

A Polynomial Chaos-Based Method for Recursive Maximum Likelihood Parameter Estimation of Load Sensing Proportional Valve

Zeyu Ma, Jinglai Wu, and Yunqing Zhang
Huazhong University of Science and Tech.

Ming Jiang
Dongfeng Motor Co. Ltd.

ABSTRACT

In this paper, a new computational method is provided to identify the uncertain parameters of Load Sensing Proportional Valve (LSPV) in a heavy truck brake system by using the polynomial chaos theory. The simulation model of LSPV is built in the software AMESim depending on structure of the valve, and the estimation process is implemented relying on the experimental measurements by pneumatic bench test. With the polynomial chaos expansion carried out by collocation method, the output observation function of the nonlinear pneumatic model can be transformed into a linear and time-invariant form, and the general recursive functions based on Newton method can therefore be reformulated to fit for the computer programming and calculation. To improve the estimation accuracy, the Newton method is modified with reference to Simulated Annealing algorithm by introducing the Metropolis Principle to control the fluctuation during the estimation process and escape from the local minima. The comparison between the introduced computational method and other estimation method indicates that the proposed method can be performed with higher convergence speed and robustness.

CITATION: Ma, Z., Wu, J., Zhang, Y., and Jiang, M., "A Polynomial Chaos-Based Method for Recursive Maximum Likelihood Parameter Estimation of Load Sensing Proportional Valve," *SAE Int. J. Commer. Veh.* 7(1):2014, doi:10.4271/2014-01-0721.

INTRODUCTION

The distribution ratio between front and rear brake force determines the performance of brake system [1]. The LSPV can adjust the proportion of brake pressure in front and rear brake circuits according to the vehicle load by automatically altering its flexible component geometry, therefore an accurate estimation for the structure parameters of its flexible component would be valuable to predict the behavior of the vehicle brake system and help to design the control strategies of the ABS controller. This paper provides a novel method using the experimental results to recursively estimate the uncertain parameters in LSPV based on polynomial chaos (PC) theory.

It is well known that Polynomial Chaos expansions (PCE) has significant computational advantage in uncertain analysis compared to Monte-Carlo method [2]. The polynomial chaos method has been attended by more and more researchers since it was first proposed by Wiener [3]. In recent years, Sandu *et al.* [4] has applied the PC theory to the multi-body dynamic systems with uncertainty, and then extended it to the uncertain analysis of vehicle dynamics [5], which shows that PC method is an efficient approach for predicting the underlying uncertainty in dynamic systems.

Since the PC method has good performance in transforming the non-linear system into state space form, the combination of the PC theory and estimation method has been used to estimate the unknown parameters, and many research papers have shown its ability in convergence and stability in parameter estimation. Blanchard E. *et al.* has developed two new computational approaches based on Bayesian framework and Kalman filter (EKF) theory respectively [6]. The first approach is performed by minimizing a cost function derived from Bayesian theorem [7], while the second approach is carried out by introducing the extended Kalman filter into PC theory [8]. These two methods can be summarized as the 'whole -set-of-data-at-once' approach, in which the objective function is constructed based on all the observations from the beginning to the end. Based on the method proposed by Blanchard [8], Ma *et al.* obtained the uncertain parameters of the load sensing proportional valve by minimizing the most likelihood function derived from polynomial chaos theory [9]. Alternately a recursive estimating method has also been developed, which could timely update the predicted value of the uncertain parameters according to the measurements and can therefore be developed into online parameter estimator. Southward [10] proposed a real-time parameter identification algorithm using Galerkin projection onto an orthogonal polynomial basis. Pence

[11] uniquely combines the base-excitation concept with the polynomial chaos estimation to identify the sprung mass of a 2-DOF quarter-car suspension model by MIT rule, and develops a recursive update law to maximize the likelihood, which is suitable for programming calculation [12].

Since the pneumatic model of the LSPV is strongly non-linear and complicated, it is difficult to derive the system into simple differential algebra equations (DAE) or ordinary differential equations (ODE). To solve this problem, a PCE approximated model is constructed by collocation method to transform the pneumatic model of LSPV into a linear and time-invariant form. Similar to the approach mentioned above, this paper recursively calculates the maximum likelihood values of the uncertain parameters based on a type of gradient estimation method, Newton method. However, compared with the works of Blanchard [8] and Ma [9], the estimation approach in this paper can be classified into the recursive estimation method. Meanwhile, unlike any of the methods above, this paper introduces the Metropolis Principle with reference to the Simulated Annealing algorithm into the Newton method to improve its convergence speed and estimation accuracy. To validate the efficiency of the proposed method, a bench test was set up and the output pressure of the LSPV was measured to serve as the observations of the estimation algorithm. The comparison between the introduced computational method and other estimation methods indicate that the proposed method can be performed with higher convergence speed and robustness.

