.

Federal states of Germany Statistics[7] show that: In 95% of the cornering maneuvers, the lateral acceleration is below 4m/s^2 , which are normal cornering maneuvers. In these maneuvers, the vehicle is far away from losing control, but the driver still expects a better vehicle handling characteristic:

1. There are hysteresis and overshoot between yaw rate and steering wheel angle input;

2. Speeding up while cornering will lead to understeering;
3. In general steering maneuvers, the slip angle of the front axle is larger than the rear axle, therefore it's easier for the front tires to reach the nonlinear region.

In this paper, a TVC algorithm for the handling improvement of the normal cornering is proposed based on state variable feedback control. The simulations performing multiple maneuvers are carried out to verify the validity of the algorithm.

CONTROL CONCEPT

Linear Single Track Model

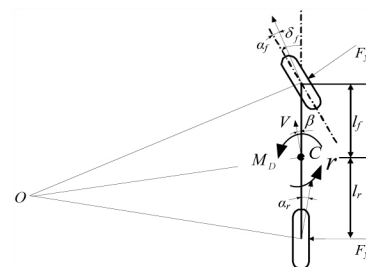


Figure 1. Single track model.

Assuming the longitudinal velocity is constant, only the lateral and yaw motion of the vehicle is taken into account. In this way, the vehicle is simplified to a model with two degree of

freedom (DOF) (Figure 1). The vehicle side slip angle and the yaw rate are selected as state variables. Also, the road friction coefficient is high enough, the lateral acceleration is below 4m/s^2 , and the tire cornering stiffness is in the linear region. The vehicle motion can be described by the followings:

$$m(\dot{\beta} + \gamma) = (C_f + C_r)\beta + \frac{l_f C_f - l_r C_r}{V} \gamma - C_f \delta_f \quad (1)$$

$$J_z \dot{\gamma} = (l_f C_f - l_r C_r)\beta + \frac{l_f^2 C_f + l_r^2 C_r}{V} \gamma - l_f C_f \delta_f + M_D \quad (2)$$

where γ is the yaw rate, β is the vehicle side slip angle, V is the vehicle longitudinal velocity, α_f , α_r are the side slip angle of the front and the rear axle, C_f , C_r is cornering stiffness of the front and the rear axle, F_{yf} , F_{yr} is the lateral force of the front and the rear axle, δ_f is the front steer angle, m is the vehicle mass, J_z is the yaw moment of inertia, C is the center of the gravity (COG), l_f , l_r are the distance from the front axle and the rear axle to COG, M_D is the direct yaw moment. The vehicle model parameters are shown in Table 1.

Table 1. Vehicle model parameters.

Parameter	Value
Vehicle mass [kg]	1022
Yaw moment of inertia [$\text{kg} \cdot \text{m}^2$]	1471.7
Cornering stiffness of front axle [N/rad]	-65203
Cornering stiffness of rear axle [N/rad]	-92017
Wheel base [m]	1.367
Distance from front axle to COG [m]	1.197
Distance from rear axle to COG [m]	1.203
Steer ratio [l]	17.5

Yaw Rate Feedback and Front Steer Angle Feedforward

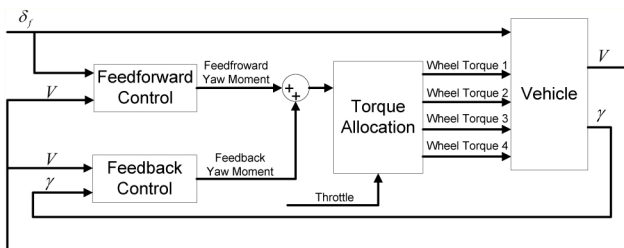


Figure 2. Block diagram of control Algorithm.

As shown in Figure 2, state variable feedback control is applied to improve the transient response of the vehicle and feedforward control is applied to increase the yaw rate steady-state gain. However, it is difficult to measure the side slip angle at the COG and the side slip angle isn't large enough to affect the vehicle stability during normal cornering maneuvers, therefore the yaw rate is regarded as the only feedback variable.

Front steer angle is selected as the feedforward variable. The feedback control coefficient is K_b , the feedforward control coefficient is K_f . The total direct yaw moment can be written as:

$$M_D = K_b \gamma + K_f \delta_f \quad (3)$$

Using equation(3), equation(2) can be transformed into:

$$J_z \dot{\gamma} = (l_f C_f - l_r C_r)\beta + \left(\frac{l_f^2 C_f + l_r^2 C_r}{V} + K_b \right) \gamma + (K_f - l_f C_f) \delta_f \quad (4)$$

According to equation (1) and (4), the state space equation can be described as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad (5)$$

where $x = [\beta \ \gamma]^T$, $u = \delta_f$, $y = \gamma$,

$$A = \begin{bmatrix} \frac{C_f + C_r}{mV} & \frac{C_f l_f - C_r l_r}{mV^2} - 1 \\ \frac{C_f l_f - C_r l_r}{J_z} & \frac{C_f l_f^2 + C_r l_r^2}{J_z V} + \frac{K_b}{J_z} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} -\frac{C_f}{mV} \\ -\frac{C_f l_f}{J_z} + \frac{K_f}{J_z} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, C = [0 \ 1], D = 0.$$

