

A Dual Grid Curved Beam Finite Element Model of Piston Rings for Improved Contact Capabilities

Camille Baelden and Tian Tian
Massachusetts Institute of Technology

ABSTRACT

Piston rings are large contributors to friction losses in internal combustion engines. To achieve higher engine efficiency, low friction ring packs that can maintain good sealing performance must be designed. To support this effort, simulation tools have been developed to model the performance of piston rings during engine operation. However, the challenge of predicting oil consumption, blow by, and ring pack friction with sufficient accuracy remains. This is mostly due to the complexity of this system. Ring dynamics, deformation, interaction with liner and piston, gas and lubricant flow must all be studied together to make relevant predictions. In this paper, a new curved beam finite element model of piston rings is proposed. Ring structural deformation and contact with the liner are treated on two separate grids. A comparison with ring models in the literature and analytical solutions shows that it can provide accurate results efficiently. The proposed model can serve as a numerical framework to couple ring structural deformation, interaction with liner and piston, and predict oil transport. The new model is used to study conformability of piston rings to distorted bore. Results show that the differences between existing conformability criteria can be explained by the singular behavior of the ring gap region.

CITATION: Baelden, C. and Tian, T., "A Dual Grid Curved Beam Finite Element Model of Piston Rings for Improved Contact Capabilities," *SAE Int. J. Engines* 7(1):2014, doi:10.4271/2014-01-1085.

INTRODUCTION

The piston assembly accounts for roughly 50% of the friction losses in an internal combustion engine, 50% of which comes from the ring pack [1]. Significant efficiency gains could be made and this potential motivates engine manufacturers and component suppliers to continue studying the behavior of piston ring packs in order to improve current designs.

Early studies of piston rings were focused on ring mechanics, in particular on the relation between ring free shape and the contact pressure distribution in the cylinder. Prescott [2] developed a formula for the ring free shape geometry in order to achieve a uniform pressure distribution. This formula is still being used in the industry as a baseline for ring free shapes. Swift [3] continued studying the relation between geometry and pressure distribution while experimental validation was later provided by Dragoni and Strozzi [4].

When it became apparent that bore out of roundness, caused by mechanical or thermal deformations, could lead to increased oil consumption and blow by, several studies of ring conformability were conducted. Mueller [5] developed an analytical criterion to calculate the maximum admissible bore distortion for a given ring configuration. A similar analytical approach and a statistical treatment of bore distortion was used by Dunaevsky *et al* to suggest different bore distortion

limits [6,7,8]. Tomanik later proposed a new conformability criterion based on experimental measurements of ring conformability limits [9,10].

Bore conformability criteria provide guidelines to select adequate ring tension and stiffness when the bore distortion is known. It can also be used to quantify how much bore distortion reduction is required to make sure that low tension ring packs can still seal properly. However, it is difficult to directly use conformability calculation results to estimate oil consumption of a given ring-bore configuration. To address this issue, several numerical models of piston rings have been developed. Modeling the ring structure using straight beam finite elements has been a method of choice [11,12,13]. With this technique, ring deformation can be coupled to nonlinear contact with the ring surroundings (piston groove and cylinder liner). Calculations over entire engine cycle are carried out to quantify the oil film thickness left on the liner along the ring path and provide some information on oil consumption.

While straight beam models of piston ring provided a first picture of piston ring cycle performance with consideration of non-axisymmetric effect such as bore distortion, they have a certain number of limitations. Large number of straight beam elements are required to approximate the ring curved geometry and calculate contact with the liner. Thus, computation time can become large, especially when several thousands of

calculation per engine cycles are required. Discontinuity of ring slope at the nodes is a concern as it affects key structural parameters such as curvature and stress. Their ability to model the behavior of ring tips, which are highly sensitive to local bore distortion, is also questioned.

To address these limitations, a curved beam finite element model of piston rings is proposed. The element is derived from elementary theory of beam bending under assumptions relevant to thin curved beams. Modeling the interaction between a ring and a liner or a piston requires an accuracy on the ring displacement and deformation in the order of the cylinder surface roughness (typically $0.1\mu\text{m}$). The curved geometry of this new element is able to provide this high accuracy for a limited number of elements while removing issues of discontinuities at the nodes. A novel method of separation of structural grid and contact mesh is implemented. The contact between ring and liner is handled on a fine grid and element shape functions are used to calculate the contact loads on the structural mesh. The fine grid is able to capture rapid changes of contact conditions around the ring circumference. Considering the ring's stiffness and that ring deformations are never too sharp, a coarse mesh can be used to model the ring structure.

