

Development of Transient Thermal Models Based on Theoretical Analysis and Vehicle Test Data

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ABSTRACT

In this paper, thermal models are developed based on experimental test data, and the physics of thermal systems. If experimental data is available, the data can be fitted to mathematical models that represent the system response to changes in its input parameters. Therefore, empirical models which are based on test data are developed. The concept of time constant is presented and applied to development of transient models. Mathematical models for component temperature changes during transient vehicle driving conditions are also presented. Mathematical models for climate control system warm up and cool-down are also discussed. The results show the significance of adopting this concept in analysis of vehicle test data, and in development of analytical models. The developed models can be applied to simulate the system or component response to variety of changes in input parameters. As a result, significant testing and simulation time can be saved during the vehicle development process.

CITATION: El-Sharkawy, A. and Uddin, A., "Development of Transient Thermal Models Based on Theoretical Analysis and Vehicle Test Data," *SAE Int. J. Passeng. Cars - Mech. Syst.* 7(1):2014, doi:10.4271/2014-01-0726.

INTRODUCTION

Theoretical models are usually developed based on the system physics. However, development of rigorous theoretical models may not be practical for complex processes if the model requires a large number of differential equations with significant number of unknown parameters. An alternative approach is to develop an empirical model directly from empirical data. Empirical models are usually referred to as black box models. In this approach, the inputs and outputs are known, but the model internals are unknown. The following section will discuss different modeling approaches based on empirical data. In this paper, the concept of time constant and its applicability to the overall vehicle thermal management systems are discussed.

Mathematical models based on test data are presented in the following sections.

Linear Regression

In linear regression, experimental data are fitted into polynomial equation. The polynomial coefficients are selected by minimizing the difference between experimental data and the developed polynomial model.

The error ε_i , which represents the difference between the experimental test data and the model estimates can be expressed as follows:

$$\varepsilon_i = y_i - (a_0 - a_1 x_i) \quad (1)$$

Where y_i represents the measured test data points, a_0 and a_1 are the polynomial coefficients of the linear regression model. x_i Represents the model input parameters. A commonly accepted method of determining the best fit is to calculate the values of a_0 and a_1 that minimize the sum of squares of the errors ξ , namely:

$$\min \xi = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (y_i - (a_0 + a_1 x_i))^2 \quad (2)$$

Non-Linear Regression

If the model is nonlinear with respect to the model parameters, then nonlinear regression rather than linear regression needs to be used. A general nonlinear model can be written as:

$$\bar{y} = f(x_1, x_2, x_3, \dots, a_1, a_2, a_3, \dots) \quad (3)$$

Where $\bar{y}$ is the empirical model output, the x 's are the model inputs and a 's the coefficients to be estimated. In this case, the coefficients are not linear multipliers of the input functions.

Fitting of the First Order Models Using Step Function

The output response of a system to a step change in input is usually referred to as the process reaction curve. If the process model can be approximated by a first order differential equation, the model parameters can be obtained by inspection of the reaction curve. From test data, the reaction curve can be constructed. The required parameters can then be estimated. A dynamic first order model can be represented as follows:

$$\tau \frac{dy}{dt} + y = Kx \quad (4)$$

The transfer function of such a system can be described in terms of Laplace transformation by the following equation:

$$G(s) = \frac{K}{\tau s + 1} \quad (5)$$

If the input x is suddenly changed from 0 to M in zero time, the response is given by:

$$y(t) = KM(1 - e^{-t/\tau}) \quad (6)$$

In the above equation, τ is the system time constant. After a time t equal to τ , the response can be represented by the following equation:

$$y(t) = KM(1 - e^{-1}) \quad (7)$$

At this condition, the response curve has a value of 63.2% of the final steady state. Therefore, if test data is available, the system or process time constant can be extracted.

K is defined as the process gain, which can be expressed as:

$$K = \frac{y(\infty)}{M}$$

M is defined as the magnitude of change in input signal. $y(\infty)$ is the steady state value of the response curve.

Once the time constant is determined from step response curve, [equation \(4\)](#) can be solved to determine the system response for other input signals such as ramp, pulse or compounded input signals. Hence, there is no need to know details about the system internal design.

Time constant can also be used for conducting sound comparisons between different system designs or comparison of thermal performance of different vehicles.

In several experimental situations, it is difficult to perform a pure step change. Therefore, in real testing environment, few experimental test data follow a first order response.

