

A Lode Angle Dependent von Mises Yield and Hardening Model for Deformation Simulation of Cast Magnesium Alloy

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ABSTRACT

Magnesium alloys are of growing research, development and commercial interest for their lightweight characteristics, notably in the automotive sector. Recent results based on experiments and simulations of beam components have shown that finite element (FE) predictions using commercial FE software may significantly overestimate the peak load and load beyond the peak load. This indicates that better deformation and failure criteria are needed for crashworthiness simulation and design of Mg alloys for the development of computer-assisted engineering (CAE) capacity for Mg alloys.

In this study, yield and hardening laws for deformation simulation of Mg alloys are reviewed. An isotropic Lode angle dependent von Mises yield and flow model originally used for soil was modified by replacing shear strength with tensile or compressive flow strength for deformation simulation of Mg alloys. In this law, the yield and strain hardening criteria depend on Lode angle (θ), a parameter related to the stress state of the material, e.g. $\theta=0^\circ$ corresponds to uniaxial tension and $\theta=30^\circ$ corresponds to pure shear. For the Lode angle to equal 0° , 30° and between 0 to 30° , the criterion for the Lode angle dependent von Mises model is equivalent to the von Mises criterion, the Tresca criterion and a criterion between the Tresca and von Mises. Comparisons of predicted deformation curves with experimental results of notch tension specimens with different notch radii, and notched and un-notched three-point bend specimens show that the Lode angle dependent von Mises law has much better prediction than the von Mises law for those materials with Lode angle dependent plastic flow, such as Mg alloy. The model may also be applied to deformation simulation of cubic alloys (such as Al alloys and steel).

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INTRODUCTION

Magnesium alloys are hexagonal close-packed (HCP) materials of growing research, development and commercial interest for their lightweight characteristics, notably in automotive sector. Because of the lower crystallographic symmetry compared to the common cubic metals, such as steel and aluminum alloys, the deformation of Mg alloys requires activation of several variants of both slip systems and twin systems, and is complex. Magnesium alloys are also relatively brittle. Fractographic examination showed typical quasi-cleavage fracture for Charpy and fracture toughness specimens tested at room temperature and quasi-static rate [1]. Finite element (FE) simulation of deformation and fracture of Mg alloys is important for the design of Mg components. Robust crashworthiness simulation may reduce 30 to 50% development time and costs for automotive manufacturers [2] and is an important enabling technology in the application of new automotive materials. For crashworthiness, it is essential to accurately simulate load vs. deflection relationships and failure behaviour of beam-type components under bending and axial crush at quasi-static and impact loading rates. However,

recent results from beam components have shown that FE predictions using commercial FE software may significantly overestimate the peak load and load beyond the peak load [3]. This indicates that better deformation and failure criteria are needed for crashworthiness simulation and design of Mg alloys for the development of computer-assisted engineering (CAE) capacity for Mg alloys.

Although it is possible to employ damage plasticity theory to deal with the over-prediction, this theory also needs to describe accurate elastic-plastic stress-strain fields to be a robust simulation algorithm. This is because the introduction of damage in the elastic-plastic deformation region contradicts the traditional elastic-plastic stress-strain plasticity theory. When the von Mises yield and hardening laws are used for deformation simulation of Mg alloys, it has been shown that the yield and hardening loads are usually overestimated [3,4].

In the present study, yield and hardening laws for deformation simulation of Mg alloys are reviewed. A Lode angle dependent von Mises yield and hardening law is proposed and applied to

model deformation of cast Mg alloys under multiaxial stresses. Cylindrical notched tension specimens, having bi-axial stress with different stress ratios and Lode angles, and bent Charpy bars were tested and simulated. The predictions by using the proposed Lode angle dependent von Mises flow model and the routinely used von Mises flow law were compared with experimental data.

The isotropic model is applied to cast Mg alloys in this work because the tensile-compressive asymmetry of cast Mg alloys, especially die-cast Mg alloys with random texture, is much smaller than that of wrought Mg alloys with strong texture. Simulation of cylindrical notched tension specimens only requires tensile flow curve. As will be shown, it is reasonable to assume that the tensile flow behaviour of the die cast AM60 alloy is close to the compressive flow behaviour for simulation of bend-bar specimens. It is intended to extend the work described in this paper in future to anisotropic wrought Mg alloys.

