

Design Analysis for Origami-Based Folded Sheet Metal Parts

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ABSTRACT

The manufacturing of Origami based sheet metal products is a promising technology, mostly in terms of reducing the tooling and process complexity. This procedure can also be called fold forming, as it depends on exclusively shaping the required geometry via sequence of bends. However, the design analysis and modeling of folded sheet metal products are not fully mature, especially in terms of determining the best approach for transferring the analysis from a three-dimensional (3D) to a two-dimensional (2D) context. This manuscript discusses the extension of the Origami technique to the fold forming of sheet metal products represented in modeling approach and design considerations for the topological variations, the geometrical validity, and the variance of stress-based performance. This paper also details the optimization metrics that were developed to reflect the design and manufacturing differences among the possible topological and geometrical options for a single part design. These metrics target five different optimization objectives: material utilization, cost, ease of manufacturability, ease of handling, and mechanical behavior estimation.

A boundary representation is first used to embody the 3D geometry of the sheet metal part. Next, the face adjacency matrix is used to translate the topological information into a generic format valid for numerical analysis. The uniqueness of this approach lies in analyzing the topological aspect of the part's 2D layout by graph modeling and a developed graph traversal algorithm. The analysis then involves a subsequent investigation of the geometrical validity using a proposed scheme for overlapping detection. Finally, the authors estimate the variance in mechanical behavior among all possible 2D layouts for a single part by adopting the structural matrix analysis and propose a modification factor suitable for sheet metal applications.

In this manuscript, three design case studies for automotive components made of Origami based sheet metal products are described: i) a battery pack tunnel for electrical vehicles, ii) a vehicle's interior (dashboard and floor pan), iii) and a shock tower.

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2. INTRODUCTION

The manufacturing of origami-based sheet metal parts, also known as fold forming, is anticipated to overcome some of the challenges in press-based stamping with respect to the ability of the process to reduce the number of components and lead time while utilizing a common platform. The technology, therefore, enables modularity and standardization between different products [1]. Press-based stamping is considered a complex process, however, because of the variety of material types and grades that are used and of the multitude of parameters that must be controlled (e.g. the part shape and the different forming modes, the forming operation in terms of tonnage control, and the uniformity of material properties). Over 40 variables can affect the quality of the stamping [2]. The stamping process includes high tooling cost for die design and validation, which can take nearly a year to develop at a cost of nearly \$5 million in cost per die. Fold forming, however,

is a much less expensive prospect in that no complex dies are required; only minimal tooling is necessary. The fold forming technology can best be implemented by creating a set of material discontinuities along the bend line that facilitate the bending and the shaping of the final geometry. Figure 1 shows different options for material discontinuities along the bend line. Fold forming applies a series of blanking and punching operations to create discontinuities, along the bend line. The 2D flat strip is then designed with multiple folds to generate the intricate 3D shapes. This novel system can lead to lower tooling cost compared to stamping since no dies are heavily involved in shaping. The ability to form most of the final geometry from one single metal strip can also reduce the joining requirements.

Additional advantages for fold forming come from the fact that the discontinuities at the bend lines affect the bending force and remove the limitations on bending radius, which

consequently controls the occurrence of tears and cracks. These material discontinuities can also reduce the punch displacement analysis that is needed in traditional bending. The use of these material discontinuities at the bend line is effective for controlling the bending defects in flanges and for obtaining bends with sharp radii [3].

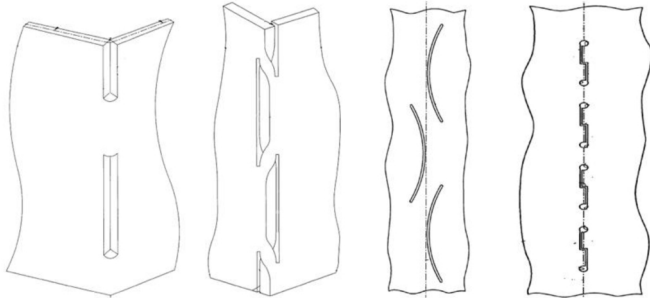


Figure 1. Various possible material discontinuities designs for sheet metal fold forming [4], [5].

