

Enhancing Decision Topology Assessment in Engineering Design

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ABSTRACT

Implications of decision analysis (DA) on engineering design are important and well-documented. However, widespread adoption has not occurred. To that end, the authors recently proposed *decision topologies* (DT) as a visual method for representing decision situations and proved that they are entirely consistent with normative decision analysis. This paper addresses the practical issue of assessing the DTs of a designer using their responses. As in classical DA, this step is critical to encoding the DA's preferences so that further analysis and mathematical optimization can be performed on the correct set of preferences. We show how multi-attribute DTs can be directly assessed from DM responses. Furthermore, we show that preferences under uncertainty can be trivially incorporated and that topologies can be constructed using single attribute topologies similarly to multi-linear functions in utility analysis. This incremental construction simplifies the process of topology construction. The methods are demonstrated using a design decision making problem of a welded beam.

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1. INTRODUCTION

The notion that engineering design is a decision making process is well accepted and documented (see Howard (1989), Thurston (1991) and Hazelrigg (1998)). However, despite all possible benefits, widespread use of decision making tools is impeded by the complexity of these tools to the average designer. Most practicing engineers are not trained in decision analysis or utility theory and as a result, they implement "best practices" when it comes to making decisions. Even when an expert attempts to elicit the DM's preferences, errors are introduced, affecting the quality of decisions to be made. These errors may result from problems associated with stated and revealed preferences (Train, 2003). It is also possible that the facilitator (expert helping the DM to make the decision) overwhelms the DM with lottery questions to assess utility functions, particularly when the DM is not entirely comfortable giving responses to lottery questions. The authors have recently proposed an alternative approach based on Decision Topologies, which alleviates these issues with classical decision analysis and also offers many significant advantages (Pandey and Mourelatos, 2013). They also showed that DTs are entirely consistent with decision analysis at the limit. In this paper, we make theoretical advances in DTs by focusing on their assessment directly. We show that DTs can be

constructed from single attribute topologies similarly to multi-linear functions in utility analysis. This makes DT assessment simple.

The paper is arranged as follows. Section 2 discusses decision topologies and Section 3 presents theoretical results in DT assessment. Section 4 describes a case study involving a welded beam demonstrating the topology assessment and the decision making process using the topology. Section 5 concludes and discusses directions for future work.

2. DECISION TOPOLOGIES

What is a Decision Topology?

A decision topology is a block diagram similar to the reliability block diagrams in reliability engineering (Figure 1) where a system is operational if its block diagram representation has a continuous path from one side of the diagram (input) to another (output). The decision topology extends this notion. If there are no paths from one end to the other, the DT is assigned a score of zero. Otherwise, all paths from one side of the diagram to the other are counted providing an overall score for the DT. This score is a positive linear transformation of the decision maker's utility function, a claim that has been substantiated in Pandey and Mourelatos (2013). We provide a sketch of the proof later for continuity.

We use a simple example to show how DTs work. Consider a car-purchasing decision where the attributes are fuel economy M in miles per gallon (mpg), acceleration A in seconds for 0 to 60 mph, roominess R in cu. ft., price P in dollars, luxury L (acceptable/unacceptable) and color C (acceptable/unacceptable). Some attributes such as luxury and color, are not easy to define mathematically because they are subjective. Figure 1 shows an example DT.

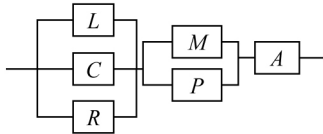


Figure 1. Reliability block diagram (decision topology) in a hypothetical vehicle buying decision.

The given DT can be interpreted using simple reliability engineering principles (Kapur and Lamberson, 1977). The algorithm for identifying topologies similar to the one above, and scoring them is based on our recent work towards identifying system topologies with limited information (Pandey and Mourelatos, 2012b) and evaluating a reliability block diagram using linear algebra operations (Pandey et al., 2012).

2.1. Proof of Consistency with Decision Analysis

Many types of Multi-Attribute Utility Functions (MAUF) exist (e.g., multi-linear, linear, Cobb-Douglas, etc.) allowing for different tradeoff behaviors. We have shown how decision topologies are consistent with decision analysis by showing that the behavior of any continuously differentiable utility function which is monotonic in attributes can be modeled by decision topologies. This is *in addition* to binary attributes, which can be easily incorporated.