The new piston ring model is validated in three steps:

- A curve representative of a distorted cylinder is interpolated by the curved finite element.
- The deflection of a circular ring under radial loading at its tip is calculated. Results of the new curved finite element are compared with existing piston ring models and analytical solutions.
- The ring - liner contact is linearized to obtain a closed form solution of the conformability problem. This analytical solution is compared to curved beam model results for a bore distortion of mid-size automotive engines.

Behavior of piston ring tips in a distorted bore is investigated using the new curved beam model. The piston ring is rotated in a distorted bore to observe how contact conditions near the ring gap are changing. The results of this analysis are used to discuss the difference between popular conformability criteria and validate the proposed model.

PISTON RING CURVED BEAM FINITE ELEMENT

Piston Ring Mechanics

A summary of the elementary theory of beam bending relevant to piston rings is presented here. It is based on the assumption that beam cross sections remain plane after bending which has been confirmed experimentally to provide accurate description of beam mechanics. Timoshenko offers a more comprehensive treatment of the mechanics of curved bars in his strength of materials text [14]. Piston rings can be considered as thin

curved beams. Their cross sectional dimensions are small compared to the bore radius (typically less than $1/10^{\text{th}}$ of the bore radius). As such, additional assumptions can be made: the distribution of normal stress in the ring section under bending can be considered linear, and the effect of axial and shear forces on the deflection of the ring can be neglected (Euler-Bernoulli beam model).

With these assumptions, piston ring deformations are due to bending only. Under the action of a bending moment the ring curvature is modified. This is illustrated in Figure 1, which represents a segment of ring under the action of a bending moment. M is the bending moment, κ_0 the natural curvature of the ring and κ its curvature after bending.



Figure 1. Ring bending

$$M = EI(\kappa - \kappa_0) \quad (1)$$

As equation (1) indicates, the bending moment is a function of curvature change and ring flexural stiffness " EI ". E is the young's modulus of the ring material (typically cast iron or steel, assumed isotropic) and I is the area moment of inertia of the ring cross section along the cylinder axis. In this analysis, only piston rings with rectangular cross section are considered. The area moment of inertia, I , can thus be calculated analytically.

The free shape curvature of the ring, κ_0 , is designed to provide a desired distribution of contact pressure around the ring circumference when it is inserted in the cylinder. The baseline free shape used in the industry for a uniform contact pressure distribution was derived by Prescott [2]:

$$\kappa_0 = \frac{1}{R} - \frac{F_t R}{EI} (1 - \cos\theta) \quad (2)$$

F_t is the ring tension, R the cylinder radius, and θ the polar angle with $\theta=0$ at the ring gap. In the cylinder, ring tension is balanced by contact pressure generated at the ring-liner interface. A relation between contact pressure and ring bending is found by studying the equilibrium of the ring.

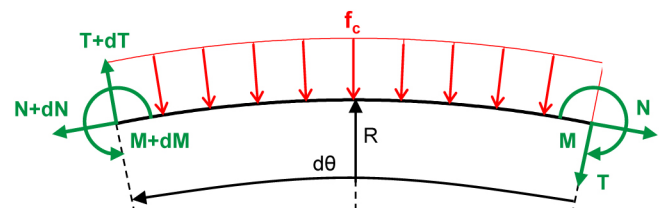


Figure 2. Equilibrium of a ring differential element

By carrying out a force balance on the differential element of ring shown in [Figure 2](#), 3 equilibrium equations can be derived (radial (3), circumferential direction (4), bending around cylinder axis (5)):

$$T' - N - f_c R = 0 \quad (3)$$

$$N' + T = 0 \quad (4)$$

$$\frac{1}{R} M' - T = 0 \quad (5)$$

N , T , and M are the axial, shear force, and bending moment respectively. The action of the cylinder on the piston ring is represented by f_c , the contact force of dimension force per unit length. The prime is used in this paper to represent derivation with respect to θ , the polar angle. It can be seen that because of the ring curvature, axial and shear forces are coupled. When calculating the ring equilibrium in the cylinder, the focus is on contact force distribution and ring deformation. Therefore, the 3 equations shown above are combined to obtain a relation between bending moment and contact force:

$$\frac{1}{R^2} (M + M'') = f_c \quad (6)$$

The above equation can be used to verify that closing a ring whose free shape is given by Prescott's formula (2) in a cylinder of radius R yields a uniform contact pressure. Combining [equation \(1\)](#) and (6) and using $\kappa=1/R$ in [equation \(2\)](#), f_c is found to be constant and equal to F_t/R .

The ring curvature can be calculated from its geometry. With the help of [equation \(1\)](#) and (6), one can relate ring deformation to ring-bore contact force distribution. The curvature of a plane curve in polar coordinates is given by:

$$\kappa(\theta) = \frac{(r^2 + 2r'^2 - rr'')}{(r^2 + r'^2)^{3/2}} \quad (7)$$

θ is the polar angle and r is the radial coordinate of the ring neutral axis. In engine operation, the ring neutral axis deformation is small compared to the cylinder radius and the curvature can be approximated by:

$$\kappa \approx \frac{1}{R} - \frac{1}{R^2} \left(y + \frac{d^2 y}{d\theta^2} \right) \text{ for } \frac{y}{R} \ll 1 \quad (8)$$

Here, y is the ring neutral axis radial displacement from its original position at radius R . The expression of ring curvature and free shape can be substituted in [equation \(6\)](#) to obtain a relation between ring deformation and contact force distribution:

$$y^{(4)} + 2y'' + y = \frac{R^4}{EI} \left(\frac{F_t}{R} - f_c \right) \quad (9)$$

The contact force distribution, f_c , is a function of ring-liner clearance as well as lubricating conditions. Piston rings experience boundary contact near the top and bottom dead centers and transition to mixed or hydrodynamic lubrication regimes depending on engine speed during the rest of the stroke. In this study, Greenwood-Tripp asperity contact model is used to calculate f_c [15]. Though it is representative of a ring in boundary lubrication regime only, it still captures the physics of ring conformability to a distorted bore. Other models taking into account the generation of hydrodynamic pressure at the ring-liner interface are available in the literature [16]. The Greenwood-Tripp asperity contact model is simplified into the following correlation [17]:

$$f_c = \begin{cases} 0 & h/\sigma \geq 4 \\ k_c \left(4 - \frac{h}{\sigma} \right)^z & h/\sigma < 4 \end{cases} \quad (10)$$

$$k_c = Cl_w \frac{E}{(1 - \nu^2)} \quad (11)$$

$$C = 5.279 \times 10^{-9} \text{ and } z = 6.804$$

The ring-liner asperity contact model is stiff. The contact force, f_c , increases from zero a large value within four σ , the standard deviation of the liner roughness. The correlation parameter k_c is a function of E Young's modulus and ν Poisson coefficient of the ring and liner material (assumed the same). The axial width of the ring running face on which contact is occurring is l_w .

The expression of the contact force can now be inserted in [equation \(9\)](#). The result is a differential equation expressed as a function of ring displacement, ring parameters (tension, radius, stiffness) and bore distortion y_b :

$$y^{(4)} + 2y'' + y = \frac{R^4}{EI} \left[\frac{F_t}{R} - k_c \left(4 - \frac{y_b - y}{\sigma} \right)^z \right] \quad (12)$$

The bore distortion, y_b , is the radial displacement between the nominal and deformed bore. It is usually caused by thermal and mechanical deformation of the engine block. The solution to the conformability [equation \(12\)](#) gives the distribution of contact pressure and clearance around the ring circumference, both of which are important when assessing the sealing performance of the ring. Since the ring-liner contact is

nonlinear, closed form solutions to the conformability equation cannot be derived with conventional methods. Therefore a curved beam finite element model is developed to provide numerical results.

Finite Element Development

Due to the complexity of contact interactions between the ring and its surroundings (piston and bore), numerical methods are required to predict piston ring performance. The derivation of a curved beam finite element is presented here. It is developed specifically to couple ring structural deformation and liner contact.