Hence, there will be some departure from the first order response curve. To account for higher-order dynamics that are neglected in a first order model, a time delay term can be included. The introduction of these dynamic parameters improves the agreement between test data and the process or system model. A transfer function, equivalent to [equation \(5\)](#) can be expressed as:

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1} \quad (8)$$

The solution to the system response for the above equation is expressed by:

$$y(\infty) - y(i) = y(\infty)e^{-\frac{t_i - \theta}{\tau}} \quad (9)$$

A linearized form of the above equation can be written in as follows:

$$\ln \left[\frac{y(\infty) - y(i)}{y(\infty)} \right] = -\frac{t_i - \theta}{\tau} \quad (10)$$

This equation represents a straight line relationship between $\ln \left[\frac{y(\infty) - y(i)}{y(\infty)} \right]$ and $(t_i - \theta)$. The slope of the straight line is equal to $-\frac{1}{\tau}$. To determine the time constant τ and the delay time θ , it was suggested to determine the system response curve at two values of t_i corresponding to 35.3% and 85.3% response times [\(4\)](#). The time delay and time constant can then be estimated from the following equations:

$$\theta = 1.3t_1 - 0.29t_2 \quad (11)$$

$$\tau = 0.67(t_2 - t_1) \quad (12)$$

Fitting Second Order Models Using Step Function

Normally, a better model can be obtained by fitting test data into a second order model. A second order model can be expressed by the following transfer function:

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (13)$$

A second order system with time delay can be expressed as:

$$G(s) = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad (14)$$

A more general model (5) is given by the following equation:

$$G(s) = \frac{Ke^{-\theta s}}{(\tau^2 s^2 + 2\zeta\tau s + 1)} \quad (15)$$

THERMAL SYSTEMS MATHEMATICAL MODELS

Transient Response of a Thermocouple

Vehicle Thermal Management System (VTMS) traditionally depends on continuous monitoring of different system component as well as fluid temperature where hundreds of thermocouples are generally used to measure these temperatures. To ensure a safe and efficient VTMS process, it's essential to measure the accurate temperatures under dynamic conditions [1]. The dynamic response of a thermocouple (TC) sensor is often an important consideration in designing a measurement system. The response of a temperature sensor known as a thermocouple may be modeled as a first-order system. When the thermocouple is subjected to a rapid temperature change, it will take some time to respond. If the response time is slow in comparison with the rate of change of the temperature that are measured, then the TC will not represent the dynamic response to the temperature fluctuations.

A model of the response of a TC is based on energy balance around the TC junction (Figure 1). Heat transferred to a thermocouple junction is represented by the following:

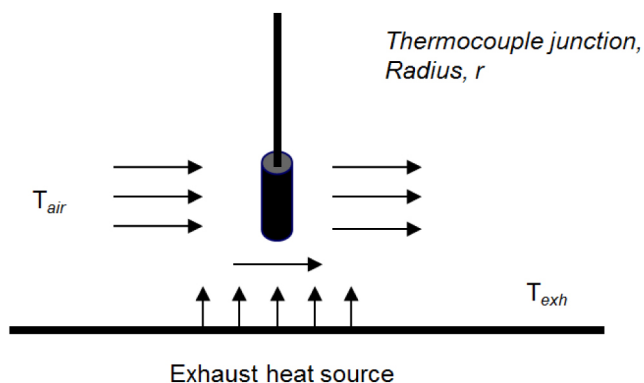


Figure 1. Thermocouple in an exhaust heat source

In a real operating environment transient energy balance around a thermocouple junction is given by:

$$m_c c_p \frac{\partial T_c}{\partial t} = hA(T_{air} - T_c) + \sigma \varepsilon FA(T_{exh}^4 - T_c^4) - k_c A \frac{\partial T_c}{\partial r} \quad (16)$$

Where m_c is the mass of thermocouple junction, and c_p is the heat capacity, h is the convection heat transfer coefficient, A is the surface area of the junction, T is the temperature, σ is the Stefan-Boltzmann constant, ε is the emissivity, and k_c is the thermal conductivity for the TC lead [4].

Conduction resistance through thermocouple wall can be ignored and therefore the equation becomes:

$$m_c c_p \frac{\partial T_c}{\partial t} = hA(T_{air} - T_c) + \sigma \varepsilon FA(T_{exh}^4 - T_c^4)$$

In absence of radiation, the heat transfer is controlled by convection and the equation can be simplified to

$$m_c c_p \frac{\partial T_c}{\partial t} = hA(T_{air} - T_c) \quad (17)$$

The transient equation reduces to:

$$\begin{aligned} m_c c_p \frac{dT_c}{dt} &= hA(T_{air} - T_c) \\ \frac{m_c c_p}{hA} \frac{dT_c}{dt} &= (T_{air} - T_c) \end{aligned} \quad (18)$$

Time constant of the system can be denoted by

$$\tau = \frac{m_c c_p}{hA} \quad (19)$$

Thus the system equation can be written as:

$$\tau \frac{dT_c}{dt} = (T_{air} - T_c)$$

At steady-state condition: $T_c = T_{air}$

For transient analysis, however, consider a step change in the surrounding temperature from T_0 to T_p . Transient temperature of the TC junction can be determined from a first order system response:

$$\frac{T_c - T_f}{T_0 - T_f} = e^{-\frac{t}{\tau}} \quad (20)$$

After a time interval equal to the time constant:

$$\frac{T - T_f}{T_0 - T_f} = e^{-1} \quad \text{or} \quad \frac{T - T_f}{T_0 - T_f} = 0.367$$

$$\frac{T - T_0}{T_f - T_0} = 0.632$$

