

A Two Degree of Freedom, Lumped Inertia Model for Automatic Transmission Clutch-to-Clutch Shift Dynamics

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ABSTRACT

This paper presents a methodology to represent automatic transmission clutch-to-clutch shift dynamics with a two degree of freedom, lumped inertia model. The method of reducing the automatic transmission to a lumped, two inertia model as a function of shift and input shaft acceleration is detailed using a full kinematic representation of the automatic transmission. For a given clutch-to-clutch shift maneuver there are two dependent equations that utilize the two lumped inertias and represent the response of the transmission system from input to output shaft. Applicability of the method is shown for planetary automatic and layshaft dual clutch transmissions. Typical clutch-to-clutch shift maneuvers are illustrated with the two inertia model for power on upshifts and downshifts. The simplification of automatic transmission shift dynamics using this method enables rapid synthesis of a given transmission powerflow for estimation of shift quality potential, required clutching torques, inertia phase engine torque intervention and output torque through all phases of a shift event.

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INTRODUCTION

Step gear automatic transmissions have continued to evolve over the previous two decades realizing a rapid increase from 3 and 4 speed units to 5, 6 or 7 speed designs, [1, 2 and 3] and more recently 8 and 9 speeds, for example [4, 5 and 6]. The increase in number of forward gear states is prompted by increasingly stringent greenhouse gas, GHG, emissions regulations regardless of application or market segment and the need to improve driving comfort with nearly transparent shift events. In general, the increase in number of forward gear states is directly associated with the complexity of the powerflow and rotating component inertias. The requirements of increased fuel economy without degradation in shift quality or shift busyness are in direct competition. As a result, development of new automatic transmission powerflows must facilitate analysis of all possible shifts for driving comfort to ensure that the proper powerflow is selected to have a balanced fuel economy and performance quality.

The objective of this paper is to present a methodology that leverages existing kinematic methods for performing dynamic analysis of a step gear automatic transmission shift event and further simplify the system into two lumped inertias and two dependent linear equations for the inertia phase. This simplification greatly reduces complexity of shift dynamic analysis and enables rapid synthesis of a given powerflow for shifting strategy. Additionally, greater insight into a particular automatic transmission powerflows shift quality potential can

be drawn from inspection of lumped inertia values determined by shift event. The simplifications associated with the proposed method facilitate incorporation into analysis tools such as energy analysis or powerflow assessment without much mathematical effort. The details of the method and corresponding equations with example shifts are provided.

Clutch-to-Clutch Shift

Nearly all modern automatic transmissions designs for passenger vehicle applications are clutch-to-clutch, C2C, with coordinated engine torque control. The only exception to this trend is single clutch automated manual transmissions, AMT, used in performance applications or as a cost effective automatic transmission substitute. Shifting comfort is substantially improved through coordinated handoff of offgoing and oncoming clutches within the transmission as well as torque intervention throughout all phases of the ratio change to achieve a desired level of output torque to the drive wheels of the vehicle as described in [7, 8, 9, 10, 11]. A typical power based C2C upshift is shown in Figure 1, indicating what occurs with engine and transmission input speeds, engine torque, offgoing and oncoming clutches torques and the resulting transmission output torque delivered to the vehicle driveline, see [9 or 11]. There are two distinct phases associated with a C2C shift. In the case of the upshift in Figure 1, the first phase is referred to as torque phase, TP, in which torque capacity is handed off from one clutch to another with no change in transmission input speed occurring. Without any change in engine

torque transmission output torque drops inversely proportional with clutch capacities and decreases by the ratio of current to commanded gear ratios. Following the torque phase in [Figure 1](#) is the inertia phase, IP, in which powertrain nodes change speed. In this phase, the oncoming clutch torque capacity and engine torque are coordinated to achieve a desired level of output torque. Ideally, the C2C shift process is coordinated to provide minimal disturbance to the driver and vehicle occupants. For a more in depth discussion on automatic transmission C2C shift mechanics the reader is referred to [7, 8, 9, 10].

