

Accelerated Fatigue and Modal Parameter Identification of Lightweight Structures

Janko Slavic, Martin Cesnik, and Miha Boltezar
University of Ljubljana

ABSTRACT

Car components are exposed to the random/harmonic/impact excitation which can result in component failure due to vibration fatigue. The stress and strain loads do depend on local stress concentration effects and also on the global structural dynamics properties. Standardized fatigue testing is long-lasting, while the dynamic fatigue testing can be much faster; however, the dynamical changes due to fatigue are usually not taken into account and therefore the identified fatigue and structural parameters can be biased. In detail: damage accumulation results in structural changes (stiffness, damping) which are hard to measure in real time; further, structural changes change the dynamics of the loaded system and without taking this changes into account the fatigue load in the stress concentration zone can change significantly (even if the excitation remains the same).

This research presents a new approach for accelerated vibration testing of real structures. The new approach bases on phase locked harmonic excitation and can be used for identification of natural frequencies and damping while the damage due to vibration is being accumulated. At the same time a real-time control loop takes into account the structural changes and adequately changes the excitation to preserve the constant amplitude of the stress at the fatigue zone of the tested structure.

An example of a real aluminum (A-S8U3) part excited close to the 4th natural frequency is presented. Due to high testing frequency the 1×10^6 cycles are reached in approx. 20 minutes. The presented measurements show the change of natural frequency and damping during the test, while the stress amplitude at the fatigue zone is kept constant.

CITATION: Slavic, J., Cesnik, M., and Boltezar, M., "Accelerated Fatigue and Modal Parameter Identification of Lightweight Structures," *SAE Int. J. Mater. Manf.* 8(1):2015, doi:10.4271/2014-01-2095.

1. INTRODUCTION

Vibration testing represents one of necessary steps when assuring the automobile component's durability to the environmental conditions [1]. During the vibration testing the component may show the signs of damage accumulation, resulting in a fatigue failure, excessive wear or instant fracture due to the amplification of the component's dynamic response. Although unwanted during the prototype testing, the fatigue failure due to the structure's dynamic response can provide useful information about the structure's fatigue properties. The most widely used approach, relating fatigue failure to the vibration load is the inverse power law [2,3], that presents a simple exponential relation between the component's fatigue life during service and during vibration test. However, in order to apply the inverse power law to the vibration testing results, the fatigue parameters of the material (fatigue exponent and fatigue strength) must be known beforehand. Consequently, this raises a need of performing an additional set of tests on specific machines for fatigue testing [4,5]. In this study a new experimental methodology is introduced that allows the

assessment of the material's fatigue properties using an electro-dynamic shaker only and is based on the dynamic response of the tested structure. In the presented study the tested structure is excited near it's natural frequency with harmonic signal. Due to the large dynamic response amplification high stresses can be achieved in the stress zone even with low excitation levels of the structure. In this way the stress cycles in the fatigue zone are easily monitored, but their accumulation is reasonably faster compared to the classical fatigue testing, since the frequency of the load cycles coincides with the structure's natural frequency.

The nature of the fatigue crack growth during the fatigue damage accumulation [6] largely influences the dynamic response of the structure, generally lowering the natural frequencies and increasing damping loss factor. The changes of the structure's dynamic properties due to the damage accumulation have been a subject of several research studies [7, 8, 9, 10, 11], where authors predicted the fatigue failure of the structure only by evaluating the changes in measured

dynamic response. Opposed to the usefulness in the fatigue failure prediction, the changes of dynamic response represent an important issue when performing the fatigue test on the electro-dynamic shaker with harmonic excitation, since the stress amplitude deviates significantly if the excitation signal remains constant in amplitude. George *et al.* [12] performed a dynamic fatigue test on an electro-dynamical shaker, where they manually adjusted the excitation frequency when the stress amplitude in the fatigue zone dropped for 5 percent of the nominal value. Additionally, George *et al.* [13] have shown that vibration testing concept can be used also for assessing the fatigue parameters from the prestressed specimens.

This research improves the approach of George *et al.* [12] by testing at near-resonance area with a real-time phase locked control loop, which greatly improves the control of the stress load level at the fatigue zone. Furthermore, with this enhancement it is possible to monitor the instantaneous modal parameters of the structure without any interruptions of the fatigue test and therefore obtaining additional information about the structure's fatigue behavior.

This script is organized as follows. In [Section 2](#) the theoretical background of the base excitation is given together with the study of the proposed Y-shaped specimen's dynamic response. In [Section 3](#) the experimental setup with the corresponding shaker-control loop is presented. In [section 4](#) the experimental results are presented and analyzed to prove the feasibility of the proposed method. Finally, the conclusions of the presented research are given in [Section 5](#).