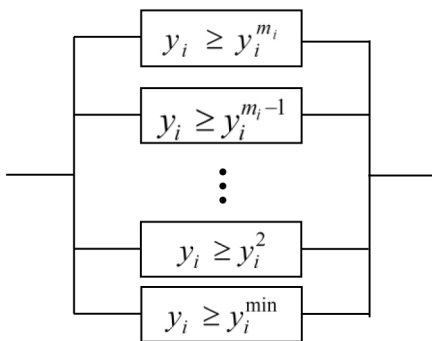


Figure 2. A decision topology for a single attribute (superscript is not an exponent).

Let us start with the single attribute case, where the objective is to find the score corresponding to an attribute level. We can represent the decision topology associated with an attribute visually as shown in Figure 2. The decision topology consists of blocks where each block tests the binary condition that the attribute level is greater than a partition y_i^j of the attribute. If this is true, the block is considered active. The *raw* score for an attribute level y_i is equal to the number of paths from right to left (or left to right) through the active blocks. For example, if

$y_i \geq y_i^{\max}$ all blocks will be active because all inequalities are satisfied. Similarly, if $y_i < y_i^{\min}$ none of the blocks will be active. For $y_i^{\min} < y_i < y_i^{\max}$, a number of blocks less than m_i will be active. Figure 3 pictorially shows the score from a DT as the attribute level is steadily increased and the blocks become progressively active.

Since utility functions are conventionally constrained between 0 and 1, we can divide the score of an attribute level by m_i to obtain a *normalized* score which we refer to as score S . It may seem that the score will be linear in attribute level (akin to a linear utility function) but this is not true. The way the score varies with respect to an attribute depends on the chosen partitioning for the attribute. A non-uniform partitioning can be used to draw the topology. If the density of partitions around a particular value of y_i is proportional to the derivative of the utility function at that value of y_i , the normalized score will be equal to the utility value (Figure 3). This is guaranteed if $m_i \rightarrow \infty$ since the score approaches the Riemann sum under the utility density function (derivative of the utility function). A rigorous proof is given in Pandey and Mourelatos (2013).

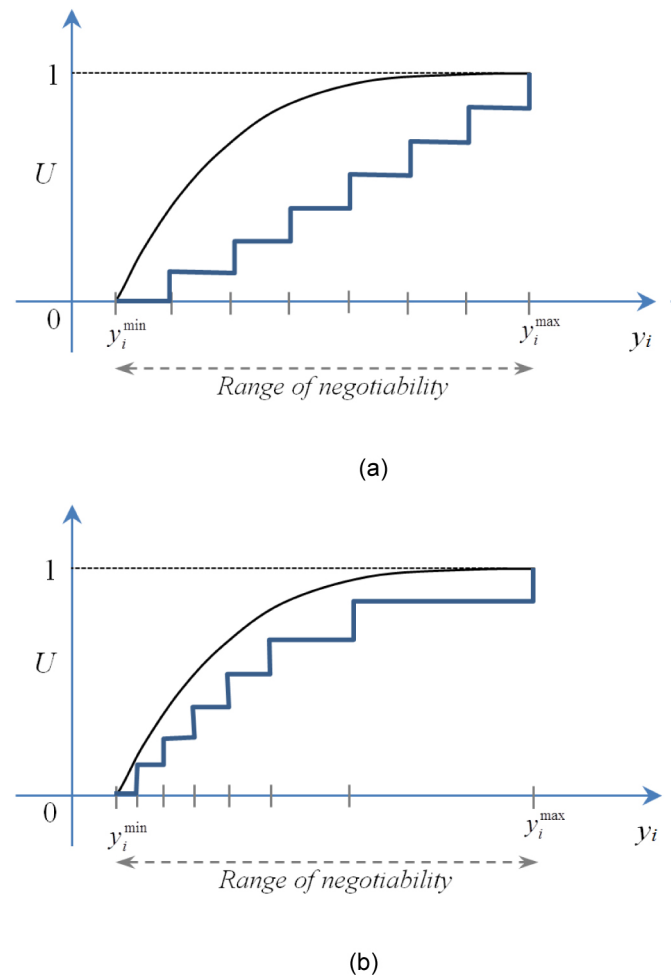


Figure 3. Comparison of a utility function with a decision topology score when the first partitioning of the attribute is uniform (a) or proportional to the local value of the derivative of the utility function (b).

Tradeoffs

The multi-attribute case is less straightforward compared to the single attribute case. For this reason, we motivate the reader by first showing that tradeoffs are also represented in a topology when multiple attributes are present. Consider the mileage and price block of Figure 1. Assume that the two attributes have three levels simplistically designated as low, medium and high. A tradeoff structure can be introduced as in Figure 4 which expands upon the price-mpg block of Figure 1. Based on Figure 4, the decision maker will buy a vehicle that meets their low price requirements even if it gives low mpg (top path). Similarly, a vehicle that provides average mpg at average initial price is also acceptable (middle path). Finally, a costly vehicle will be acceptable if it provides high mpg (bottom path).