POLYNOMIAL CHAOS APPROACH

For many practical problems, one is only interested in knowing the state of a system at a discrete set of times $t \in \{t_T | T = 1, 2, \dots, N\}$. These discrete times may correspond to the times at which the outputs of a system are sampled. The mathematical expression of dynamic system for time varying nonlinear discrete model can be defined as a recursive type as follow:

$$\begin{aligned} y(t_T) &= f(t_{T-1}, y(t_{T-1}), u(t_{T-1}), \Theta) \quad T = 1, 2, \dots, N \\ y(t_0) &= y_0 \end{aligned} \quad (1)$$

Where the vector $y(t_T) \in R^{n_s}$ represents the system states with known initial conditions y_0 , and the vector

$\Theta = [\Theta_1, \Theta_2, \dots, \Theta_n]^T$ contains the unknown parameters.

The vector $u(t_T)$ denotes the input signal of the dynamic system, varying with respect to time, and f denotes the discrete model solution operator which derives $y(t_T)$ at time t_T from known y_{T-1} at time t_{T-1} .

Generally, the observations of dynamic system available at discrete time t_T can be defined as a nonlinear and time-varying output model:

$$z(t_T) = h(t_{T-1}, z(t_{T-1}), u(t_{T-1}), \Theta) \quad T = 1, 2, \dots, N \quad (2)$$

However, in this paper, we simplify it to a linear, time-invariant, discrete model, described as follow:

$$z(t_T) = Hy(t_T) + v(t_T) \quad T = 1, 2, \dots, N \quad (3)$$

H is the linearization of the observation vector and the vector $z(t_T) \in R^{n_z}$ contains the output observations. The vector $v(t_T) \in R^{n_z}$ represents the observational error which is caused by measurement noise with Gaussian distribution with zero mean and a known covariance matrix $R_T \in R^{n_z \times n_z}$.

The basic theory of PC approach is that a random process of interest can be approximated by sums of orthogonal polynomial chaos of random independent variables. This indicates that any uncertain variable u with known distribution can be represented in terms of an n -dimensional random variable $\xi = [\xi_1, \dots, \xi_n]^T$. Therefore, the uncertain parameter vector $\Theta = [\Theta_1, \Theta_2, \dots, \Theta_n]^T$ and system state $y(t_T) = [y_1(t_T), y_2(t_T), \dots, y_{n_s}(t_T)]^T \in R^{n_s}$ can be approximated as the expansion of orthogonal polynomial basis as follows:

$$\Theta_k = \sum_{i=1}^s \Theta_k^i \Phi_i(\xi), \quad 1 \leq k \leq n \quad (4)$$

$$y_k(t_T) = \sum_{i=1}^s y_k^i(t_T) \Phi_i(\xi), \quad 1 \leq k \leq n_s \quad (5)$$

These approximate equations can be more accurate with $s \rightarrow \infty$ in the L^2 sense. However, it needs a large amount of computation, which seems to be unavailable, so the truncated expression has to be used.

The coefficients Θ_k^i and $y_k^i(t_T)$ with respect to time can be derived from either Galerkin or collocation method. However, the Galerkin method is not suitable for large and complicated dynamic system, which could not be expressed by explicit formulations. On the other hand, the stochastic collocation requires less computationally expensive than the traditional Galerkin approach, which turns out to be an advantage in recursive parameter estimation process, so the stochastic collocation approach is selected to approximate the random process in the discrete dynamic system.

Substitute Eq. (4) and (5) into Eq. (1), leading to:

$$\begin{aligned} & \sum_{i=1}^s y_k^i(t_T) \Phi_i(\xi) \\ &= f\left(t_{T-1}, u(t_{T-1}), \sum_{i=1}^s y^i(t_{T-1}) \Phi_i(\xi), \sum_{i=1}^s \Theta^i \Phi_i(\xi)\right) \\ & T=1, 2, \dots, N \quad 1 \leq k \leq n_s \end{aligned} \quad (6)$$