Despite this aforementioned discussion of the potential of fold forming technology, a thorough understanding is still needed to create a design and analysis method for sheet metal parts. More specifically, the design and optimization of components from the 3D to the 2D level necessitates structural and scientific techniques to judge the validity of using a 3D geometry in a fold forming process. In the balance of this paper, we describe a possible approach for analyzing and optimizing fold formed shapes for sheet metal applications using computational geometry mathematics and engineered optimization metrics.

3. FLAT PATTERN ANALYSIS

The first step in our proposed Flat Pattern analysis (FPA) for sheet metal folding is to define the adequate representation scheme to extract the sufficient geometrical and topological information of a 3D part, and then manipulate the important information only. Boundary representation (B-rep) [6], is used to translate all needed geometric and topological information, where geometric data communicates the shape of structure, and topological data describes the connectivity between surfaces. Figure 2 illustrates the used B-rep elements.

3.1. Representation of 3-D Structure

In order to apply the FPA for sheet metal parts with a defined thickness, some assumptions are necessary to simplify the investigation. This work adopts the construction principles and the manufacturing constraints proposed in [7]. The construction principles determine the nominal layout by assuming that the flat pattern layouts are a group of facets connected to each other along the bend lines. A facet, which is the major entity in the analysis, is a planar entity bounded by bending lines, at least one of which is straight.

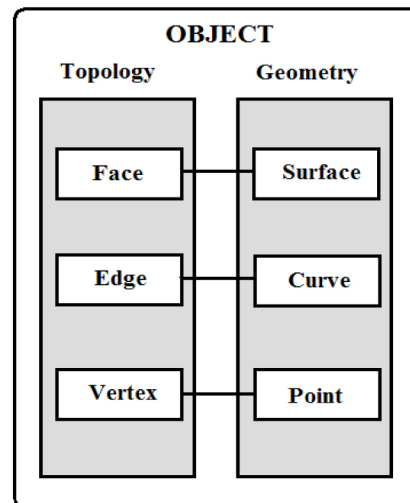


Figure 2. Elements of the B-rep used for FPA

Sharp or zero bending radius tools are used to these facets, which are paper-thin walls during the topological analysis phase. Hence, sheet metal parts are represented schematically without bending allowances. This principle is valid for bending metal sheets along a bend line with material's discontinuities, because no bending punches are required or assumed. Moreover, these construction principles assume that the sheets are manufactured by folding operations only. In terms of the manufacturing constraints, this set of rules describes the transformation of the proposed layouts into an actual 3D part. A manufacturing constraint sets the dimensions of the nominal layout as either inner or outer dimensions. This assigned attribute for the layout can be useful once the final geometry is considered and the material thickness is added.

3.1.1. Used Anatomy

The selected parameters and terms referring to sheet metal parts are shown in Figure 3. The sheet metal part is a collection of surfaces in different planes, or faces, connected to each other by common edges and surrounded by both free and common edges. The point where any two edges meet is a vertex. In order to unfold or unroll all the surfaces of the part and place them on one plane without being stretched or distorted a 2D pattern is generated. However, this unfolding procedure requires some common edges to be broken. These edges are broken ties of faces connections known as seam or weld lines. The free edge refers to any edges that cannot be a bend or a seam line. The different combinations of seam lines produce multiple flat layouts that all correspond to the 3D structure. Two types of opening internal features can be seen in sheet metal parts those are rings and holes, where a ring is an edge loop interior on a face that is disconnected from the face boundaries. A ring is a local feature and independent from the part's topology, which can be discounted, while a hole is an opening that intersects with one bend line or more [8]. The number of holes in a part is the genus of the flat pattern and is denoted by g in Euler-Poincaré's formula for manifold objects and g_{nm} for non-manifold objects. A manifold geometry is the structure that satisfies the following topological conditions: all

edges separate exactly two faces, and all vertices are surrounded by a single loop of faces. The non-manifold shape is the geometry that violates some of the topological conditions.