From equation(5), the transfer function of the side slip angle and the yaw rate can be written as follows:

$$\frac{\beta(s)}{\delta_f(s)} = \frac{b_1 s + (a_{12} b_2 - a_{22} b_1)}{s^2 - (a_{11} + a_{22})s + (a_{11} a_{22} - a_{12} a_{21})} \quad (6)$$

$$\frac{\gamma(s)}{\delta_f(s)} = \frac{b_2 s + (a_{21} b_1 - a_{11} b_2)}{s^2 - (a_{11} + a_{22})s + (a_{11} a_{22} - a_{12} a_{21})} \quad (7)$$

When $s=0$, steady gains of the side slip angle and the yaw rate can be expressed as follows:

$$\left. \frac{\beta}{\delta_f} \right|_s = \frac{a_{12} b_2 - a_{22} b_1}{a_{11} a_{22} - a_{12} a_{21}} \quad (8)$$

$$\left. \frac{\gamma}{\delta_f} \right|_s = \frac{a_{21} b_1 - a_{11} b_2}{a_{11} a_{22} - a_{12} a_{21}} \quad (9)$$

YAW RATE FEEDBACK CONTROL ALGORITHM

Influence of Feedback Coefficient on Transient Response Characteristics

The transient response characteristics of the vehicle are changing with different velocities (Table 2), so it is necessary to calculate feedback coefficients according to the velocity to improve the yaw rate response under different velocity pertinently. According to equation(7), the system natural frequency ω_n and damping ratio ζ can be written as follows:

$$\omega_n = \sqrt{a_{11}a_{22} - a_{12}a_{21}} \quad (10)$$

$$\zeta = -\text{sgn}(a_{11} + a_{22}) \sqrt{\frac{(a_{11} + a_{22})^2}{4(a_{22}a_{11} - a_{12}a_{21})}} \quad (11)$$

Table 2. Yaw rate transient response characteristics under different velocity.

Velocity [m/s]	Natural frequency [rad/s]	Damping ratio
5	30.68	1.003
15	11.15	0.920
25	7.678	0.801
35	6.399	0.686

As shown in Figure 3. When the feedback coefficient is less than -14000, the system damping ratio exceeds 1, it becomes an overdamping system. There is no overshoot at all, but the rise time is infinite. When the feedback coefficient exceeds 13500, there will be singular values in the damping ratio. Therefore, a large range of feedback coefficient range [-14000, 13500] is selected to study the Influence of the feedback coefficient on the transient response characteristics.

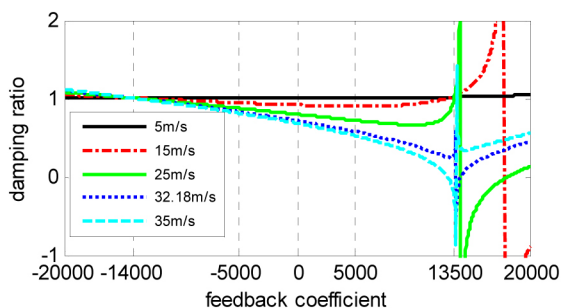


Figure 3. The relationship between damping ratio and feedback coefficient.

Also, as shown in Figure 3. When the vehicle velocity is 5m/s, the feedback coefficient has little influence on the damping ratio. As the velocity rises, the influence increases. With a velocity below the characteristic velocity V_{ch} (32.18m/s), the damping ratio never goes below zero for the chosen value of the feedback coefficient. When the velocity exceeds the characteristic velocity, the damping ratio may become zero. As

the velocity reaches 35m/s, a large feedback coefficient will lead to a zero or even negative damping ratio, and the vehicle system is no longer convergent.

At different velocities, influences of the feedback coefficient on system natural frequency are the same: It decreases as the feedback coefficient increases (Figure 4).

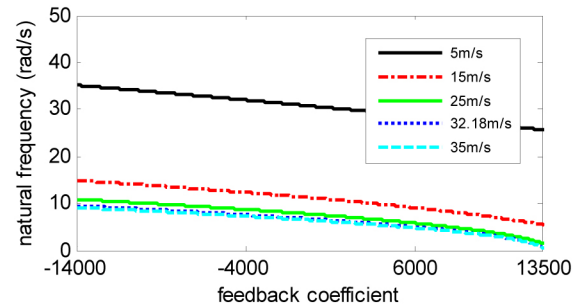


Figure 4. The relationship between system natural frequency and feedback coefficient.

Control Interventions

The first indicator of the need for control intervention is the lateral acceleration. The handling improvement control algorithm is designed based on the linear 2DOF single track model, but the model is no longer accurate when the tire characteristic enters the nonlinear region. On a dry asphalt road with a side slip angle of no more 3°, tire cornering stiffness remains in linear region, the corresponding lateral acceleration is under 0.4g[7].

The longitudinal velocity is also a significant indicator to determine the control intervention. The damping ratio will not be sensitive to the feedback coefficient's variety with a small velocity, so the actuator efficiency will be extremely low. On the contrary, when the velocity is too high, a large feedback coefficient will lead to a non-convergent system. Therefore, the intervention will be conducted between the velocity of 25 ~110km/h.

Optimal Feedback Coefficient

On the basis of the feedback coefficient influence and the control interventions, according to the optimization objects, the optimal natural frequency and damping ratio are chosen. The corresponding feedback coefficient is regarded as the optimal feedback coefficient. The feedback coefficient restriction should be taken into consideration, so the calculation of the optimal feedback control coefficient is converted to a optimization problem.

Transient response can be evaluated by the rise time, the peak time, the maximum overshoot, the delay time and the number of oscillation. Optimization objects should balance both the rapidity and stability of the transient response, so the peak time t_p and the maximum overshoot M_p are chosen as optimization objects.