Now we prove that the score determined from the topology is the same as that from a utility function. As before, we assume that the multi-attribute utility function is continuously differentiable and increasing in individual attributes. Also, each attribute is partitioned within its range of negotiability. The partition of each attribute results in a block of the form

$\{y_i \geq y_i^j\}$ in the topology. Since we have m_i blocks for an attribute i , the total number of block combinations is equal to $\prod_{i=1}^n m_i$. In this case, the most intuitive way to draw the decision topology is to use a tuple of blocks in each row of the decision topology as shown in Figure 5. The decision topology

will then have $\prod_{i=1}^n m_i$ rows in parallel. The score from such a topology mimics the multi-attribute utility function and is monotonic in each attribute, as expected. Moreover,

$$S(y_1 \geq y_1^{\max}, \dots, y_n \geq y_n^{\max}) = 1 \text{ and}$$

$$S(y_1 = y_1^{\min}, \dots, y_n = y_n^{\min}) = 0.$$

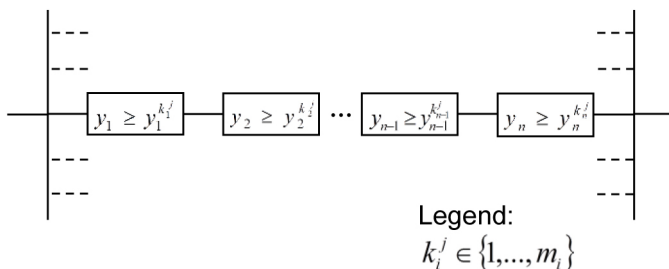


Figure 4. An example block representing tradeoff between price and mpg for the vehicle buying example.

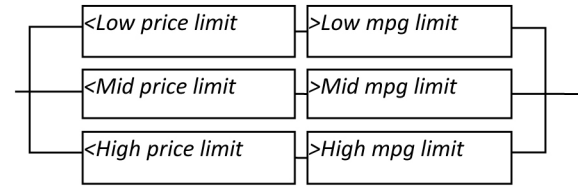


Figure 5. Decision topology for multi-attribute case (only one row is shown).

If the CDF analogy is used, the proof that DTs are consistent with DA can be derived using the arguments for the single attribute case. We have

$$S(y_1^t, \dots, y_n^t) = \int_{y_1^{\min}}^{y_1^t} \dots \int_{y_n^{\min}}^{y_n^t} u_{y_1, \dots, y_n}(y_1, \dots, y_n) dy_1 \dots dy_n =$$

$$U(y_1^t, \dots, y_n^t) \quad (1)$$

where $u_{y_1, \dots, y_n}(y_1, \dots, y_n)$ is the joint utility density function as defined in Abbas (2009) which is analogous to probability density functions. The analogy however, is not applicable to utility functions which are not n -increasing (i.e., having a negative mixed derivative; see Nelsen, 1999). Because such utility functions are common, we provide below a general proof that is valid for all continuously differentiable utility functions.

Assume that we start with the point $\{y_1^{\min}, \dots, y_n^{\min}\}$ where the utility function and the score $S(y_1^{\min}, \dots, y_n^{\min})$ are both zero. If the attribute vector is perturbed by a small amount $\{dy_1, \dots, dy_n\}$ in any general direction that keeps the attributes in the range of negotiability, the new utility can be approximated by

$$U(y_1^{\min} + dy_1, \dots, y_n^{\min} + dy_n) \approx \frac{\partial U}{\partial y_1} dy_1 + \dots + \frac{\partial U}{\partial y_n} dy_n. \quad (2)$$

Similarly to the single attribute case, if the first partitioning at every point along any attribute is proportional to the partial derivative along that attribute, we have

$$S(y_1^{\min} + dy_1, \dots, y_n^{\min} + dy_n) \approx$$

$$U(y_1^{\min} + dy_1, \dots, y_n^{\min} + dy_n). \quad (3)$$

As the partitioning gets finer, the approximation of Equation (3)

keeps improving. As we move on a path from $\{y_1^{\min}, \dots, y_n^{\min}\}$ towards any point $\{y_1^t, \dots, y_n^t\}$, the decision topology makes successive linear approximations of the utility function, and