2. THEORETICAL BACKGROUND

When a dynamic system is excited in the resonant area a response amplification can be observed. By taking advantage of this response amplification it is possible to achieve high stresses in the specimen by applying relatively small excitation forces, which makes an electro-dynamic shaker suitable for performing fatigue tests. The idea of the accelerated vibrational fatigue test is based on the dynamic response of the specimen; therefore, it is necessary to research the dynamic properties of the specimen before the actual fatigue test.

2.1. Specimen's Dynamic Response

The dynamic response of the specimen depends on the modal properties of the specimen; *i.e.*, natural frequencies, mode shapes and damping loss factors. The natural frequencies and mode shapes are easily obtained using the finite-element method. The damping loss factor of the particular mode shape can only be determined experimentally; however, in the case of

lightly damped structures the value of the damping loss factors do not noticeably change the positions of the natural frequencies [14]. For an in-resonance fatigue test the modal properties must comply with three main guidelines [12]:

1. the specimen's natural frequency of interest must be well separated from the remaining natural frequencies and be within the shaker's frequency range,
2. the specimen's mode shape of interest must be excitable with a translational movement in the axis of the shaker,
3. the fatigue zone, where the maximum stresses in the specimen occur, must be located on a surface that is appropriate for strain-gauge measurements.

Regardless of the specimen geometry, the numerical modal analysis should always be performed to confirm that the proposed geometry complies with the stated guidelines. In the case of the biaxial fatigue zone stress state the ratio of the monitored principal stress σ_1 to the equivalent stress σ_{eq} [15] must also be determined.

In this research the Y-shaped specimen shown in [Fig. 1](#) is used. The main three beams are arranged at 120° around the main axis and have a rectangular cross-section of 10×10 mm. The Y-shaped specimen was made from aluminium alloy A-S8U3 [16,17] by casting and with a surface finish performed by milling. The fatigue zone was additionally fine-ground in order to remove any scratches that could cause the premature start of an initial crack. Two additional features are included in the Y-shaped design: steel dead-weights of 52.5 grams and a round hole through the main axis, which are used to adjust the initial natural frequency and to position the fatigue zone on a suitable surface, respectively.

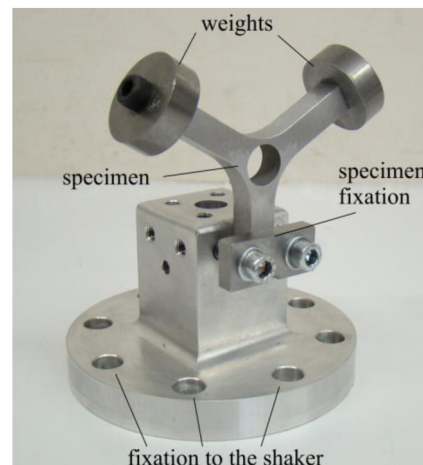


Figure 1. Fixed Y-shaped specimen.

By evaluating the dynamic response of the Y-shaped specimen according to the stated guidelines, the fourth-mode shape at $\omega_1 = 794$ Hz (Fig. 2) was recognized as the most suitable for the near-resonance fatigue test. For the sake of simplicity the fourth natural frequency is denoted as ω_1 rather than ω_4 .

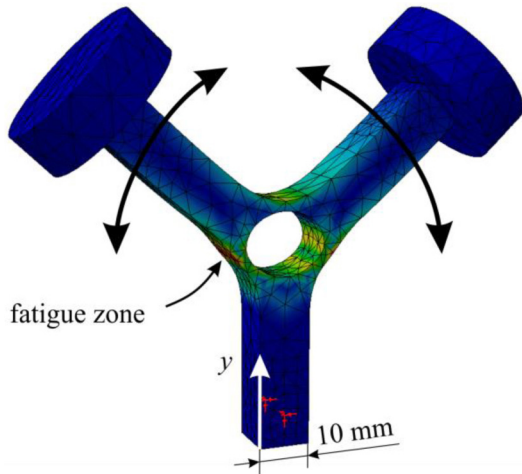


Figure 2. Specimen's fourth mode shape - σ_{eq} .

The stress state in the fatigue zone, marked in Fig. 2, is biaxial. The ratio of the equivalent to the principal stress is $\sigma_{eq} / \sigma_1 = 0.902$, which must be taken into account when constructing the Wöhler curve, since σ_{eq} represents the proper fatigue criterion for biaxial stress states [18].