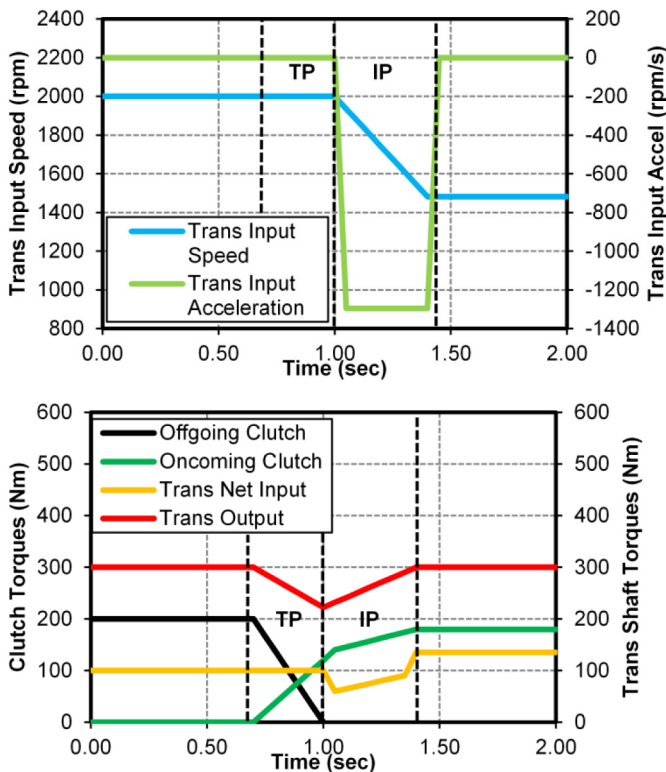


Figure 1. Time traces describing a typical clutch-to-clutch shift in an automatic transmission.

Automatic Transmission Shift Modeling

There exists an extensive amount of published literature regarding techniques to model and analyze C2C shift dynamics for planetary or layshaft automatic transmissions. A few investigations, [12 and 13] for example, have reported methods to reduce the complexity of C2C modeling by reducing the transmission system to a set of equivalent nodal inertias. The objective of this investigation is identical, however, the simplification technique presented still relies on foundation techniques such as the kinematic approach shown in [Figure 2](#) (Lagrangian mechanics), or 1-D lumped parameter models based upon bond graph modeling similar to the example

shown in [Figure 3](#). For detailed descriptions of these techniques, the reader is referred to [7, 8 and 9].

$$I\alpha = \sum T_{external} + \sum T_{constraints}$$

$$\begin{bmatrix} \phi_1 & \phi_2 & \phi_m \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_n \end{bmatrix} = 0$$

The figure shows a stick diagram of a planetary gear set on the right and a set of equations on the left. The stick diagram includes components like input shaft (S1), output shaft (S2), planet carrier (C1), and various gears (R1, R2, R3, C2, C3, C456, C35R, CBR, CB26). The equations relate angular positions (phi) and angular velocities (omega) for different components, with a constraint equation for the planet carrier (C1).

Figure 2. Traditional stick diagram and generalized equations of motion and constraint equations for planetary automatic transmission.

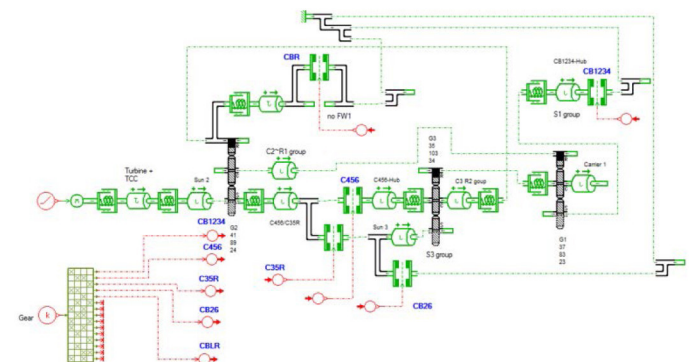


Figure 3. Example lumped parameter planetary automatic transmission model utilizing bond graph solution.

LUMPED, TWO INERTIA SHIFT MODEL

The lumped, two inertia shift model is detailed by describing the free body diagram, lumped inertia determination method and the two dependent, linear equations that govern inertia phase behavior of an upshift or downshift event.

Free Body Diagram

The lumped, two inertia model is based upon the free body diagram sketch shown in [Figure 4](#). At the fundamental level, the transmission is essentially discretized into two lumped inertias, one that can be considered upstream of the shifting clutches, I_3 , and one that can be considered downstream of the shifting clutch, I_4 . The lumped inertia I_3 is imposed upon the oncoming clutch and input shaft as an inertia torque while the lumped inertia I_4 is imposed on the torque acting at the transmission output shaft.

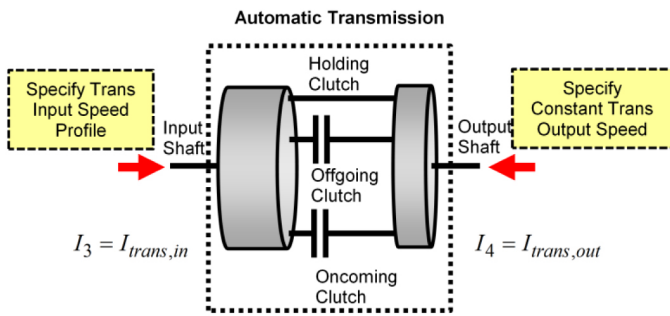


Figure 4. Free body representation of a generic, lumped, two inertia shift model for a C2C automatic transmission powerflow.