The piston ring is subdivided in finite elements. The free and deformed geometry of a ring element are represented in [Figure 3](#).

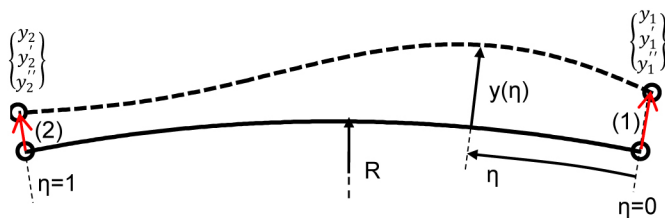


Figure 3. Piston ring finite element

Each node of the ring element has 3 degrees of freedom: the ring radial displacement, its first, and its second derivative. The radial displacement, y , is positive for increased ring radius and its zero corresponds to the ring neutral axis location in a cylinder of radius R . The ring displacement within the element is interpolated from the nodal displacements using polynomial shape functions:

$$y(\eta) = \sum_{k=1}^6 N_k(\eta) u_k \text{ where } \{u\} = \{y_1, y_1', y_1'', y_2, y_2', y_2''\}^T \quad (13)$$

The ring displacement is decomposed in Hermite polynomials, N_k , called the element shape functions [18]. The expression of the 6 element shape functions is given in [appendix](#). The isoparametric coordinate, η , is related to the polar angle, θ , and the ring length, s , in the following way:

$$s = R\theta = R\theta_{elt}\eta = L_{elt}\eta \quad (14)$$

θ_{elt} in the above equation refers to the polar angle between the two nodes of an element and L_{elt} the element length. 5th order polynomial interpolation was chosen over the 3rd order interpolation traditionally used in beam finite elements. As explained in the previous section, ring displacements are due to curvature changes from bending. The ring curvature is a function of the radial displacement second derivative which

becomes continuous when the order of interpolation is increased from 3 to 5. A good compromise between model accuracy and calculation time is thus achieved.

The stiffness and load matrices of the ring element are derived from the principal of virtual work:

$$\delta \Pi^{(e)} = \delta U^{(e)} - \delta W^{(e)} = 0 \quad (15)$$

$\Pi^{(e)}$ is the potential energy of the ring element, $U^{(e)}$ its strain energy, and $W^{(e)}$ the work of external forces. For bending only, the element strain energy can be expressed as:

$$U^{(e)} = \int_0^{L_{elt}} \frac{1}{2} M(\kappa - \kappa_0) ds = L_{elt} \int_0^1 \frac{1}{2} EI (\kappa - \kappa_0)^2 d\eta \quad (16)$$

To simplify derivation of the ring finite element matrices, κ_0 is taken to be $1/R$ in the above equation. In reality, the ring free shape curvature for uniform pressure distribution is given by [equation \(2\)](#). Ring natural geometry is considered circular and free shape is modeled by adding an external force acting on the ring and pushing it against the cylinder. For Prescott's free shape, this force is uniformly distributed around the ring circumference and equal to F_c/R . Other free shapes and pressure distribution can be modeled. Substituting [equation \(8\)](#) and $\kappa_0 = 1/R$ in (16) yields:

$$U^{(e)} = \frac{L_{elt}}{2R^4} \int_0^1 EI \left\{ \sum_{k=1}^6 (N_k + N_k'') u_k \right\}^2 d\eta \quad (17)$$

The above equation is a bilinear form of the ring nodal displacement u_k . It can be expressed in matrix form, with $[K]^{(e)}$ the element stiffness matrix:

$$U^{(e)} = \frac{1}{2} \{u\}^T [K]^{(e)} \{u\} \quad (18)$$

The stiffness matrix is of dimension $[6 \times 6]$ and its term on the i^{th} row and j^{th} column is:

$$K_{ij}^{(e)} = \frac{L_{elt}}{R^4} \int_0^1 EI (N_i + N_i'')(N_j + N_j'') d\eta \quad (19)$$

The ring element stiffness matrix is found by carrying out the integration in the above equation. It is interesting to note that ring with properties varying around the circumference can also be modeled. For instance, a variable I can be substituted in the above integral to calculate the stiffness matrix of ring with non-uniform cross-section. One can use a similar method to account for the draining slots of an oil control ring. The work of external forces on the ring element can now be determined.