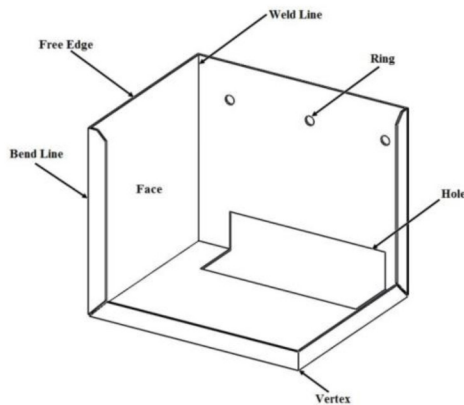


Figure 3. Components of the sheet metal structure

3.1.2. Determining Flat Patterns by Unfolding

The unfolding process of a 3D structure can produce many potential 2D layouts, which all can be successfully folded to the desired shape. To generate these possible 2D designs the topological data of a 3D structure is extracted and represented using a graph model, where each face of a 3D part is symbolized as a node and each edge connecting two faces is denoted as a link. Figure 4 illustrates this representation methodology. In graph theory, the graph $G(V, E)$ is defined as a mathematical structure with two finite sets of vertices (V) and edges (E). The elements in V can be interpreted as nodes and each E has a set of one or two associated vertices associated. The type of the graph associated with unfolding applications is an undirected graph since there is no difference between linking face 1 to face 2 or vice versa. The conceptual interpretation of the set of connected nodes that refers to a 3-D structure results, can be referred to Face Adjacency Graph (FAG), which is numerically represented by adjacency matrix. The rows and columns represent the faces of geometry or nodes in a FAG, the entries of such matrix can be either 0 or 1 such that:

$$A(a,b) = \begin{cases} 1, & \text{if } a \text{ and } b \text{ connected} \\ 0 & , \text{ otherwise} \end{cases} \quad (1)$$

Where $A(a,b)$ is the entry of the adjacency matrix for the row of face a and the column of face b . For purposes of discussion, the adjacency matrix for FAG shown in Figure 6 is shown below:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

3.1.3. Possible Number of Seam/Weld Lines

The number of possible layout arrangements is determined by extracting all spanning trees of the FAG associated with the 3-D structure. Equation (2) represents a formula that is developed based on spatial mechanisms to calculate how many edges in a 3-D geometry will be broken to flatten a thin-walled structure [9]; these edges are known as seam lines. Hence, the equation can be adapted in this work to predict how many welded or joined edges will be in a sheet metal part formed by folding.

$$N_s = N_e - N_f + 1 \quad (2)$$

Where, N_s is the number of seam lines. N_e is the total number of common edges. N_f is the number of faces. Equation (3) indicates that the number of edges that will be subjected to folding, denoted by N_{fold} , equals the number of seam lines subtracted from total number of common edges [10], such that:

$$N_{fold} = N_e - N_s = N_f - 1 \quad (3)$$

This result is consistent with the fact that for each tree of an undirected graph with n vertices, the number of edges equals $n-1$.

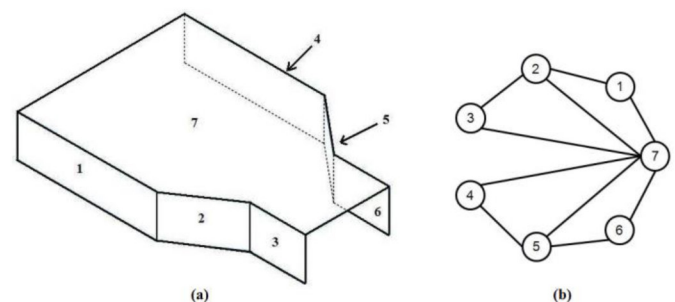


Figure 4. Representation of structure topology: (a) Faces of the 3D geometry; (b) FAG for the 3D structure.

3.1.4. Counting Number of Flat layouts

As previously discussed, the number of flat layouts that exist for a specific 3D structure depends on how many spanning trees can be extracted from the FAG. This can be calculated by applying matrix-tree theorem that involves generating a reduced matrix of the Laplacian matrix. The determinant of the

reduced matrix is the number of possible spanning trees, which alternatively means the number of possible flat layouts in physical world [11].

3.1.5. Flat Pattern Generation

The flat generation process, shown in Figure 5, demonstrates the proposed procedure followed in this work to generate the flat layouts. The overall procedure can be categorized into two phases, topological analysis and geometrical representation.

