

Analytical Solutions of Resilience Based on Tri-Parameters Constitutive Model under Different Cold-Forming Cases

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ABSTRACT

Ti-alloy sheet is a high-modulus elastic-plastic material, about which the resilience in the cold-forming process is quite difficult to control. As a matter of fact, the procedure of cold-forming is composed of many cases such as tensile case and bending case. An expression of ultimate radius for bending-curvature is obtained based on tri-parameters elastic-plastic constitutive model. By classifying the cold-forming process, some typical cases of cold-forming are presented in this paper, and analytical solutions with a high precision of resilience for high-modulus elastic-plastic material such as Ti-alloy sheet under the different typical cases are obtained in this paper. The accurate analytical solutions of resilience for high-modulus elastic-plastic material presented in this paper will contribute to cold-forming process both in the mould-designing and optimizing of cold-forming procedure.

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INTRODUCTION

The cold-forming process of sheet is a cost-effective processing method of modern industrial which is widely applied in machinery manufacturing, aviation industry, vehicle engineering, shipbuilding industry as well as other industry fields. It is generally carried out by stamping, which includes various complex processes such as cold-bending and cold-drawing process[1, 2, 3]. Due to the resilience, especially of high-modulus elastic-plastic sheet, after the cold-forming process, the precision control of products is a technical problem. Beside of depending on the material properties, the control of resilience of cold-forming-parts is also severely hampered by the process and procedure of cold forming. By classifying various cases in the cold forming process, three typical cases are presented in this paper, which are the case of simply transverse cold-bending, the case of bending after cold-drawing and the case of drawing after cold-bending (including the case of cold-bending companied with cold-drawing). Through the analytical analysis of resilience for high-modulus elastic-plastic material such as Ti-alloy sheet under different typical cases in the cold forming process, a

theoretical reference for design of stamping tool and optimization of cold-forming procedure is presented in this paper.

THE CASE OF SIMPLY COLD-BENDING

Schematically shown in Fig. 1 is the process of the cold forming of a sheet. The upper die is a puncher and the lower one is a punching die. When driving the upper die down onto the sheet which is placed horizontally on the lower punching die, the sheet will be bent naturally. Under the case of simply cold-bending, keep the axial tensile stress, denoted by σ_t , as $\sigma_t \equiv 0$.

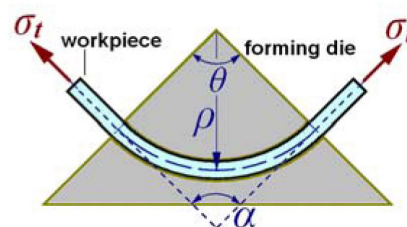


Figure 1. The process of the cold forming

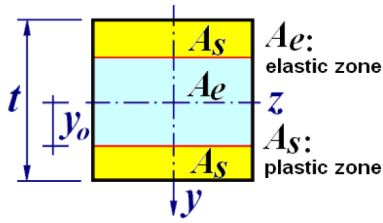


Figure 2. Plastic zone in the cross-section of work piece under the case of cold-bending

The tri-parameter elastic-plastic constitutive model (Fig.3) is given as:

$$\begin{cases} \sigma = E\varepsilon & \varepsilon \in [0, \varepsilon_s) \\ \sigma = \sigma_{so} [1 + A(\varepsilon - \varepsilon_s)^B \cdot e^{C(\varepsilon - \varepsilon_s)}] & \varepsilon \in [\varepsilon_s, \delta_s) \end{cases} \quad (1)$$

where E and σ_{so} are the modulus of elasticity and initial yield stress of the material respectively. δ_s is the elongation rate of the sheet before the tensile fracture occurs. A , B and C are constants of the material. By substituting Eq. 1 into $\frac{d\sigma}{d\varepsilon} = 0$, the ultimate strength of strain of the material can be expressed as:

$$\varepsilon_b = \varepsilon_s - \frac{B}{C} \quad (2)$$

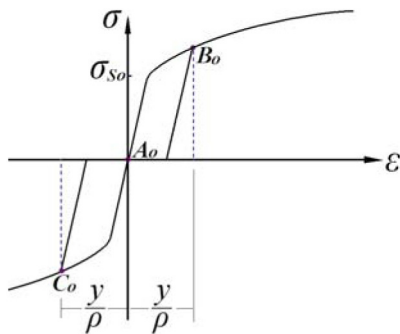


Figure 3. Historical stress-strain path under the case of simply cold-bending

Assuming the sheet has synchronous curvature in z direction under the case of cold-bending, the elementary beam theory can be applied to study the bending stress of sheet.

Define the bending radius of curvature of the ultimate strength as $\rho_b = \frac{t}{2\varepsilon_b}$ and the bending radius of curvature of the yield limit $\rho_s = \frac{Et}{2\sigma_{so}}$. When the neutral radius of the curvature of the bending sheet, denoted by ρ , satisfies $\rho_b < \rho < \rho_s$ it will exist a plastic zone in the cross-section, as shown in Fig.2. The spring-back occurs after the bending forces are removed and the sheet material will contain the residual stress. When $\rho_b < \rho$, the strain of the cross-section is $\varepsilon = \frac{y}{\rho}$, and the stress-strain trajectories of the top layer as well as bottom layer of the

sheet will move along with the curves A_0C_0 and A_0B_0 (shown in Fig. 3). The boundary layer of yielding zone is $y_o = \frac{\sigma_{so}}{E} \rho$. Then distribution of the normal-bending-stress on the cross-section before unloading (Fig.4) can be expressed as:

$$\sigma_{bend} = \begin{cases} -\sigma_s(-y) & y \in [-\frac{t}{2}, -y_o] \\ E \frac{y}{\rho} & y \in [-y_o, y_o] \\ \sigma_s(y) & y \in [y_o, \frac{t}{2}] \end{cases} \quad (3)$$

where $\sigma_s(y) = \sigma_{so} [1 + A(\frac{y}{\rho} - \varepsilon_s)^B \cdot e^{C(\frac{y}{\rho} - \varepsilon_s)}]$. The residual stress on the cross-section after unloading can be expressed as:

$$\sigma_{res} = \begin{cases} -\sigma_s(-y) - \frac{\Delta M}{I_z} y & y \in [-\frac{t}{2}, -y_o] \\ E \frac{y}{\rho} - \frac{\Delta M}{I_z} y & y \in [-y_o, y_o] \\ \sigma_s(y) - \frac{\Delta M}{I_z} y & y \in [y_o, \frac{t}{2}] \end{cases} \quad (4)$$

where ΔM is the equivalent bending moment of unloading.

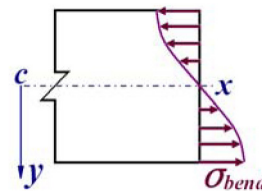


Figure 4. The distribution of the normal-bending-stress before unloading

After unloading, the total bending moment vanishes, $M_{res} = \int_A \sigma_{res} dA \cdot y = 0$, thus we can finally have the equation about the equivalent bending moment of unloading

$$\frac{E}{\rho} I_{zAe} - \Delta M + 2\overline{\sigma_s} S_{zAs}^* = 0$$

where $\overline{\sigma_s} = \frac{1}{2} [\sigma_{so} + \sigma_s(\frac{t}{2})] = \sigma_{so} [1 + \frac{A}{2} (\frac{t}{2\rho} - \varepsilon_s)^B \cdot e^{C(\frac{t}{2\rho} - \varepsilon_s)}]$, I_{zAe} and I_{zAs} are, respectively, the inertia moment about z -axis for the elastic zone and the plastic zone and S_{zAs}^* is the static moment about z -axis for the upper half of the plastic zone.

We can easily obtain the expression

$$\Delta M = \frac{E}{\rho} I_{zAe} + 2\overline{\sigma_s} S_{zAs}^*$$

