

Analytical Modelling of Diesel Powertrain Fuel System and Consumption Rate

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ABSTRACT

Vehicle analytical models are often favorable due to describing the physical phenomena associated with vehicle operation following from the principles of physics, with explainable mathematical trends and with extendable modeling to other types of vehicle. However, no experimentally validated analytical model has been developed as yet of diesel engine fuel consumption rate. The present paper demonstrates and validates for trucks and light commercial vehicles an analytical model of supercharged diesel engine fuel consumption rate. The study points out with 99.6% coefficient of determination that the average percentage of deviation of the steady speed-based simulated results from the corresponding field data is 3.7% for all Freeway cycles. The paper also shows with 98% coefficient of determination that the average percentage of deviation of the acceleration-based simulated results from the corresponding field data under negative acceleration is 0.12 %. Under positive acceleration, the average percentage of deviation of the acceleration-based simulated results from the corresponding field data is 1.77 % with 93.5% coefficient of determination. The simulated results for all standard cycles show the developed models outperform widely recognized models such as the CMEM and VT-Micro.

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INTRODUCTION

The rate of fuel consumption is one of the significant vehicle operating costs [1, 2, 3, 4]. Modeling has emerged as an effective approach to estimate the rate of vehicle fuel consumption [5, 6, 7, 8]. Modeling the rate of vehicle fuel consumption initially had been based on coarse empirical data to produce fuel consumption models [9]. These models were then superseded by empirical studies that related the rate of vehicle fuel consumption to operating conditions, such as roughness, gradient, and vehicle speed [10].

An et al. [11] developed a modal emissions model, called CMEM, which is based on a simple parameterised empirical approach and consists of six modules that predict engine power, engine speed, air/fuel ratio, fuel use, engine-out

emissions, and catalyst pass fraction [5]. The deviation of the simulated results of the microscopic model CMEM from experimental data is more than 10% [12]. More recently, Rakha and Ahn developed a microscopic vehicle fuel consumption and emission model called VT-Micro that predicts the vehicle instantaneous fuel consumption and emission rates of HC, CO, NOx and CO2 based on instantaneous speed and acceleration levels [13, 14, 15, 5]. The model accounts for the accelerating, idling, cruising and decelerating driving modes [12]. The VT-Micro model has been evaluated against ORNL's field data and its simulated results estimate vehicle fuel consumption to within 2.5% of actual measured field values [12]. Yet, the VT-Micro model needs recalibration with each dataset. In addition, it is based on empirical formulae and thus provides unexplainable mathematical trends.

A more recent approach has been to utilize analytical modeling to estimate the rate of vehicle fuel consumption. In the analytical modeling approach, vehicle fuel consumption is related to the fuel conversion efficiency of the engine and to the forces opposing motion [10]. Analytical models are superior to empirical models in the following senses: (i) an analytical model directly accounts for the individual vehicle characteristics and for the forces acting on the vehicle, (ii) an empirical model is only valid for the set of measurements based on which it was built, (iii) an empirical model is unexplainable and thus can not be extended to other types of vehicles. In spite of the fact that diesel engines, which exhibit advantageous features such as low rate of fuel consumption, are a major source of air pollution no experimentally validated analytical model has been developed as yet of the actual rate of diesel fuel consumption.

The present study presents following from the principles of physics an analytical model of the actual rate of fuel consumption in diesel engines. The study commences with presenting the materials and methods used in this study. The research results and discussion follow. Finally, the research conclusion is demonstrated.

MATERIALS AND METHODS

The study adopts an analytical approach to model the diesel fuel consumption rate. The developed analytical models are simulated using MATLAB and the simulated results are experimentally validated through comparison with corresponding field data. The results of the comparison conducted in the experimental validation are then statistically analyzed. Finally, a sensitivity analysis is conducted on the developed analytical models. This section presents the developed models, experimental validation, statistical analysis, and sensitivity analysis, respectively.

Analytical Model of Diesel Fuel Consumption Rate

The direct injection common rail diesel fuel system that is widely adopted in vehicular diesel technology consists mainly of the following components: (i) fuel tank, (ii) fuel filter, (iii) fuel pump, (iv) pressure sensor, (v) common rail, (vi) governor, (vii) injectors. The fuel mass actual flow rate of this diesel fuel system under both steady speed and acceleration-based operating conditions will be analytically modelled in this section, respectively.