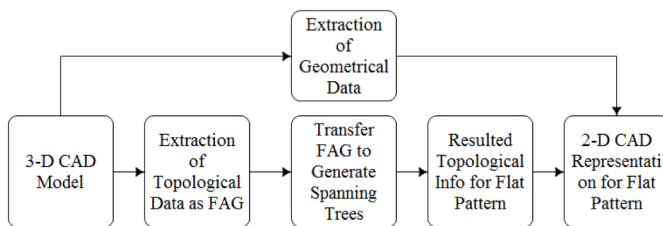


Figure 5. Overall view of the procedure followed for the flat pattern generation

3.1.6. Topological Analysis

The authors used their Flat Pattern Enumeration Algorithm (FPEA), established in this work and integrated within the FPA tool, to list all possible spanning trees. The FPEA is based on investigating all the possible routes that cross through the FAG by permutations. The FPEA structure is illustrated in the pseudo code shown in Figure 7, where the input for the algorithm is the FAG of a 3D structure with N number of nodes and L number of links. The algorithm lists the available links for each node and stores them as sets. The enumeration procedure considers all different combination of links within all sets. While running, the algorithm also detects the physically impossible spanning trees, which actually represent unconnected 3D sheet metal part. More details about the developed enumeration scheme can be found in [11]. For the part in Figure 6, there are 108 possible combinations, whereas only 45 of them represent a connected sub-graph, i.e. a physically valid 3D structure. Examples of the 45 generated spanning trees for open box part are listed below in the matrix format below.

Like any other traversing algorithm, the FPEA serves as a mathematical transformation that maps a possible path, which in this case is the path between the 3D and 2D geometries. Also, there are available graph traversal algorithms, such as the Depth First Search (DFS), the Breadth First Search (BFS), the A* search, Prim's search, and the minimum spanning trees search, that are useful in partially solving the unfolding problem. However, the practical implication for each of those approaches is not the same when used in sheet metal folding application. There are engineering differences in the resulted flat patterns related to the used algorithm, specifically, manufacturability, joining, and cost variations [12]. The developed algorithm is considered efficient for applications with small number of nodes, as with a folded sheet metal application, and can scan all possible spanning trees with

adequate time and memory. The FPEA is also exponentially related to the number of links and nodes, which significantly increases the number of possible combinations as the number of nodes or/and links of a graph increase.

$$\text{Spanning Tree 1} = \begin{bmatrix} 1 & -1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Spanning Tree 28} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

$$\text{Spanning Tree 40} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

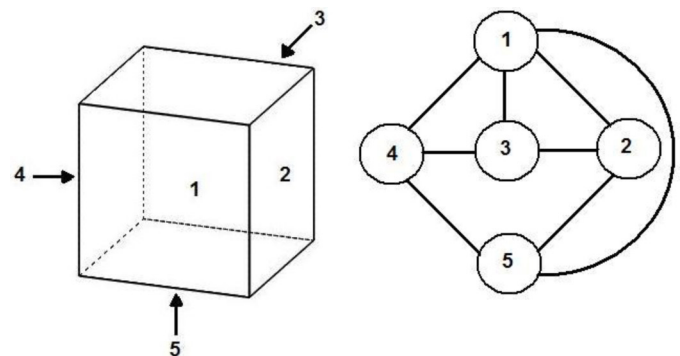


Figure 6. Sheet metal part with an open box geometry and FAG

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Insert graph  $G(V, N)$ 
Collect links for each node
Establish possible links by permutations of all possible sets of links in a spanning tree  $s$ 
Collect all generated spanning trees in set  $S$ 
For each composed set of links check
If  $\forall s \in S$ , composes a connected graph
Then store it in enumerated spanning tree set
Otherwise, delete  $s$ 
  
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Figure 7. Pseudo-code for the developed enumeration algorithm FPEA

3.1.7. Geometrical Representation

The geometrical information that was extracted from the 3D CAD file is used to translate each spanning tree into geometrical flattened patterns, where the results of graph traversal phase only conveys the topological permutations. The flattening process employs the information stored in the bending arrangement, i.e. spanning trees, for projection on the part geometry. This analysis phase detects the geometrical overlapping and excludes any flat pattern that violates the intended 3-D part geometry. Figure 8 illustrates the geometrical analysis phase.