

Discrete Flow Mapping - A Mesh Based Simulation Tool for Mid-to-High Frequency Vibro-Acoustic Excitation of Complex Automotive Structures

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ABSTRACT

Modelling the vibro-acoustic properties of mechanical built-up structures is a challenging task, especially in the mid to high frequency regime, even with the computational resources available today. Standard modelling tools for complex vehicle parts include finite and boundary element methods (FEM and BEM), as well as Multi-Body Simulations (MBS). These methods are, however, robust only in the low frequency regime. In particular, FEM is not scalable to higher frequencies due to the prohibitive increase in model size. We have recently developed a new method called Discrete Flow Mapping (DFM), which extends existing high frequency methods, such as Statistical Energy Analysis or the so-called Dynamical Energy Analysis (DEA), to work on meshed structures. It provides for the first time detailed spatial information about the vibrational energy of a whole built-up structure of arbitrary complexity in this frequency range. The response of small-scale features and coupling coefficients between sub-components are obtained through local FEM models integrated in the global DFM treatment. The computational cost of DFM is largely frequency independent making it possible to get results from the mid-to-high frequency regime. This tool will be important when considering the vibrational response of a structure as a whole moving away from modelling vibrations only in sub-parts of the mechanical body.

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INTRODUCTION

A large range of numerical methods are available for solving noise and vibration problems in mechanical structures. Popular approaches include finite and boundary element methods, and multi-body simulations. There are, however, basic limitations when numerically approximating the solutions of wave equations directly: the size of the associated linear system increases with decreasing wavelength and numerical schemes become inefficient when the local wavelengths are orders of magnitude smaller than typical dimensions of the physical system. In addition, high frequency noise and vibration patterns are very sensitive to the structural uncertainties introduced during the manufacturing process. This means that simulations using standard wave equation based methods are only of statistical significance.

One therefore moves to high frequency methods, such as ray tracing or statistical approaches. Both options have disadvantages, however. Tracking rays including multiple reflections on boundaries can become cumbersome, in particular on curved surfaces and including mode conversion at interfaces. As a result, ray tracing has not yet been widely adopted for mechanical engineering problems. Statistical Energy Analysis (SEA) [1] involves dividing a structure into a set of substructures where diffuse wave fields and quasi-equilibrium conditions are postulated. SEA has found widespread applications in the automotive and aviation industry, as well as in architectural acoustics. The underlying assumptions are, however, often hard to verify a-priori or are only justified when an additional averaging over equivalent subsystems is considered. Improved methods addressing some of the shortcomings of SEA include wave intensity analysis (see Langley [2]) and radiosity methods (see Le Bot [3, 4]) amongst others.

A computational tool based on a transfer operator approach for propagating ray densities - *Dynamical Energy Analysis* (DEA) - has been proposed in [5]. DEA includes SEA, wave intensity analysis and radiosity methods as special cases, depending on the form of the numerical approximation. The name points towards the similarities with SEA, but stresses at the same time the importance of non-diffusive transport along the ray dynamics. In particular, in DEA we have much more freedom in sub-structuring the total system and variations of the energy density across sub-structures can be resolved [6]. An efficient implementation of DEA on meshes called *Discrete Flow Mapping* (DFM) has been presented in [7]. DFM makes possible the simulation of high-frequency vibro-acoustics on complex mechanical structures including multi-mode wave propagation and curved surfaces. Coupling at material interfaces is described in terms of reflection/transmission matrices. These matrices are obtained by solving the wave equation locally in the coupling region.

The new method has been tested in collaboration with Germanischer Lloyd (ship building, see [8]) and Jaguar Land Rover. We can in particular show that DFM is capable of modelling the frequency response of complex car-body parts such as car-floor panels or highly structured aluminium castings.

DYNAMICAL ENERGY ANALYSIS

Transport of Wave Energy Densities

In this section we give an overview of the theory of dynamical energy analysis, more detailed descriptions can be found in [5], [9] and [10].