REVIEW OF YIELD CRITERIA AND FLOW LAWS

The physical mechanisms that cause plastic deformation can vary widely. For Mg alloys, plasticity is usually a consequence of dislocation slip and/or twinning. In brittle materials, plasticity may be caused predominantly by slip at microcracks [5]. Automobile parts and other engineering components are in multiaxial stress status when their shapes are complex or their loading conditions are multiaxial. When the uniaxial stress-strain properties determined from standard tensile and compression tests are used to predict the deformation of the components, the yield surface of the material needs to be a function of individual stress components. For cast Mg alloys (approximately isotropic material), the yield surface is usually a convex shape defined by six stress components, namely σ_x , σ_y , σ_z , τ_{xy} , τ_{xz} and τ_{yz} , where σ_x , σ_y , σ_z , τ_{xy} , τ_{xz} , and τ_{yz} are normal stresses and shear stresses in an arbitrary three dimensional Cartesian coordinate system. The yield surface can also be expressed in terms of the three principal stresses σ_1 , σ_2 , and σ_3 . When a combination of stress components lies on the yield surface, the material reaches its yield point and starts to flow plastically. After yielding, Mg alloys become strain-hardened with deformation as most metals and alloys do. As strain hardening occurs, the shape and size of the yield surface changes. The law for describing this change is called flow law. The available yield and flow criteria are reviewed as a basis for the introduction of the Lode angle dependent von Mises yield and flow model.

Yielding Criteria

There are several theories to explain plasticity and to mathematically express the isotropic yield surface [6]. Among these theories, the most widely used ones are the distortion energy theory (i.e. the theory with the von Mises yield criterion, or the von Mises criterion) and the maximum shear stress theory (i.e. the theory with the Tresca yield criterion, or the

Tresca criterion). The distortion energy theory divides the total strain energy into hydrostatic (volumetric) and distortion (shape) energy components. It defines yield as that point when the distortion energy component reaches the yield point of a tensile test.

The yielding criterion in the von Mises yield criterion [7] is expressed in following forms related to the yield strength (σ_Y or τ_Y) of a tensile test:

$$\sqrt{\frac{1}{2}(\sigma_x - \sigma_y)^2 + \frac{1}{2}(\sigma_x - \sigma_z)^2 + \frac{1}{2}(\sigma_y - \sigma_z)^2 + 3(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2)} = \sigma_Y \quad (1)$$

or in a principle stress-based formula:

$$\sqrt{\frac{1}{2}(\sigma_1 - \sigma_2)^2 + \frac{1}{2}(\sigma_1 - \sigma_3)^2 + \frac{1}{2}(\sigma_2 - \sigma_3)^2} = \sigma_Y \quad (2)$$

or in a distortion stress-based formula:

$$\sigma_e^{Mises} = \sqrt{\frac{3}{2}s_{ij}s_{ij}} = \sigma_Y = \sqrt{3}\tau_Y \quad (3)$$

where the deviatoric stress tensor (s_{ij}) is defined by:

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk} \quad (4)$$

where σ_{ij} is the Cauchy stress tensor ($i, j=1, 2, 3$) and $\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33}$.

The Tresca yielding criterion [8] assumes that yield occurs when the maximum shear stress reaches the yield shear strength (τ_Y):

$$\tau_{\max} = \tau_Y = \frac{1}{2}\sigma_Y \quad (5)$$

Yield Surfaces and Flow Laws

For multiaxial stress status, yield condition is usually expressed as a 2-dimensional (2D) or 3-dimensional (3D) surface in the principal stresses space. Generally speaking, the von Mises criterion is the best criterion for materials with $\tau_Y=0.577\sigma_Y$ (Eq. 3) while the Tresca criterion provides a reasonable calculation for material with $\tau_Y=0.5\sigma_Y$ (Eq. 5). In a multiaxial stress state, the von Mises criterion based stress-strain relationship has been well established and can be written in incremental or deformation form. For example, the Prandtl-Reuss incremental form [9]:



$$d\varepsilon_{ij}^p = \frac{3}{2} \frac{d\varepsilon_e^p}{\sigma_e} s_{ij} \quad (6)$$

where ε_{ij} , ε_e and σ_e are the strain tensor, the von Mises equivalent strain and von Mises equivalent stress respectively. The superscript p in the equation means the plastic part of the strain components. The von Mises equivalent strain increment is defined as:

$$d\varepsilon_e^p = \sqrt{\frac{2}{3} d\varepsilon_{ij}^p d\varepsilon_{ij}^p} \quad (7)$$