Analytical Modeling of the Steady Speed-Based Fuel Mass Actual Flow Rate in Diesel Engines

In diesel engine testing, the fuel consumption is measured in terms of the fuel mass flow rate. In order to analytically model the fuel mass flow rate in diesel engines, let us apply a control volume to the fuel pump in the diesel fuel system. Let us first recall the definition of the actual fuel-to-air ratio, $R_{F-A \text{ actual}}$ [16] under steady speed condition:

$$\dot{m}_{AFSS} = R_{F-A \text{ actual}} \dot{m}_{air \text{ actual SS}} \quad (1)$$

By combining the definition of equivalence ratio, Φ , with [equation \(1\)](#), it follows that [16]:

$$\dot{m}_{AFSS} = \Phi R_{F-A \text{ stoich}} \dot{m}_{air \text{ actual SS}} \quad (2)$$

In order to analytically formulate the actual mass flow rate of drawn air into the diesel engine under the steady speed condition in (kg/s), $\dot{m}_{air \text{ actual SS}}$, the following follows from the fundamental definition of the mass flow rate and from the fundamental definition of volumetric efficiency [17, 18, 19]:

$$\dot{m}_{All \text{ Cyl}} = \rho_{air} \eta_V \dot{V}_{Cyl \text{ Theoretical}} \quad (3)$$

Thus, by combining [equation \(3\)](#) with the definition of the theoretical air volumetric flow into cylinder, the following follows [17]:

$$\dot{m}_{Cyl} = \frac{\rho_{air} \eta_V V_d N_s n}{N_{Cyl}} \quad (4)$$

Therefore, [equation \(4\)](#) can be rewritten as follows:

$$\dot{m}_{Cyl} = \frac{\rho_{air} \eta_V V_d N_m n}{60 N_{Cyl}} \quad (5)$$

For all the cylinders of the engine it follows from [equation \(5\)](#) that:

$$\dot{m}_{All \text{ Cyl}} = \frac{\rho_{air} \eta_V V_d N_m n}{60} \quad (6)$$

Taking into consideration that in diesel engines the actual mass flow rate of air primarily equals the theoretical mass flow rate of air since what primarily varies with load and accelerator pedal position is the mass flow rate of fuel, it thus follows that:

$$\dot{m}_{AFSS} = \Phi R_{F-A \text{ stoichiometric}} \frac{\rho_{air} \eta_V V_d N_m n}{60} \quad (7)$$

The engine crankshaft rotational speed in (rad/s), w , can be as well analytically formulated as follows [20]:

$$w = \frac{V_S G_t G_d}{r_{wr}} \quad (8)$$

Thus, following from [equation \(8\)](#), the actual mass flow rate of fuel in the diesel engine under the steady speed condition in (kg/s), $\dot{m}_{AFSS}$, indicated in [equation \(7\)](#) can be analytically reformulated as follows:

$$\dot{m}_{fuel \text{ actual}} = \Phi R_{F-A \text{ stoich}} \frac{\rho_{air} \eta_V V_d V_S G_t G_d n}{2\pi r_{wr}} \quad (9)$$

The V_d term in [equation \(9\)](#) can be rewritten in terms of the number of cylinders of the engine as follows:

$$V_d = V_{d-Cyl} N_{Cyl} \quad (10)$$

Substituting [equation \(10\)](#) into [equation \(9\)](#) leads to the final formulation of the actual mass flow rate of fuel in the diesel engine under the steady speed condition in (kg/s), $\dot{m}_{AFSS}$:

$$\dot{m}_{AFSS} = \Phi R_{F-A \text{ stoich}} \frac{\rho_{air} \eta_V V_{d-Cyl} N_{Cyl} V_S G_t G_d n}{2\pi r_{wheel}} \quad (11)$$

The key explanatory variable in [equation \(11\)](#) is V_S .

Analytically modeling the steady speed-based fuel mass actual flow rate in diesel engines paves the way for analytically modeling the acceleration-based fuel mass actual flow rate in diesel powertrain fuel system.