Any automatic transmission powerflow can be reduced to the two inertia model represented in Figure 4. However, this method is particularly well suited for planetary automatic transmissions and dual clutch transmissions as will be demonstrated in subsequent sections.

Lumped Inertia Determination

To calculate the lumped inertias of the simplified automatic transmission shown in Figure 4, a detailed automatic transmission model can be setup with whatever method the analyst chooses such as those presented previously. For this investigation a 1-D, lumped parameter model using bond graph methods is used similar to that shown in Figure 3. The model is setup with speed constraints at the input and output shafts and follows a constant output speed upshift profile as indicated in Figure 5. Instead of executing the shift with typical offgoing or oncoming clutch torque capacities, the offgoing clutch is instantly dropped at onset of inertia phase and oncoming is instantly ramped on at the conclusion of inertia phase. During inertia phase there is no torque capacity on the oncoming clutch. During the inertia phase, reaction torques occur at the input, $T_{net,trans,in}$, and output, $T_{net,trans,out}$, shafts as a result of the ratio change event. These torques represent the inertia torque associated with the kinematic behavior of the automatic transmission's powerflow for the particular shift event modeled. This analytical setup can be visualized as an electric dynamometer setup in which the electric motors execute the ratio change with no clutch work during the inertia phase. The torques that would be measured at each electric motor of the dynamometer setup are the inertia torques associated with the transmission specific to the shift being executed.

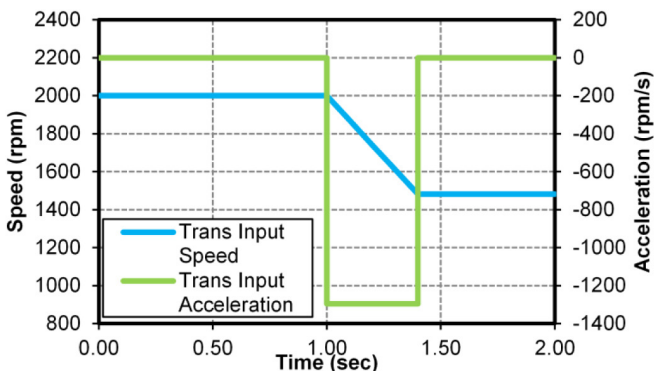


Figure 5.

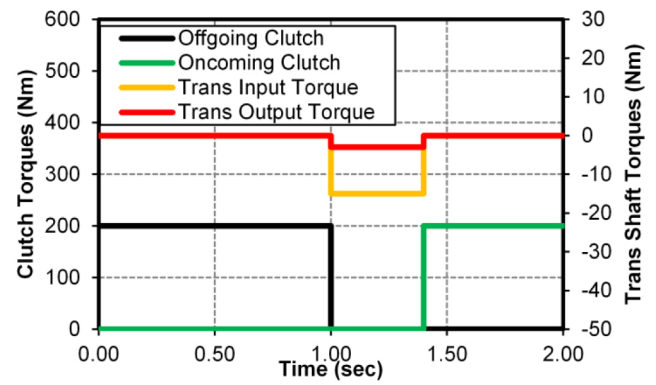


Figure 5. (cont.) Typical transmission signals from a single step upshift for the methodology used to determine lumped inertias for simplified shift dynamics model.

When the inertia torques that result during the inertia phase are normalized by the transmission input shaft acceleration rate, α_{input} , as in Eq (1) and (2), they become reflected inertias of the transmission and are a property of the transmission owing to defined kinematic behavior for a particular shift event. The lumped inertia values determined using Eq (1) and (2) are valid whether considering an upshift or downshift and can be determined for sequential step or skip shifts.

$$I_3 = \frac{T_{net,trans,in}}{\alpha_{input}} \quad (1)$$

$$I_4 = \frac{T_{net,trans,out}}{\alpha_{input}} \quad (2)$$

Typical Lumped Inertia Values

A brief summary of lumped inertias for I_3 and I_4 are provided in this section for comparison purposes for various automatic transmission configurations currently available in passenger vehicle applications.

Planetary 6 Speeds

Table 1 contains lumped transmission input and output inertias, I_3 and I_4 from Eq (1) and Eq (2), respectively, for 6 speed FWD and RWD transmissions by sequential step shifts. The 6 speed FWD design is a three simple planetary gearset, 5 clutch design while the 6 speed RWD design is a Lepelletier powerflow with 5 clutches.