Another way to express the polynomial chaos expansion is to rewrite the Eq. (5) as the form of matrix manipulation, shown in Eq. (7). Here, the vector $\ell = [\Phi_1(\xi), \Phi_2(\xi), \dots, \Phi_s(\xi)]^T$ is a s dimensional multi-index, and $\chi_k(t_T) \in R^s$ is denoted as

$$\chi_k(t_T) = [y_k^1(t_T), y_k^2(t_T), \dots, y_k^s(t_T)]^T \text{ for.}$$

$$y_k(t_T) = \ell^T \times \chi_k(t_T) \quad 1 \leq k \leq n_s \quad (7)$$

For simplifying the notation in the following sections, we stack the coefficients vector of the polynomial chaos basis $\chi_k(t_T)$ into matrix $X \in R^{s \times n_s}$, so Eq. (1) can be expressed as follow:

$$y(t_T) = L(\xi) \times X(t_T) \text{ for}$$

$$X(t_T) = \left[(\chi_1(t_T))^T, (\chi_2(t_T))^T, \dots, (\chi_{n_s}(t_T))^T \right]^T \quad (8)$$

Here, $L \in R^{n_s \times s \times n_s}$ can be defined as:

$$L(\xi) = \begin{bmatrix} \ell^T & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \ell^T \end{bmatrix} \quad (9)$$

The number of simulations solved for the PC basis coefficients is determined by the number of collocation points, which is typically much more efficient compared with Monte Carlo simulations in order to assess uncertainties [13]. As the dynamic system of interest is complicate and nonlinear, expressing the dynamic system in DAE formulations would be time-consuming and difficult. Therefore, the PC theory is employed and the PC functions are constructed to approximate the random process of the dynamic system by independent variables, which could be regarded as uncertain parameters in parameter estimation process.

THE MAXIMUM LIKELIHOOD ESTIMATION

We can use the difference between the model function or 'signal' and the observation to construct the likelihood. If the true signal is known, this difference would be just the noise or observational error. For parameter estimation, it would be

convenient to consider that the uncertain parameters without changing in the dynamic model evaluation. Substituting Eq. (8) into Eq. (3), we have the following equation

$$z(t_T) = H(\xi) L(\xi) X(t_T) + v(t_T) \quad T=1, 2, \dots, N \quad (10)$$

Where $z(t_T) \in R^d$ is the observation vector at t_T , and the observational error is induced by the experimental uncertainty associated with the measurements. According to the central limit theorem of probability, if the noise is known to be the result of many small independent effects, it is usually considered to have a Gaussian distribution with zero mean and a known covariance matrix $R(t_T)$, and the probability density function of v_T can be expressed as:

$$\begin{aligned} & P(v(t_T) / z(t_T)) \\ &= \frac{1}{\sqrt{2\pi} \det(R(t_T))} \times \\ & \exp \left[-\frac{1}{2} (z(t_T) - H(\xi) L(\xi) X(t_T))^T R_k^{-1} (z(t_T) - H(\xi) L(\xi) X(t_T)) \right] \end{aligned} \quad (11)$$

Use the product rule form probability theory to obtain the probability of vector $\{v(t_1), \dots, v(t_N)\}$, so the joint probability density function of the noise values is expressed as:

$$\begin{aligned} & P(\Theta / z(t_1), \dots, z(t_N)) \stackrel{\text{def}}{=} P(v(t_1), \dots, v(t_N) / z(t_1), \dots, z(t_N)) = \frac{1}{\text{den}} \times \\ & \exp \left[-\frac{1}{2} \sum_{T=1}^N (z(t_T) - H(\xi) L(\xi) X(t_T))^T R_k^{-1} (z(t_T) - H(\xi) L(\xi) X(t_T)) \right] \end{aligned} \quad (12)$$

The denominator can be evaluated by multiplying the Probability Density Function (PDF) $P(v(t_T)/z(t_T))$ of all states. However, it is not necessary to compute this scaling factor for this factor does not change the extreme point of the function which needs to be minimized.

According to the maximum likelihood principle, the point $\hat{\Theta}$ which maximizes the probability $P(\Theta(\hat{\xi}) / z(t_1), \dots, z(t_N))$ is the closest point to the true point Θ_0 , which, equivalently, minimizes the element $\ln(P(\Theta(\hat{\xi}) / z(t_1), \dots, z(t_N)))$ and could be attained by minimizing $J(t_T)$ defined as:

$$\begin{aligned} & J(t_T) = (z(t_T) - H(\xi) L(\xi) X(t_T))^T \times \\ & R_k^{-1} (z(t_T) - H(\xi) L(\xi) X(t_T)) \\ & T=1, 2, \dots, N \end{aligned} \quad (13)$$