

Components Durability, Reliability and Uncertainty Assessments Based on Fatigue Failure Data

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ABSTRACT

Road vibrations cause fatigue failures in vehicle components and systems. Therefore, reliable and accurate damage and life assessment is crucial to the durability and reliability performances of vehicles, especially at early design stages. However, durability and reliability assessment is difficult not only because of the unknown underlying damage mechanisms, such as crack initiation and crack growth, but also due to the large uncertainties introduced by many factors during operation. How to effectively and accurately assess the damage status and quantitatively measure the uncertainties in a damage evolution process is an important but still unsolved task in engineering probabilistic analysis. In this paper, a new procedure is developed to assess the durability and reliability performance, and characterize the uncertainties of damage evolution of components under constant amplitude loadings. The linear and two nonlinear probabilistic damage accumulation models are briefly described first. Then, fatigue failure life data are combined with the damage accumulation rules to derive the probabilistic damage evolution models. Subsequently, a new uncertainty measure recently developed by the authors is introduced. Finally, several examples are provided to demonstrate the effectiveness and robustness of the new damage and life assessment procedure.

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1. INTRODUCTION

The durability and reliability performance of vehicle components and systems, which experience cyclic fatigue loading, thermal loading or other loadings, are of significant interest to product and validation designers [1, 2]. Durability describes the ability of a product to sustain required performance over a period of time or cycles without undesirable failure. Reliability describes the ability of a component or system to function or simply probability of failure under stated conditions. Considerable efforts have been made to quantitatively assess the fatigue life of components and systems with the purpose of accurate damage and life assessment. However, fatigue failure data such as cycles to failure often exhibit big scatter bands. The scatter in test data comes from many sources, such as material uncertainty, loading uncertainty, and the uncertainty of the initial microcrack (damage) distribution. As a consequence, safety factors on predictions of parameters, such as mean curves, obtained from the deterministic life analyses are often required for engineering application purposes [3]. These factors have to be judiciously chosen, based on experience, information of data and spectra, knowledge about the governing equations, statistical

variations, and sequence of fatigue failures [3]. In certain cases, with the introduction of safety factors, conservative, sometimes over-conservative, fatigue predictions can be made [3]. Eventually, more accurate assessment approaches are required for better understanding the failure data patterns and the underlying failure mechanisms as well as providing reliable predictive tools for design and validation engineers.

Since fatigue failure process is intrinsically a probabilistic process, and therefore probabilistic failure and risk analysis is a natural approach for fatigue damage and life assessment. Probabilistic risk assessment (PRA) is such a systematic and comprehensive methodology to evaluate risks associated with complex engineering components or systems [4]. Continuous probability density functions (PDFs) and cumulative distribution functions (CDFs) and other statistical concepts are commonly used for characterizing the scattered test data and eventually used for probabilistic durability and reliability analyses [2, 5]. Fig.1 illustrates, with the help of PDFs, the concepts of risk (unreliability and durability) and the uncertainty in three consecutive program/project/product development and improvement processes [4]. As the state of knowledge and design improve, the risk is lowered and the uncertainty is

reduced as the design phases going from Milestone 1 to Milestone 2 to Milestone 3. Clearly, the risk is represented by the shadowed portion of the distribution on the left tail. Obviously, the bigger the shadowed part in Fig.1, the higher the risk; the narrower the distribution, the lower the uncertainty. Fig.1 also clearly reflects the uncertainty nature of a specific test, in other words, a test is a random process and the tested data could be ended up anywhere in the range, and the accurate location and fate of the tested data are totally unknown a priori. Again, clearly, the bigger the range, the higher the uncertainty.



Fig.1. Decreasing uncertainty and risk over time [4]

How to quantitatively measure the uncertainty of damage and life is an important but still unsolved task in probabilistic risk analysis. Intuitively, the uncertainty parameter in a distribution function should be controlled by the spread of the distribution. Parameters with such capabilities include the standard deviation in the normal distribution and the shape parameter in the Weibull distribution. However, these parameters belong to their respective specific distributions and cannot be used interchangeably. Therefore, a unified parameter that gives a consistent uncertainty measurement regardless of specific distribution functions is needed. In a recent paper [6], inspired by the development in complexity and uncertainty measurements in computational complexity, classical statistical physics and information theory, an uncertainty measure has been proposed.