The deformation form of the von Mises based stress-strain relationship [e.g. 10] is:

$$\frac{\varepsilon_{ij}^p}{\varepsilon_0} = \frac{3}{2} \alpha \left(\frac{\sigma_e}{\sigma_f} \right)^{n-1} \frac{s_{ij}}{\sigma_f} \quad (8)$$

where σ_f and n are the flow stress and strain hardening coefficients, respectively, as determined from uniaxial tensile testing. Based on the above equations, the von Mises yield surface is a smooth elliptical sphere, its effective stress can be simply expressed as the function of six stress components (Eq. 1) and there is a relatively simple relationship between stress and strain in deformation or incremental formats. As a result, the von Mises yield criterion and the related stress-strain relationship with deformational or incremental formats are coded into most commercial finite element packages, such as ABAQUS, ANSYS and ADINA for simulating yield and hardening.

The Tresca yield surface F can only be written as the function of 3 principal stresses:

$$F = [(\sigma_1 - \sigma_2)^2 - \sigma_y^2][(\sigma_1 - \sigma_3)^2 - \sigma_y^2][(\sigma_2 - \sigma_3)^2 - \sigma_y^2] = 0 \quad (9)$$

The solution for principal stresses from six stress components can be solved from the results in [11].

For simulation of isotropic strain hardening, the Tresca equivalent stress-strain based relationship needs to be defined. The Tresca equivalent stress is defined as:

$$\sigma_e^{Tresca} = \max[|\sigma_1 - \sigma_2|, |\sigma_1 - \sigma_3|, |\sigma_2 - \sigma_3|] \quad (10)$$

and the Tresca equivalent strain may be defined by following formula given by Matvienko et al [12]:

$$\varepsilon_e^{Tresca} = \frac{1}{1+\nu} \max[|\varepsilon_1 - \varepsilon_2|, |\varepsilon_1 - \varepsilon_3|, |\varepsilon_2 - \varepsilon_3|] \quad (11)$$

However, the development of the Tresca hardening law is difficult. Although some success has been achieved on developing maximum shearing stress (Tresca)-based flow law in the plane strain condition [14], the full 3D Tresca-based flow law has not been well developed. The Tresca criterion has not been coded in most commercial finite element packages, and this is probably because Tresca stress can only be expressed as the function of the three principal stresses, and shows the discontinuity between the 6 yield surfaces and the complexity in developing the stress-strain relationship in yielding and strain hardening for multiaxial loading.

In the following section, a Lode angle dependent von Mises flow law is proposed for deformation simulation of Mg alloys.

LODE ANGLE DEPENDENT VON MISES FLOW MODEL

As it is difficult to develop deformational or incremental stress-strain relationship based on the Tresca yield law, a combination of the von Mises and Tresca criteria called the Tresca-Mises model [14] has been proposed and used in finite element simulation (e.g. in ABAQUS [10]) for soil. This model allows for the use of shear stress-strain as material property input for soil. The Tresca-Mises surface [14] is written as:

$$F = \sqrt{J_2} - \frac{\tau_u}{\cos\theta} = 0 \quad (12)$$

where J_2 , τ_u and θ are the second stress invariant, undrained shear strength of the soil and the Lode angle,

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \quad (13)$$

and

$$\theta = \frac{1}{3} \cos^{-1} \left(\frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}} \right) \quad (14)$$

and J_3 is the third invariant:

$$J_3 = \frac{1}{3} s_{ij} s_{jk} s_{ki} = \begin{vmatrix} \sigma_x - I_1/3 & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - I_1/3 & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - I_1/3 \end{vmatrix} \quad (15)$$