Table 1. Typical 6 speed FWD and RWD lumped inertias for two inertia shift model, inertia units are kg-m².

Shift		6 Spd FWD		6 Spd RWD	
Up	Down	I3 - Input	I4 - Output	I3 - Input	I4 - Output
1-2	2-1	0.0358	0.0786	0.0649	0.0675
2-3	3-2	0.0358	0.0786	0.0649	0.0675
3-4	4-3	0.0358	0.0786	0.0649	0.0675
4-5	5-4	0.1568	0.1115	0.2112	0.1130
5-6	6-5	0.1568	0.1115	0.2112	0.1130
Input Torque Limit		425 Nm		800 Nm	
Transmission Powerflow		3 simple gearsets, 5 clutches		Lepelletier, 5 clutches	

Planetary 8 Speeds

Table 2 contains lumped inertia values determined for two 8 speed RWD planetary automatic transmission configurations. The first configuration is for a powerflow containing one compound gearset and one ravigneaux gearset. The second configuration is based upon a four simple planetary gearset and five clutch powerflow. Although both transmissions have nearly equivalent input torque capacities, the differences in architecture lead to vastly different lumped inertia values for identical shift events. This will be shown to have a profound influence on C2C behavior.

Table 2. Typical 8 speed RWD lumped inertias for two inertia shift model, inertia units are kg-m².

Shift		8 Spd RWD 1		8 Spd RWD 2	
Up	Down	I3 - Input	I4 - Output	I3 - Input	I4 - Output
1-2	2-1	0.0696	0.1527	0.0800	0.0065
2-3	3-2	0.0695	0.1522	0.5364	0.7294
3-4	4-3	0.0695	0.1524	0.5364	0.7294
4-5	5-4	0.0695	0.1524	0.0896	0.0755
5-6	6-5	0.4179	0.2599	0.5134	0.4290
6-7	7-6	0.4179	0.2599	0.5134	0.4290
7-8	8-7	0.4179	0.2599	0.1095	0.0546
Input Torque Limit		800 Nm		900 Nm	
Transmission Powerflow		1 compound and 1 ravigneaux gearsets, 6 clutches		4 simple gearsets, 5 clutches	

Layshaft Dual Clutch

Table 3 contains lumped inertias for a 7 speed FWD dual clutch transmission, DCT, which are noted to be significantly less than those of a planetary design. For this type of automatic transmission, the bulk of the inertia is allocated at the engine and launch device rather than the gear and shafting of the

DCT. Although DCT's will not be a focus for discussion in this paper, the inertias are included for reference and comparison purposes. It is worth noting that the I_4 terms for the DCT are all zero for every shift. This is a result of gear preselection and that the transmission internal rotating parts up to the pre-selected or engaged gear are already synchronized with the speed of the driveline.

Table 3. Example 7 speed FWD dual clutch transmission lumped inertias for two inertia shift model, inertia units are kg-m².

Shift		7 Spd FWD	
Up	Down	I3 - Input	I4 - Output
1-2	2-1	0.0100	0
2-3	3-2	0.0074	0
3-4	4-3	0.0100	0
4-5	5-4	0.0074	0
5-6	6-5	0.0100	0
6-7	7-6	0.0074	0
Input Torque Limit		250 Nm	
Transmission Powerflow		Concentric input shafts, 2 countershafts	

Other Powertrain Inertias

The latter half of this paper will present equations that require inertias of the engine, torque converter or other type of launch device to predict clutch torques, engine torque and transmission output torque during various phases of C2C shifts. Table 4 summarizes some typical ranges of values for these components as reference to the reader.

Table 4. Typical engine and launch device inertias for various engine configurations, inertia values in kg-m².

Component	Engine Cylinder Count		
	4	6	8
Engine + Accessory Drive	0.7 – 0.12	0.10 – 0.14	0.19 – 0.33
Torque Converter Pump	0.04 – 0.08	0.07 – 0.11	0.10 – 0.17
Torque Converter Turbine	0.025 – 0.04	0.03 – 0.065	0.05 – 0.08
DMF Pri&Sec Masses + Press Plate	0.16 – 0.22	0.22 – 0.28	0.24 – 0.30
DMF Clutch Disk	0.004 – 0.008	0.005 – 0.009	0.007 – 0.01

Inertia Phase Equations

The prediction of the clutch torque governing inertia phase input shaft behavior and the corresponding transmission output torque are the desired results of any dynamic shift analysis technique. With the input, I_3 , and output, I_4 , lumped inertias established using the method described in the previous section, a new free body diagram comprehending the entire