Overall, the damage distribution, reliability and associated uncertainty at any applied cycles of components are crucial in product design and validation. However, the damage status, damage evolution law, and the statistical properties of damage at any applied cycles are essentially unknown that makes the damage assessment very difficult. Therefore, a new simplified engineering approach must be adopted. In this paper, these hurdles are properly solved by using a reverse engineering approach with the help of the information gained from fatigue failure data and the assumed damage accumulation (evolution) law (linear or nonlinear damage rules). The uncertainty measure is also added to address the uncertainty of damage and life assessment processes for a specific test.

2. A NEW DAMAGE AND LIFE ASSESSMENT PROCEDURE

The new damage and life assessment procedure essentially consists of the following three parts:

1. Collect fatigue life data and obtain their probabilistic distribution;

2. Convert the life distribution into damage distribution at a given cycle with a transformation formula;
3. Conduct durability, reliability and uncertainty analyses and make recommendations based on the observations.

The procedure can be schematically illustrated with the damage-cycle plot as shown in Fig.2(a) and (b), respectively, for linear and nonlinear damage rules. When $N = 0$, the damage $D = D_0$, which is initial damage (could be 0 or any finite values, which are usually much less than unity). $N = N_F$ indicates the failure status of the component with $D = 1$. For the linear damage rule, damage vs cycle plot is simply a cluster of straight lines, while curved lines for nonlinear damage rule.

Clearly, the damage evolution and final failure are totally controlled by the initial condition, applied load and the damage evolution (accumulation) rules. However, the initial condition such as the microcrack size distribution is very difficult to be accurately and completely characterized. Even with some information about the initial condition, it usually cannot generate reliable predictive models for engineering applications because the significant uncertainties involved. By contrast, fatigue life data, especially the failure data distribution resulted from tests with a large sample size (number), are more reliable, direct and convenient for engineering applications. Additionally, the failure data are the final results of the damage evolution of initial microcracks (or defects) with some initial distributions. Theoretically, if the damage accumulation rules are known and the distribution of the failure data are available, the damage, life distribution and uncertainty status at any cycles can be uniquely determined.

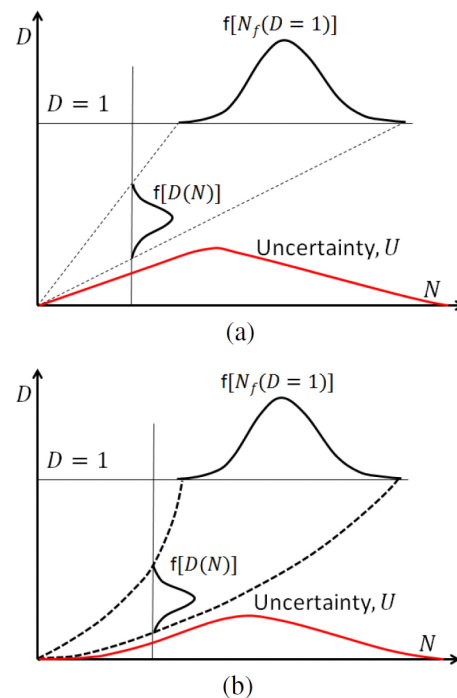


Fig.2. Damage distribution, failure data distribution and uncertainty as functions of cycle or time for (a) linear damage rule, and (b) nonlinear damage rules

Intuitively, the value of the uncertainty measure should show an up-and-down trend because we are certain that at very low cycles the component is most likely in “safe” state, and at very high cycles, the component is most likely in “failure” state. However, somewhere in the middle, the uncertainty is peaked, indicating we are not very certain about the “safe” and “failure” status, and the chances could be close to half-half.

All of the technical issues will be addressed in the following sub-sections (sub-sections 2.1, 2.2 and 2.3).

2.1. Damage Accumulation Rules

For fatigue damage and life assessment and uncertainty characterization of components at any applied cycles, the damage accumulation mechanisms and the mathematical models must be known. It is well known that fatigue damage increases with applied load cycles in a cumulative manner but the ways damage accumulates with the applied cycle and how to describe the damage processes with mathematical modeling are subjects with significant controversies [7]. A key step in analyzing the damage accumulation is to define a damage and find the relationship between the defined damage measure and the applied cycles/cycles to failure, mathematically, $D = f(N/N_F)$. It should be noted that, as compared to other measures, such as crack length and stiffness reduction, it is more natural and convenient to use applied cycles and cycles to failure for conducting fatigue damage and life assessment in engineering applications.