2.2. Stress Measurement during the Changes of the Modal Properties

During the accelerated fatigue test the changes to the modal parameters of the specimen occur [9, 11], which consequently alter the specimen's response. In order to maintain a constant stress during the accelerated fatigue test it is necessary to design a special control methodology, which is explained in detail in Section 3. However, the constant stress control, further explained in Section 3.2, requires some preliminary theoretical analysis, which follows.

The basic outline of the accelerated vibration fatigue experiment is illustrated in Fig. 3. The base of the Y-shaped specimen is excited with a ground motion of:

$$y(t) = Y e^{i\omega t}, \quad (1)$$

where Y denotes the excitation amplitude and ω the excitation frequency. The specimen's response to harmonic excitation can be defined by the total response displacement $x_1^T(t)$ (see Fig. 3) as:

$$x_1^T(t) = \bar{X}_1^T e^{i\omega t}, \quad (2)$$

where $\bar{X}_1^T = X_1^T e^{i\phi}$ is a phasor, X_1^T denotes the total response displacement amplitude and ϕ is the phase angle.

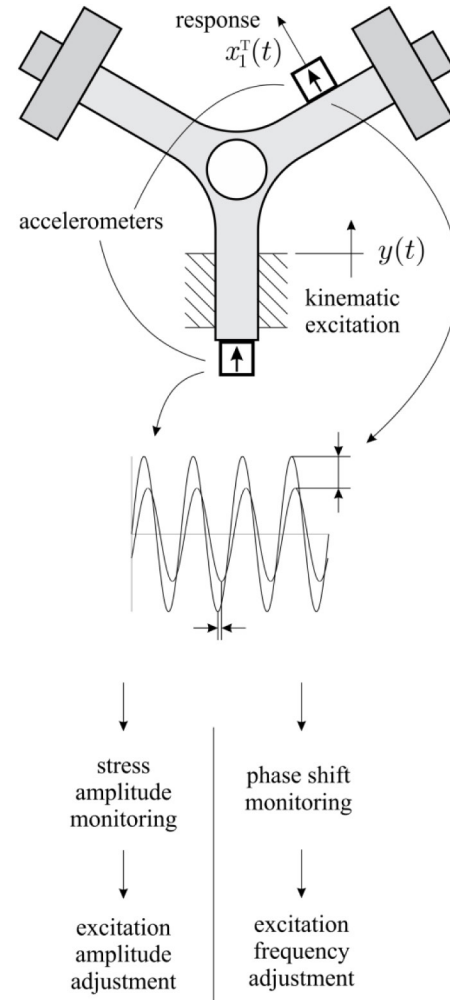


Figure 3. Experimental setup outline

Traditionally, the principal stress σ_1 is measured with a strain gauge. However, while performing the accelerated fatigue test we found that the strain gauge suffered fatigue failure before the failure of the specimen. Furthermore, the installation of the strain sensor is very sensitive to the position on the specimen and is also very time consuming. Consequently, an alternative, indirect method for the stress measurement is proposed in this research.

The proposed indirect stress-measurement method is based on the fundamentals of a modal analysis, described in the following deduction. Continuous structures, such as a Y-shaped specimen, are generally described as MDOF systems; in the case of a hysteretic damping mechanism the equilibrium equation can be written as [19]:

$$\mathbf{M} \ddot{\mathbf{x}}^T + i \mathbf{D} \dot{\mathbf{x}}^T + \mathbf{K} \mathbf{x}^T = \mathbf{f}, \quad (3)$$

where $\mathbf{x}^T$ denotes the total displacement vector, $\mathbf{f}$ is the force vector and $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively.

In the case of a harmonic excitation of an MDOF system at an arbitrary position c the excitation force is written as:

$$f_c(t) = F_c e^{i\omega t}, \quad (4)$$

where $i = \sqrt{-1}$ and F_c is the excitation force amplitude.

The complex response amplitude $\bar{X}_a^T$ for the position a is defined as [19]:

$$\bar{X}_a^T = F_c \alpha_{Xac}(\omega). \quad (5)$$

The receptance $\alpha_{Xac}(\omega)$ is introduced and can be written as:

$$\alpha_{Xac}(\omega) = \sum_{r=1}^N \frac{r A_{Xac}}{\omega_r^2 - \omega^2 + i\eta_r \omega_r^2}, \quad (6)$$

where A_{Xac} represents the mass-normalized modal constant for the positions a and c ; ω_r is the r -th natural frequency and η_r is the hysteretic damping loss factor for the r -th mode shape. These values are known as the modal model, and their derivation is explained in detail in [19]. If a dynamic system is observed when excited near the p -th natural frequency the receptance can be approximated as [19]:

$$\alpha_{Xac}(\omega) \approx \frac{p A_{Xac}}{\omega_p^2 - \omega^2 + i\eta_p \omega_p^2} + B_{Xac}, \quad (7)$$

In Eq. (7) the term B_{Xac} denotes the contribution of the remaining modes.