Analytical Modeling of the Acceleration-Based Fuel Mass Actual Flow Rate in Diesel Powertrain Fuel System

As a matter of fact, the speed and acceleration data on the majority of trucks and light duty commercial vehicles are based on steady state conditions and acceleration ranges from -0.3 m/s^2 to 0.3 m/s^2 [12]. Following from the analytical modeling of the $\dot{m}_{AFSS}$, this section thus will present the analytical models of the actual mass flow rate of fuel in diesel engines under negative acceleration ranges from -1.5 m/s^2 up to 0 m/s^2 for both low speed cycles, $\dot{m}_{NLSAF}$, and high speed cycles, $\dot{m}_{NHSAF}$. It will also present the actual mass flow rate of fuel in diesel engines under positive acceleration ranges from 0.1 m/s^2 up to 0.9 m/s^2 for both low speed cycles, $\dot{m}_{P1LSAF}$, and high speed cycles, $\dot{m}_{P1HSAF}$. This section will present as well the actual mass flow rate of fuel in diesel engines under positive acceleration ranges from 1 m/s^2 up to 2 m/s^2 , for both low speed cycles $\dot{m}_{P2LSAF}$, and high speed cycles, $\dot{m}_{P2HSAF}$.

Under the acceleration-based operating condition, the influence of the number of power strokes per cycle becomes more noticeable on the rate of fuel consumption. In addition, it has been recently found that the acceleration-based actual rate of vehicle fuel consumption is exponentially proportional to vehicle acceleration [12]. The exponential function, e^x , can be considered basically dimensionless since the basic representation of e^x in the mathematical expansion of e^x is "1" which is a dimensionless quantity. Therefore, the actual mass flow rate of fuel in diesel engines under negative acceleration ranges from -1.5 m/s^2 up to 0 m/s^2 for low speed cycles, $\dot{m}_{NLSAF}$, and for high speed cycles, $\dot{m}_{NHSAF}$, can be formulated as follows following from [equation \(11\)](#):

$$\dot{m}_{NLSAF} = \Phi R_{F-A \text{ stoich}} \frac{\rho_{air} \eta_V V_{d-Cyl} N_{Cyl} V_S G_t G_d n^{1.19}}{2\pi r_{wheel}} e^{\frac{a_{1S} V_S}{(n)^{1.8}}} \quad (12)$$

The key explanatory variables in [equation \(12\)](#) are V_S and a_{1S} . [Equation \(12\)](#) is dimensionally correct which supports the validity of this developed model. In a similar fashion, $\dot{m}_{NHSAF}$ can be formulated as follows:

$$\dot{m}_{NHSAF} = \Phi R_{F-A \text{ stoich}} \frac{\rho_{air} \eta_V V_{d-Cyl} N_{Cyl} V_S G_t G_d n^{1.19}}{2\pi r_{wheel}} e^{\frac{a_{1S} V_S}{(n)^{1.8}}} \quad (13)$$

The key explanatory variables in [equation \(13\)](#) are V_s and a_{VS} . [Equation \(13\)](#) is dimensionally correct which supports the validity of this developed model.

The actual mass flow rate of fuel in diesel engines under positive acceleration ranges from 0.1 m/s² up to 0.9 m/s² for

low speed cycles, $\dot{m}_{P1LSAF}$, and for high speed cycles,

$\dot{m}_{P1HSAF}$, can also be formulated following from [equation \(11\)](#) as follows:

$$\dot{m}_{P1LSAF} = \Phi R_{F-A \text{ stoich}} \frac{\rho_{air} \eta_V V_{d-Cyl} N_{Cyl} V_S G_t G_d n^{0.78}}{2 \pi r_{wheel}} e^{a_{VS} + 0.25} \quad (14)$$

The key explanatory variables in [equation \(14\)](#) are V_s and a_{VS} . [Equation \(14\)](#) is dimensionally correct which supports the

validity of this developed model. Similarly, $\dot{m}_{P1HSAF}$ can be analytically modeled as follows:

$$\dot{m}_{P1HSAF} = \Phi R_{F-A \text{ stoich}} \frac{\rho_{air} \eta_V V_{d-Cyl} N_{Cyl} V_S G_t G_d n^{0.78}}{2 \pi r_{wheel}} e^{a_{VS}} \quad (15)$$

The key explanatory variables in [equation \(15\)](#) are V_s and a_{VS} . [Equation \(15\)](#) is dimensionally correct which supports the validity of this developed model.