There are a lots of damage accumulation rules available for engineers to choose [7]. Generally, the rules can be divided into two categories: linear and nonlinear. It should be noted that the damage accumulation rules are more commonly used in fatigue life assessment of components/systems under variable amplitude loadings. Here, for simplicity purpose, we use it only for constant amplitude loading, which is a special case of more general variable amplitude loadings.

2.1.1. Linear Damage Rule (LDR)

LDR is the first, simplest, and the most widely used damage model as referred to as Miner's rule (1945) [8]. For variable amplitude loadings, the linear rule can be expressed as $D = \sum_{i=1}^m (N_i/N_{Fi})$. For constant amplitude loading, we have

$$D = N/N_F \quad (1)$$

In Eq.(1), the measure of damage is simply the cycle ratio with basic assumptions of constant damage accumulation per cycle. In other words, the underlying assumption of the LDR is that the damages made for each cycle are the same: $D_i = 1/N_F$, i here represents the i th cycle. It is clear from Eq.(1) that there is no damage at all ($D = 0$) when $N = 0$ and failure is deemed to occur when $D = 1$, or, equivalently, $N = N_F$. Damage vs cycle ratio plot for this rule is simply a straight line as shown in Fig.2(a), and independent of loading levels.

In reality, the LDR with a summation of cycle ratios equal to unity, is not completely accurate. The limitations of the linear damage rule include the effect of loading sequence, the effect of residual stress, effect of fatigue limit, coxing effect, etc. [9]. Various nonlinear rules have been proposed as alternatives to the linear damage rule. However, none overcomes all the deficiencies, and many introduce additional complexities that either preclude or make their use extremely difficult in practice [7]. Therefore, because of its simplicity and its agreement with experimental data for certain cases, as compared with the nonlinear counterparts, the LDR is more often used in design/product validation and other engineering applications [7]. In fact, one of the biggest advantages of the LDR is that no fitting parameter from testing is required, which significantly reduces the damage and life assessment process.

2.1.2. Nonlinear Damage Rule (NDR)

Many nonlinear damage rules or models have been developed to overcome the drawbacks of the linear model [7]. Marco and Starkey[10] proposed the first nonlinear load-dependent damage theory in 1954, represented by a power relationship $D = \sum_{i=1}^m (N_i/N_{Fi})^{\epsilon_i}$, where ϵ_i is a variable quantity related to the i th loading level. Miner's rule is a special case of the model. Similar formula has also been proposed by Manson and Halford [11, 12]. It should be noted that these nonlinear damage curves are related to loading sequence only and, therefore, the models are essentially empirical. A clear physically meaningful definition of damage is needed. Clearly, the damage parameters, such as effective crack length, area or volume, as defined in continuum damage mechanics can provide more physically meaningful damage definitions. Two such damage mechanics based nonlinear models are provided below.

2.1.2.1. Chaboche Type Nonlinear Damage Accumulation Rule

For one-dimensional case, Chaboche et al. [13, 14] postulated that fatigue damage evolution per cycle can be generalized by a function of the load condition and damage state. They formulated several nonlinear damage evolution equations and all these models are based on the effective stress concept in continuum damage models. All of the models have similar characteristics, and the differences are the number of variables assumed in the damage rate equation and boundary conditions. One of these models is listed in Eq.(2)[13].

$$D = 1 - \left[1 - (N/N_F)^{1/(1-\alpha)} \right]^{1/(1+\gamma)} \quad (2)$$

Where γ is a material constant and α is a function of the stress state. N and N_F are number of cycles and the number of cycles to failure. Clearly, $D = 0$ when $N = 0$, and $D = 1$ when $N = N_F$. This damage model is highly nonlinear in damage evolution and is believed to be able to account for the mean stress effect.