Consider a linear wave problem driven by a distribution of sources at a fixed angular frequency ω . The total system is divided into sub-domains $j = 1, \dots, M$, (such as the elements of an FEM grid), whereby the material parameters and thus the local wave speeds c_j are assumed to be constant in each sub-domain, but may vary between domains. Damping is incorporated through a complex wavenumber with an imaginary part proportional to a damping coefficient μ_j , which may depend on ω . In general, one wants to determine the solution u of a reduced wave equation, (where u is, for example, the displacement within a solid or pressure variations within a fluid), that is,

$$(H + \omega^2)u(r) = f(r). \quad (1)$$

Here, H corresponds to a linear operator describing the vibro-acoustic dynamics including damping and $f(r)$ represents an excitation driving the system. Boundary conditions are

specified at the outer boundaries and the interfaces between sub-domains. The wave energy density ε at a point r is then proportional to the modulus-square of the wave amplitude, that is

$$\varepsilon(r, \omega) \propto |u(r, \omega)|^2. \quad (2)$$

The linear wave equation (1) can in a natural way be associated with a ray dynamics via the Eikonal approximation expressing the wave function in (2) in terms of ray contributions [9]. After averaging over a small frequency band centred at ω , the mean wave energy density is proportional to the density of rays ρ going through a point r , that is,

$$\bar{\varepsilon}(r, \omega) \propto \int \rho(r, p; \omega) dp, \quad (3)$$

where p is the direction (or momentum) vector which is related to the wave number. Rays emerge uniformly from source points and undergo reflection and dissipation depending on the properties of the structure and its (internal) interfaces and outer boundaries. This makes it possible to relate wave energy densities to classical flow equations and thus thermo-dynamical concepts, which are at the heart of SEA or DEA treatments. Note that different mode types such as shear, pressure or bending modes in plates are treated as rays with different local wave speed. Mode coupling at boundaries or interfaces leads to mode-conversion of rays, that is, the classical flow of rays will undergo ray-splitting. The conversion rates between rays corresponding to different modes are related to the modulus square of the entries of an interface scattering matrix. Likewise, the transmission/reflection probabilities between sub-domains are given by the ratios of the outgoing normal power fluxes to the incoming normal power flux at the interface, which can again be obtained from the scattering matrix.

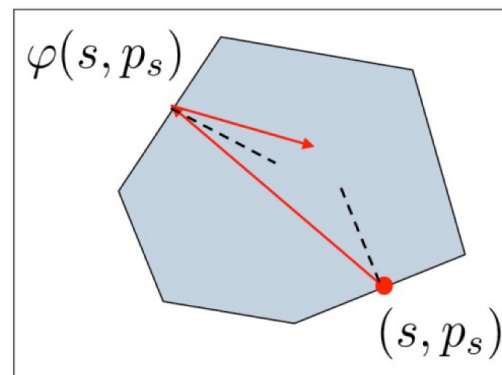


Figure 1. Boundary map in a single subsystem.

The theory underlying DEA is based on the observation that the transport of ray densities, and hence wave energy densities, can be described using transfer operators. In particular, the propagation of these densities along ray trajectories can be written as a boundary integral equation if one uses a boundary map (see Fig. 1) to describe the propagation.

We first consider a problem with a single domain with boundary Γ such as shown in Fig. 1. The density of rays emanating from interior source points is first mapped onto the boundary to give ρ^0 , which generates the same ray field in the interior as the original field after one reflection. Ray densities ρ emanating from the boundary are then transported to the next intersection with the boundary by a transfer operator $\mathbf{B}$,

$$\mathbf{B}[\rho](X^s) = w(X^s) \rho(\varphi^{-1}(X^s)), \quad (4)$$

see Fig. 1. Here $X^s = (s; p_s)$ represents a phase-space coordinate on the boundary, that is s parameterises the boundary Γ and p_s denotes direction or momentum components tangential to Γ at s . Also φ is the boundary map shown in Fig. 1; it takes a ray from a boundary point s with tangential direction p_s along a straight ray to the next intersection with the boundary. The weight function $w(X^s)$ contains the absorption factors. The final density on the boundary (after all reflections) induced by the initial distribution ρ^0 is then obtained using

$$\rho = \sum_{n=1}^{\infty} \mathbf{B}^n[\rho^0] = (\mathbf{I} - \mathbf{B})^{-1}[\rho^0]. \quad (5)$$

The density distribution in the interior region can then be obtained from the boundary density $\rho(X^s)$. One obtains the density (3) after projecting down onto coordinate space. Note that this treatment is formally equivalent to ray tracing.