The equation of motion (3) is applicable when an excitation force $f_c(t)$ is known. In the case of a structure exposed to a kinematic excitation $y(t)$ (1), such as a fixed Y-shaped specimen (Fig. 1), the excitation force cannot be measured directly; however, the excitation motion can be easily obtained with an accelerometer. Therefore, the specific case of an MDOF system with the kinematic excitation $y(t)$ must be analyzed. For kinematic excitation $y(t)$ of the structure, the vector of total displacements $\mathbf{x}^T(t)$ is divided into [20]:

$$\mathbf{x}^T(t) = \mathbf{x}(t) + \mathbf{l} y(t), \quad (8)$$

where $\mathbf{x}(t)$ is a vector of the relative displacements between the masses and the base, and $\mathbf{l}$ is a directional vector of size $[N \times 1]$.

With the base excitation applied in the form of acceleration $\ddot{y}(t)$, the system's inertial forces $\mathbf{f}_I$ can be written as [20]:

$$\mathbf{f}_I = \mathbf{M} \ddot{\mathbf{x}}^T = \mathbf{M} \ddot{\mathbf{x}} + \mathbf{M} \mathbf{l} \ddot{y}(t). \quad (9)$$

Because the system is in dynamic equilibrium for the hysteretic damping we can write:

$$\mathbf{M} \ddot{\mathbf{x}} + i \mathbf{D} \dot{\mathbf{x}} + \mathbf{K} \mathbf{x} = -\mathbf{M} \mathbf{l} \ddot{y}(t). \quad (10)$$

By comparing Eqs. (3) and (10) it is obvious that the base-excited MDOF system is basically a special case of a general MDOF system, where the excitation forces $\mathbf{f}$ directly depend on the mass matrix $\mathbf{M}$ and the base excitation amplitude Y ; the base-excited MDOF system can be considered as an MDOF system excited by an additional inertial force in every degree of freedom.

According to equation of motion (10) the system's relative response $\bar{X}_a$ at the a -th position in the case of the base excitation $y(t)$ is deduced in a similar way to Eq. (6) as:

$$\bar{X}_a = \alpha_{Xa}(\omega) \mathbf{m}_a \mathbf{l} \omega^2 Y = \mathbf{m}_a \mathbf{l} \omega^2 Y \sum_{r=1}^N \frac{r A_{Xa}}{\omega_r^2 - \omega^2 + i\eta_r \omega_r^2}, \quad (11)$$

where $\mathbf{m}_a$ denotes the a -th row in the mass matrix $\mathbf{M}$. Here it should be noted that since the excitation and response are single harmonics with a known frequency ω the relations $\ddot{x}(t) = -\omega^2 \cdot x(t)$ and $\ddot{y}(t) = -\omega^2 \cdot y(t)$ between the measured accelerations and displacements are valid. In the case of an excitation near the p -th natural frequency (Eq. (7)) the ratio $\bar{X}_a/Y$ is:

$$\frac{\bar{X}_a}{Y} = \mathbf{m}_a \mathbf{l} \omega^2 \alpha_{Xa}(\omega) = \mathbf{m}_a \mathbf{l} \omega^2 \left(\frac{p A_{Xa}}{\omega_p^2 - \omega^2 + i\eta_p \omega_p^2} + B_{Xa} \right), \quad (12)$$

If the Y-shaped specimen is observed, a similar deduction to that in Eq. (12) can be made for the frequency response of the measured principal stress $\bar{\sigma}_1$ to the base excitation $y(t)$ as:

$$\frac{\bar{\sigma}_1}{Y} = \mathbf{m}_{\sigma_1} \mathbf{l} \omega^2 \alpha_{\sigma_1}(\omega) = \mathbf{m}_{\sigma_1} \mathbf{l} \omega^2 \left(\frac{p A_{\sigma_1}}{\omega_p^2 - \omega^2 + i\eta_p \omega_p^2} + B_{\sigma_1} \right), \quad (13)$$

where the mass-normalized modal constant and the contribution of the remaining modes are denoted as $p A_{\sigma_1}$ and B_{σ_1} , respectively. $\mathbf{m}_{\sigma_1}$ is a vector of constant values, related to the mass matrix.