2.1.2.2. Paris-Law Type Nonlinear Damage Accumulation Rule

The power-law type nonlinear damage accumulation rule, Eq. (3), can be used as an alternative formula for the relationship between cycles to failure and damage,

$$D = [1 - \nu(N_F - N)]^p \quad (3)$$

Eq. (3) can be justified by examining the fatigue crack growth based on the Paris law [15]. For pure fatigue damage in metallic materials, the Paris-law type crack growth can be expressed as $da/dN = C(\Delta K)^m$, $\Delta K = \Phi \Delta \sigma \sqrt{\pi l}$, where l is crack length, ΔK and $\Delta \sigma$ are stress intensity factor range and stress range, respectively. C , Φ and m are material or geometry related constants. For a given applied constant far field stress range $\Delta \sigma$, if both initial crack length l_0 and crack length at failure l_F are known, we can get the following simple closed-form solution between the crack length and the applied cycles $l^{(1-m/2)} - l_0^{(1-m/2)} = (1-m/2)C\Phi^m(\Delta \sigma)^m \pi^{m/2} N$. In deriving this equation, we assume that $l = l_0$ when $N = 0$; similarly, we have

$l_F^{(1-m/2)} - l_0^{(1-m/2)} = (1-m/2)C\Phi^m(\Delta \sigma)^m \pi^{m/2} N_F$. With some mathematical manipulations and the introduction of the damage parameter $D = l/L$, where L is characteristic length such as the thickness of the specimen, Eq. (3) can be derived. Again, the continuum damage mechanics approach is adopted here. In continuum damage mechanics, damage is usually defined as the crack length/area/volume divided by the virgin length/area/volume. Clearly, from Eq. (3), $D = 1$ when $N = N_F$ and $D = D_0$ when $N = 0$, which is different from the Chaboche-type nonlinear damage models.

A significant drawback of the NDR is that fit parameters can be obtained only from tests and related data analysis, and in most cases, the parameters are usually not immediately available, especially for new materials. Additionally, related testing and parameters determination process are usually time consuming and costly. Therefore, the application of the nonlinear models is limited. Definitely, if the model is properly established and calibrated, the nonlinear models can produce more accurate results and thus has more predictive powers. The nonlinear models are often used in critical applications such as nuclear power plants and aerospace turbines.

2.2. Failure Data and Distribution Transformation

Repeated fatigue failure data with large sample size or number are routinely conducted in practice, and the distribution of the data can be easily obtained by fitting the test data with certain distribution functions, such as lognormal or Weibull distribution functions. Intuitively, with the information about the failure data distribution and an assumed damage accumulation rule, the damage distribution at any applied cycles can be uniquely

determined. Mathematically, the problem is equivalent to seeking a target distribution function $F_Y(y)$ for a given initial distribution function $F_X(x)$ and a transformation function $y = \varphi(x)$. Similar problems have been solved for seeking the possible underlying driving forces of the observed failure data distributions [16, 17]. The procedure [18] is well developed and is briefly described below.

The target distribution function $F_Y(y)$ can be expressed as

$$F_Y(y) = P(Y \leq y) = P[\varphi(X) \leq y] \quad (4)$$

First, consider the case where $y = \varphi(x)$ is a strictly monotone increasing function. $x = \psi(y)$ is then a unique inverse function and

$$F_Y(y) = \int_{-\infty}^{\psi(y)} f_X(x) dx = F_X[\psi(y)] \quad (5)$$

The target PDF, $f_Y(y)$, of Y is obtained as

$$f_Y(y) = \frac{dF_Y(y)}{dy} = f_X[\psi(y)] \frac{d\psi(y)}{dy} \quad (6)$$

For cases where $\varphi(x)$ is a strictly monotone decreasing function,

$$F_Y(y) = \int_{\psi(y)}^{\infty} f_X(x) dx = 1 - F_X[\psi(y)] \quad (7)$$

$$f_Y(y) = \frac{dF_Y(y)}{dy} = -f_X[\psi(y)] \frac{d\psi(y)}{dy} \quad (8)$$

The two cases in Eq. (6) and (8) can be combined as

$$f_Y(y) = \frac{dF_Y(y)}{dy} = f_X[\psi(y)] \left| \frac{d\psi(y)}{dy} \right| \quad (9)$$

Various PDFs/CDFs have been derived for various initial PDFs/CDFs and transformation functions for both single and multiple-modal failure mechanisms and modes [16, 17].

For the damage accumulation rules shown in Eq. (1), Eq. (2), Eq. (3), the analytical expressions for the transformation process can be derived and some of the analytical results are