

Estimation of Nonlinear Viscoelastic Parameters from Estimated Linear Models of Behavior around Multiple Settling Points of a Foam-Mass System

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ABSTRACT

Flexible polyurethane foam is the main cushioning element used in car seats. Optimization of an occupied seat's static and dynamic behavior requires models of foam that are accurate over a wide range of excitation and pre-compression conditions. In this research, a method is described to estimate the parameters of a global model of the foam behavior from data gathered in a series of impulse tests at different settling points. The estimated model is capable of describing the responses gathered from all the impulse tests using a unique set of parameters. The global model structure includes a nonlinear elastic term and a hereditary viscoelastic term. The model can be used to predict the settling point for each mass used and, by expanding the model about that settling point, local linear models of the response to impulsive excitation can be derived. From this analysis the relationship between the local linear model parameters and the global model parameters is defined. A series of experiments are conducted using different sized masses on the foam block. For each mass, the settling point is measured and the free response after an impulsive excitation is modeled as a Prony series whose parameters can be related to the parameters in the local linear dynamic model. By using the relationship between the local and global model parameters and estimates of the local models' parameters, the parameters of the global model are estimated. The estimation method is first applied to simulation data and then used to identify models of the uniaxial dynamic behavior of polyurethane foam blocks.

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INTRODUCTION

Flexible polyurethane foam is used in a wide range of engineering applications including seating, cushioning, sports, medical equipment, and vibration and sound isolation. In order to use this material into systems such as car seats, foam's static and dynamic behavior needs to be characterized. Foam exhibits both nonlinear and viscoelastic mechanical behavior. Most of the current models, which describe the mechanical behavior of foam undergoing large strain compression, have only been validated for a specific type of loading [1] or in certain regions [2,3]. Therefore, the estimated models, while often of almost the same structure, have different parameter values depending on compression levels and the form of the excitation. This leads to the estimation of different models for a certain material which is not realistic. Therefore, developing a

comprehensive model, which can be used to predict the foam's behavior around different levels of mean compression and different forms of excitation is a need.

Widdle, Bajaj and Davies [1] studied a nonlinear and viscoelastic model that predicts the foam behavior under slowly changing uniaxial compression. In each of their experiments strain rates were kept constant and the resulting stress measured. The foam's elastic contribution was modeled with Ogden's hyperelastic model while the viscoelastic contribution was modeled as the convolution of a hereditary (sum of exponentials) kernel and strain. In an attempt to account for the rate dependency of polyurethane foam, Puri [4] used a linear hereditary type viscoelastic model that is a convolution of a hereditary kernel with strain rate rather than strain. In his model, to simplify parameter estimation from the uniaxial

stress-strain data, he used a polynomial to model the elastic behavior of the foam. Widdle and Puri observed that they still needed different foam models to capture the responses of foam compressed at different strain rates.

Azizi, Davies and Bajaj [5] made improvements by developing a foam model in which the viscoelastic component has two parts: a convolution of a relaxation kernel with a nonlinear function of strain and a convolution of a second relaxation kernel with strain rate. Typically the hereditary kernels contained only one or two terms and the estimated model parameters still changed with the compression rate. However, they showed that by using a much higher order viscoelastic kernels, i.e., with close to 20 terms in both the viscoelastic components, they could develop a global model that can describe the uniaxial behavior of foam compressed at different rates. They also incorporated this global foam model into a one-dimensional foam-mass system model to study the dynamic behavior of a foam-mass system constrained to move uniaxially while subjected to harmonic base excitation [6].

Deng [2] also studied the dynamic response of a foam-mass system subjected to harmonic base excitation. She used the harmonic balance technique to estimate the system parameters, including nonlinear elastic and viscoelastic parameters, using data from harmonic base excitation of a foam-mass system. She showed that the estimated parameters are functions of different levels of mean compression in the foam. She also compared the performance of estimated hereditary and fractional derivative viscoelastic models [2,3].

Singh [7] developed a linear model for a single-degree of freedom foam-mass system subjected to an impulsive excitation. He used free response data from impulse tests on a foam-mass system with different masses to identify models at various pre-compression levels (settling points). The free response of the system was modeled as a Prony series (sum of exponentials) whose parameters can be related to the parameters in the foam-mass system model. Models identified from tests at one settling point performed poorly when used to predict the response at other settling points.

The overall goal of this research is to build on previous modeling research to develop global nonlinear viscoelastic models of the foam that can be used in models of systems incorporating foam to predict settling points and the dynamic responses around those settling points. One approach to the development of such a global model is described in this paper.

Impulse tests are conducted on a foam-mass system and parameters of local linear models are estimated. Different masses are used to achieve different compression levels. The global foam model includes a nonlinear elastic component and a hereditary type viscoelastic model as well as a viscous velocity proportional damping term. The nonlinear model is linearized about a settling point, and this is used to establish the relationship between the local linear model parameters and the global model parameters. Building on the work of Singh [7],

the free responses around each settling point after an impulsive excitation are modeled as Prony series. The Prony series parameters are used to derive estimates of the parameters of the local linear dynamic models. The relationship between the parameters of the local models around each of the settling points and the global model parameters is used to derive estimates of the parameters of the global model.

This paper is organized as follows. First, the experimental setup, test protocol and the instrumentation used to acquire the impulse response are described. Next the global foam-mass system model is described and the linearized equations about a settling point for a given mass are given. The parameter estimation techniques for both the local models and the global models are described. The method is then applied to two sets of data from simulations of a foam-mass system, and finally, to a set of data from a series of experiments on a foam-mass system. The paper concludes with a summary of the results and recommendations for future studies.