The actual mass flow rate of fuel in diesel engines under positive acceleration ranges from 1 m/s² up to 2 m/s² for low speed cycles, $\dot{m}_{P2LSAF}$, and for high speed cycles, $\dot{m}_{P2HSAF}$, can as well be modeled following from [equation \(11\)](#):

$$\dot{m}_{P2LSAF} = \Phi R_{F-A \text{ stoich}} \frac{\rho_{air} \eta_V V_{d-Cyl} N_{Cyl} V_S G_t G_d n^{0.78}}{2 \pi r_{wheel}} e^{a_{VS}} \quad (16)$$

The key explanatory variables in [equation \(16\)](#) are V_s and a_{VS} . [Equation \(16\)](#) is dimensionally correct which supports the

validity of this developed model. In a similar fashion, $\dot{m}_{P2HSAF}$ can be analytically modeled as follows:

$$\dot{m}_{P2HSAF} = \Phi R_{F-A \text{ stoich}} \frac{\rho_{air} \eta_V V_{d-Cyl} N_{Cyl} V_S G_t G_d n^{0.78}}{2 \pi r_{wheel}} e^{(n)^{0.25} a_{VS}} \quad (17)$$

The key explanatory variables in [equation \(17\)](#) are V_s and a_{VS} . [Equation \(17\)](#) is dimensionally correct which supports the validity of this developed model. Having analytically modelled the fuel mass flow rate in diesel engines, let us now turn hence to explore its experimental validation.

Experimental Validation

The experimental validation in this study compares the simulated results of the analytical model of the steady speed-

based fuel mass actual flow rate $\dot{m}_{AFSS}$ following from [equation \(11\)](#) with the corresponding field data of Oak Ridge National Laboratory (ORNL) and U.S. Environmental Protection Agency (EPA) [12, 21, 22]. It compares as well the simulated results of the analytical models of the acceleration-

based fuel mass actual flow rate $\dot{m}_{NLSAF}$, $\dot{m}_{NHSAF}$, $\dot{m}_{P1LSAF}$,

$\dot{m}_{P1HSAF}$, $\dot{m}_{P2LSAF}$, and $\dot{m}_{P2HSAF}$ following from [equations \(12\), \(13\), \(14\), \(15\), \(16\), and \(17\)](#) respectively with the corresponding field data of ORNL and U.S. EPA [12, 21, 22].

The field data of ORNL and EPA are provided for the average speed of all the Freeway cycles [12, 21, 22]. The data for

making the calculations on the $\dot{m}_{AFSS}$ analytical model results are based on the following data provided on diesel light trucks and light commercial vehicles with four-stroke engines under steady speed condition with Φ of 0.7, ρ_{air} of 1.7 kg/m³, η_V of 0.9, V_d of 0.00033 m³/d-cycle, r_{wr} of 0.323 m, and stoichiometric fuel/air ratio of 0.0696. In addition, the data on G_t and G_d vary with the cycles as follows: (i) G_t of 1.43 and G_d of 2.5 for Freeway LOS G cycle (vehicle average speed 20.96 km/hr), (ii) G_t of 1.43 and G_d of 2.5 for Freeway LOS F cycle (vehicle average speed 29.76 km/hr), (iii) G_t of 1 and G_d of 2.5 for Freeway LOS E cycle (vehicle average speed 48.8 km/hr), (iv) G_t of 1 and G_d of 2.5 for Freeway LOS D cycle (vehicle average speed 84.64 km/hr), (v) G_t of 1 and G_d of 2.5 for Freeway LOS A-C cycle (vehicle average speed 95.52 km/hr), (vi) G_t of 1 and G_d of 2.5 for Freeway High Speed cycle (vehicle average speed 101 km/hr) [12, 21, 22]. The value of the equivalence ratio, Φ , depends primarily on the mode of running the vehicle engine. Vehicle diesel engines normally operate at $\phi \leq 0.80$ [17]. For idling mode, accelerating mode, cruising mode, and decelerating mode the value of Φ is almost 0.55, 0.80, 0.70, and 0.60, respectively [17, 23, 24]. This is in accord with the fact that as the value of Φ decreases, the fuel availability lost in exhaust stroke decreases; thus, engine efficiency increases which significantly interprets why the efficiency of diesel engines is relatively high [17]. The results of the simulated results are statistically analyzed.

