

Acoustic Simulation of Vehicle Exhaust System using High Order Transfer Matrix Method Coupled with Finite Element Method

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ABSTRACT

This paper addresses the numerical simulation of motorcycle exhaust system noise using a transfer matrix method (TMM) supporting high order analytical acoustic modes representation combined with finite element method (FEM) included in the Actran software, R15. In the state-of-the-art of hybrid TMM/FEM approach the main assumption consists in a 1D plane wave acoustic propagation in the components connections which is intrinsically limiting the maximum frequency of the analysis. In motorcycle exhaust systems this limitation is even stronger because typical geometries exhibit strong curvatures and bends causing the scattering of the acoustic wave into higher order modes. Therefore, results might be erroneous even at frequencies at which only the plane wave is expected to be propagating.

The improved transfer matrix method presented in this paper overcomes this limitation allowing to increase the range of applicability of this method. Specifically, the method is theoretically described and then validated on a set of test cases directly derived from a typical motorcycle exhaust system. Therefore, the method is validated on all the geometrical peculiarities that can be found in motorcycle exhaust networks such as strong bends and bifurcations. Results obtained by the TMM/FEM approach are compared with results provided by pure FEM modeling, in order to demonstrate the accuracy of the hybrid method.

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INTRODUCTION

In the design of motorcycle exhaust system, the acoustic performance is often bounded by several criteria. While the criterion of quietness is usually driving the design of urban two wheelers, louder noise emission could be sought for sporty motorcycles. Therefore, many manufactures look for developing their own exhaust noise signatures related to particular models, taking into consideration the noise regulation imposed by different states as a constraint.

Compared to physical prototyping, numerical simulation offers several advantages for the design of motorcycle exhaust insuring the expected noise level (overall dB level) and quality (frequency content). Design by numerical modeling is generally cheaper than physical testing and allows easy and quick evaluation of acoustic performance by varied design parameters.

Motorcycle exhaust systems consist usually of complex volume components (mufflers, catalyzers ...) connected by simpler ducts. The noise propagation through such systems can be efficiently addressed by means of the Transfer Matrix Method (TMM). The TMM is an analytical approach which allows assessing the acoustic propagation through an entire exhaust

network by combining acoustic transfer matrices of each of its constituent sub-components (i.e., volume components: mufflers, catalyzers, etc.) as done for example by Gerger et.al. [1]. However, TMM methods are based on a set of assumptions which are limiting its usage in real case computation:

1. Only the plane wave is propagating. This assumption leads to a strong limitation in the maximum frequency of analysis which is the cut off frequency of the first higher order mode present in any one of the exhaust line components.
2. Analytical formula used to compute the transfer matrix of each sub component are complex to assess for specific component containing :
 - a. Very complex shape with several internal sub cavities;
 - b. Porous materials;
 - c. Complex background flow (not constant over the sub-component) including temperature gradients;
3. It is not possible to accurately model the "shell noise" (e.g. the structure-borne noise radiated by the vibration of shell structure of different components).

To overcome these limitations and to exploit 3D acoustic modeling in volume components, techniques combining FEM and TMM can be used as done by Bilawchuk and Fyfe [2] where FEM and BEM methods are used to compute the transmission loss of mufflers and FEM is identified as the most suitable method. Other techniques are also possible for modeling 3D complex elements, as suggested by Fairbrother *et. al.* [3]. However, all these types of implementations remove the plane-wave assumption in the volume components only, and are still limited by the same assumption in the connecting pipes. It is worth to mention that, to the author's knowledge, while possible from a theoretical point of view, the Transfer Matrix Method is rarely extended to include higher order modes. Therefore most of the previous works available in literature exploiting such approach still limits the analysis to frequencies where a plane wave is present at connecting pipes or at the inlet and outlet of mufflers.

In this paper, we present an improved method where the plane wave assumption in the connecting pipes is further removed and the FEM/TMM method is therefore compatible with high order duct modes in the connecting pipes. This leads to a method with no intrinsic upper frequency limitation.