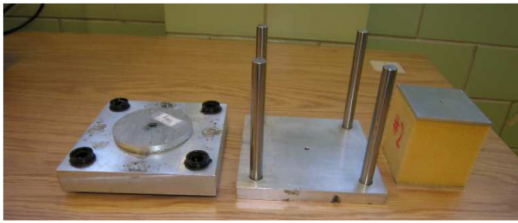
EXPERIMENTS

The foam-mass system experimental rig and instrumentation are first described and then the experimental procedure followed to produce the data used in the model parameter estimation is given. The open cell foam used in this experiment is commonly used in car seats and the foam has a relative density= 0.027. Because of the long memory of the foam, it is necessary to let the foam rest between experiments to produce repeatable experimental results.

Experimental Setup and Instrumentation

The test rig is shown in [Fig. 1](#). The foam-mass system test rig consists of a 3" cube of polyurethane foam with thin aluminum plates glued to the bottom and top surfaces, a four post fixture to constrain the motion to be uniaxial, and a top plate with four holes lined with low friction linear bearings for the posts to pass through. Additional masses can be attached to the top plate as well. The bottom plate is attached to a table or a shaker. The thin plates on the top and bottom of the foam block have threaded holes that are used to attach to the foam block to the base plate and the top plate with screws.

An electric impulse hammer, model 086C09 manufactured by PCB Piezotronics with a soft tip (084C11) was used to impart impulse on the foam-mass system. The force was measured using a PCB 208M51 force transducer. As shown in [Fig. 2](#), the impulse hammer was mounted on a post which was adjustable in the vertical direction. Adjustments in the vertical direction help in controlling the amplitude of the impulsive excitation force. The goal is to keep the impulse response close to being linear about each settling point by adjusting the impulsive force. However, the impulse response shouldn't be so small so that the signal to noise ratio in the early part of the response signal is not too low.



(a)



(b)

Figure 1. The experimental fixture: (a) the top plate with linear bearings around the holes (left) that slide over the four posts shown attached to the base plate (center); and (b) the assembled setup.

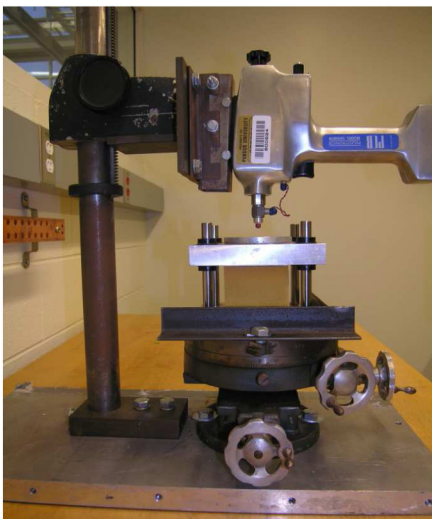


Figure 2. Experimental setup for the impulse test with the test fixture bolted to a base plate and the impulse hammer whose position can be adjusted vertically.

The foam-mass system was fixed to an adjustable x-y table to ensure that the generated impulsive force consistently hit closest to the center of the top plate, to make any rocking of the top plate as small as possible. Although, variations in friction in the four posts and in the homogeneity of the foam samples make it difficult to completely eliminate rocking. The x-y table was adjusted manually to get an impact point where the four accelerometer responses were found to be very close to each other. In order to monitor the rocking of the top plate and to reduce its effect on the uniaxial motion measurement, four accelerometers were used on the top plate to measure its motion. These were placed half way between adjacent posts.

The average of the four responses was used as the uniaxial response measurement in subsequent parameter estimation procedures.

The four accelerometers and the input force transducer signals were passed through an analog 48 dB/Octave antialiasing filter with a cut off frequency of 512 Hz. The signals were then sampled using a National Instruments 9821series Data Acquisition Card at a rate of 4096 samples/second. After inspection of the signals' temporal and spectral characteristics and initial attempts to model the free responses, it was found to be advantageous to pass the sampled signals through a 5th order digital Butterworth low pass filter with a cut off frequency 128 Hz. The damped natural frequency of the strongest components in the signal was typically below 20 Hz.

In this study, it is assumed that the fixture doesn't contribute to the results and the measured responses represent the foam behavior.

Testing Procedure and Sample Results

As discussed earlier, foam has a memory and the foam's response is highly dependent on the loading history. Thus it is important to allow sufficient recovery time between successive tests. Considering the entire range of load values to be used for impulse testing, it was found that duration of 48 hours between loading the foam and performing impulse test gave repeatable measurements. This was true for all the masses used. After the test, the foam was left unloaded and allowed to relax for another 48 hours (allowing the foam to recover to its initial height) before performing the next test.

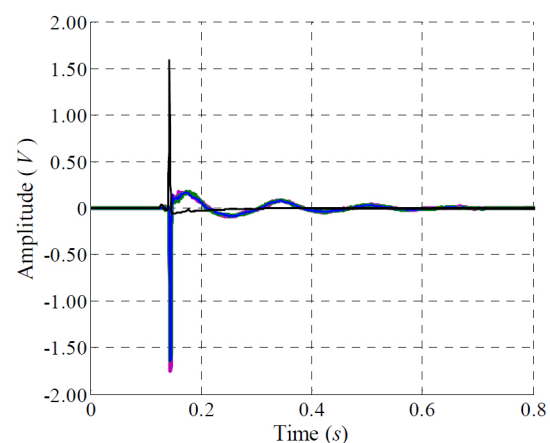


Figure 3. Signals measured during an impulse test on the foam-mass system. These are the un-calibrated signals (voltages). Force (black); and acceleration (blue, red, green, purple).

A sample test result is shown in Fig. 3. After removing the abrupt response during the forcing provided by the impulse hammer (at around 0.15 seconds), the four acceleration measurements are seen to be very close to each other as shown in Fig. 4. Here the time has been adjusted so that $t = 0$