We now consider multiple subsystems Ω_i , $i = 1, \dots, M$. Coupling between sub-elements is treated as losses in one subsystem and source terms in another. Typical subsystem interfaces are surfaces of reflection/transmission due to sudden changes in material parameters or local boundary conditions. However, as DEA takes into account the full ray dynamics, the final results will not depend critically on the choice of the sub-systems; in fact, we can always split a given sub-system into smaller sub-domains if necessary - a big advantage compared to SEA where the choice of sub-domains is often critical. We describe the full ray dynamics in terms of subsystem boundary operators $\mathbf{B}_{ij}$; flow between Ω_i and Ω_j is possible only if subsystem i and j have a common boundary, see Fig. 2. We replace the weight function w in Eq. (4) by w_{ij} , which contains (in addition to the usual damping term) reflection, transmission and mode conversion coefficients characterising the coupling between subsystems i and j at the interface.

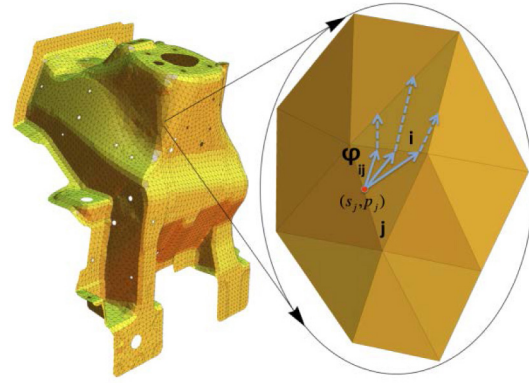


Figure 2. Discrete flow mapping defined on a triangulation of a Range Rover shock tower.

Numerical Approximation Methods

In order to evaluate Eq. (5), a finite dimensional approximation of the operator $\mathbf{B}$ is constructed. Note that in all cases Eq. (5) will be solved in its weak (Galerkin) form since the operator $\mathbf{B}$ is only well defined in a distributional sense. The finite dimensional approximation of Eq. (5) then reduces to a linear system of the form

$$(\mathbf{I} - \mathbf{B})\vec{\rho} = \vec{\rho}^0, \quad (6)$$

where $\mathbf{I}$ and $\mathbf{B}$ are matrix representations of the operators $\mathbf{I}$ and $\mathbf{B}$, respectively, and $\vec{\rho}^0$, $\vec{\rho}$ are the vectors corresponding to the initial and final boundary densities. Depending on the assumptions made on the ray density ρ , the solution of Eq. (6) will be equivalent to several of the high frequency methods discussed in the introduction. The reasons for this and the links to DEA are outlined below.

Statistical Energy Analysis and Wave Intensity Analysis

Let L_i be the length of the boundary of Ω_i and k_i be the local wavenumber. Then the SEA approximation is given by

$$\rho(s, p_s) \approx \sum_{l=1}^M \frac{\rho_l}{\sqrt{2L_l k_l}} \chi_l(s), \quad (7)$$

where χ_l is the indicator function for Ω_i , $s \in [0, L_i]$ and $p_s \in (-k_i, k_i)$. The pre-factor in the denominator is simply a scaling to make the basis approximation orthonormal. The coefficients $\vec{\rho} = \{\rho_l\}$ are the sought unknowns and represent the mean ray density in the sub-system Ω_i . Note that the approximation is independent of the ray direction p_s (i.e. the ray field is assumed to be diffuse) and the solution is assumed to be in local equilibrium in each subsystem. The operators $\mathbf{B}_{ij}$ are each represented by a single number giving the mean subsystem transmission rates, or coupling loss factors. One therefore obtains the SEA prediction after mapping the final boundary