Statistical Analysis

The statistical analysis of the experimental validation results consists of a couple of measures: (1) coefficient of determination, r_{COD} , which should be close to '1' [25], (2) relative error of the model, ϵ_R , which should be close to '0%' [25]. The coefficient of determination, r_{COD} , can be evaluated as follows [25]:

$$r_{COD} = \left(\frac{S_{xy}}{S_x S_y} \right)^2 \quad (18)$$

The standard deviation of the dataset of the interval variable x_i that is the expected value analytically, S_x , can be evaluated as follows [25]:

$$S_x = \sqrt{\frac{\sum_{i=1}^z (x_i - \bar{x})^2}{z-1}} \quad (19)$$

Similarly, the standard deviation of the field dataset of the interval variable y_i that is the measured value, S_y , can be evaluated as follows [25]:

$$S_y = \sqrt{\frac{\sum_{i=1}^z (y_i - \bar{y})^2}{z-1}} \quad (20)$$

The sample covariance, S_{xy} , can be evaluated as follows [25]:

$$S_{xy} = \frac{\sum_{i=1}^z (x_i - \bar{x})(y_i - \bar{y})}{z-1} \quad (21)$$

The relative error of the model, ϵ_R , can be evaluated as follows [25]:

$$\epsilon_R = \sum_{i=1}^z \left| \frac{y_i - x_i}{y_i} \right| \frac{100\%}{z} \quad (22)$$

Sensitivity Analysis

This section aims to simplify the analytical models developed in this study on the diesel powertrain for use in a traffic simulation environment. The simplification conducted in this

research is based on a sensitivity analysis on the developed analytical models. The sensitivity analysis relies on applying the sensitivity ratio, $R_{Sensitivity}$, formulation to each of the independent variables. While evaluating $R_{Sensitivity}$ for each independent variable, the values of the remaining independent variables remain unchanged from the baseline values. This $R_{Sensitivity}$ formulation is as follows [26]:

$$R_{Sensitivity} = \frac{\frac{V_{Dependent\ Changes} - V_{Dependent\ Baseline}}{V_{Dependent\ Baseline}}}{\frac{V_{Independent\ Changes} - V_{Independent\ Baseline}}{V_{Independent\ Baseline}}} \quad (23)$$

The actual mass flow rate of fuel in diesel engines under the steady speed condition, $\dot{m}_{AFSS}$, is analytically modeled in equation (5.22). The key explanatory variable in the developed analytical model of $\dot{m}_{AFSS}$ is V_s . The key explanatory variables in the developed analytical models of $\dot{m}_{NLSAF}$, $\dot{m}_{NHSFAF}$, $\dot{m}_{P1LSAF}$, $\dot{m}_{P1HSFAF}$, $\dot{m}_{P2LSAF}$ and $\dot{m}_{P2HSFAF}$ are V_s and a_{VS} . The baseline value of V_s is 13.56 m/s and of a_{VS} is 0 m/s². Having seen the materials and methods used in this study, let us now turn to exploring the simulated results.

RESULTS

Results of the simulation of the developed analytical models are presented in this section for both the steady speed-based fuel mass actual flow rate and the acceleration-based fuel mass actual flow rate in the diesel engines, respectively. This section presents as well the results of the statistical analysis of the simulated results. The discussion of the results is pointed out in the end of this section.

Results of the Simulation of the Developed Analytical Models

This section demonstrates the results of the simulation of the developed analytical models are presented in this section for both the steady speed-based fuel mass actual flow rate and the acceleration-based fuel mass actual flow rate in the diesel engines, respectively.

Results of the Simulation of the Analytical Model of the Steady Speed-Based Fuel Mass Actual Flow Rate in the Diesel Engines

The comparison of the results of the developed analytical model against field data on $\dot{m}_{AFSS}$ is shown in Table 1 based on equation (11).