$$W^{(e)} = \int_0^{L_{elt}} \mathbf{f}_{ext} \cdot \mathbf{u} \, ds = L_{elt} \int_0^1 f_{ext} \mathcal{V}(\eta) d\eta \quad (20)$$

External forces acting on the ring, $\mathbf{f}_{ext}$, are assumed to act only in the radial direction. In this problem, $\mathbf{f}_{ext}$ has two components: the ring-liner contact force acting inwards and the effect of ring tension, which is modeled as a radial force acting outwards. For a ring of free shape given by [equation \(2\)](#), this force is uniform and equal to F/R . The expression of the ring radial displacement from [equation \(13\)](#) can be substituted in [equation \(20\)](#):

$$W^{(e)} = L_{elt} \int_0^1 f_{ext} \sum_{k=1}^6 N_k(\eta) u_k \, d\eta \quad (21)$$

This can be written as a product of two vectors, with $\{F\}^{(e)}$ the element load vector:

$$W^{(e)} = \{u\}^T \{F\}^{(e)} \quad (22)$$

The i^{th} term of the load vector is thus:

$$F_i^{(e)} = L_{elt} \int_0^1 f_{ext} N_i \, d\eta \quad (23)$$

[Equation \(23\)](#) shows how contact between the ring and the liner can be treated on a different grid than the ring mesh. Nodal forces are calculated by integrating the product of the external forces and shape function over the element. This integration is carried out numerically on a grid fine enough to describe the distribution of contact between the ring and the liner with accuracy (ten to hundred times finer than ring mesh). The size of this contact grid can be chosen independently of the ring mesh. Additionally this reduces the computational cost of refining the contact grid since it is less demanding to increase the size of the integration grid than the size of the ring stiffness matrix.

The finite element form of the equilibrium equations are found by applying the principle of virtual work to the ring element:

$$[K]^{(e)} \{u\}^{(e)} = \{F\}^{(e)} \quad (24)$$

Consequently, the following relation between strain and stiffness matrix, and work and load matrix exists:

$$K_{ij}^{(e)} = \frac{\partial}{\partial u_j} \left(\frac{\partial U^{(e)}}{\partial u_i} \right) \quad (25)$$

$$F_i^{(e)} = \frac{\partial W^{(e)}}{\partial u_i} \quad (26)$$

To model the entire piston ring, ring elements stiffness and load matrices are assembled similarly to other finite element models to form the global stiffness and load matrices.

Note that the contact force in $\mathbf{f}_{ext}$ is a non-linear function of the ring radial position. Ring deformation and contact force are coupled and the ring equilibrium solution is found through Newton-Raphson iterations.

MODEL VALIDATION

In this section, accuracy and validity of this new curved beam model of piston rings are verified. Its ability to interpolate ring geometry, model ring structural behavior and solve conformability problems where ring deformation and ring-bore interaction are coupled are assessed.

Results from the validation cases demonstrate that the proposed curved beam model is able to accurately solve structural deformation of the ring and its interaction with a distorted bore. 20 ring elements seem to offer the best compromise between model accuracy and calculation time. The optimal size of contact grid depends on the distribution of contact around the circumference and how quickly it changes.

Geometry Interpolation

The curved beam model of piston rings must describe the geometry of deformed rings with sufficient accuracy. To validate this, a curve representative of a distorted bore horizontal section is interpolated with the 5th order Hermite polynomial presented earlier. It is assumed that during engine operation ring deformation will be similar to bore distortion. Therefore, sufficient accuracy when interpolating bore distortion should mean sufficient accuracy on ring deformation.

Fourier series are commonly used to describe the distorted bore geometry:

$$y_b(\theta) = A_0 + \sum_{k=1}^n A_k \sin[k(\theta + \phi_k)] \quad (27)$$

In [equation \(27\)](#), θ is the position along the circumference of the cylinder. The radial displacement between nominal and distorted bore is y_b . A_k and ϕ_k are the magnitude and phase of the k^{th} distortion order. 0th order distortion corresponds to a uniform expansion of the bore whereas 1st order distortion displaces the axes of the cylinder without modifying its radius. The shape of the k^{th} order of distortion will contain k lobes equally distributed around the bore circumference.