Thereafter, the method is validated on a set of test cases directly derived from a typical motorcycle exhaust system. Therefore, all the geometrical peculiarities that can be found in a motorcycle exhaust network, such as strong bends and bifurcations, will be tested. Results obtained by the high order TMM/FEM approach will be compared with results provided by pure FEM modeling in order to demonstrate the accuracy of the hybrid method.

This paper is organized as follows: first the theory of the higher order Transfer Matrix Method is described. Thereafter, the exhaust system used for the validation is described. Specifically, in this section both the reference FEM models and TMM models used for the validation are enlisted. Thereafter, the results of both the methodologies are provided, including the computational statistics in terms of CPU time and RAM requirements. Finally, conclusions about the investigations performed are provided.

HIGH-ORDER TMM METHOD

Let's consider a linear acoustic system made of an assembly of ducts and cavities. This system is split into a set of non-overlapping subsystems P_i . The junction between the subsystem P_i and the subsystem P_k is called the section S_{ik} .

The way the system is divided into subsystems is arbitrary, as long as the sections are planar and normal to the duct walls. The sections can be positioned at weld lines or assemblies, or any convenient position. It is however better to keep the sections relatively small, this will improve the efficiency of the method as shown later. Let's separate the subsystem P_i from its n_i neighbors through n_i sections $S_{ik1}, S_{ik2}, \dots$

In addition to these inner sections, entry/exit sections S_e are also defined and they connect the system to the parts which are not modeled directly (anechoic terminations, engines, ...). Within each of those sections we will define a local coordinate system $\mathbf{e}_x, \mathbf{e}_y$ (perpendicular to the wall) and $\mathbf{e}_z$ (perpendicular to the cross-section, exiting the subsystem) as depicted in Figure 1.

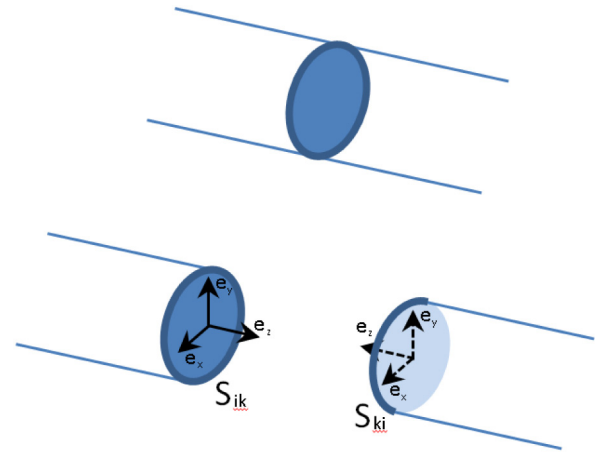


Figure 1. Duct local coordinate system definition at splitting section S_{ik} and S_{ki} .

Within this section, the acoustic pressure fluctuation is decomposed into a superposition of duct modes propagating in $\mathbf{e}_z$ direction (outgoing modes) and $-\mathbf{e}_z$ direction (incoming modes), using the classical variable separation of the Helmholtz equation [4]:

$$p_{ik} = e^{j\omega t} \sum_{m=1}^{g_{ik}} \varphi_m^{ik}(x, y) \left(A_m^{ik+} e^{jk_m^{ik+} z} + A_m^{ik-} e^{-jk_m^{ik-} z} \right) \quad (1)$$

It is worth to note that the incoming and outgoing wave numbers k_m^{ik+} and k_m^{ik-} are equal, except if a mean flow is present.

The g_{ik} incoming modes of amplitude A_m^{ik+} and wavenumber k_m^{ik+} can be considered as the excitation, while the outgoing modes of amplitude A_m^{ik-} and wavenumber k_m^{ik-} are the subsystem response.

Using those amplitudes as main Degree Of Freedom (DOF), one can describe the acoustic behavior of each section by means of $2g_{ik}$ DOFs.

Since the subsystem P_i is linear, the outgoing modes amplitudes are linked to the incoming mode amplitudes through the relation

$$\begin{bmatrix} A_m^{ik_1-} \\ A_m^{ik_2-} \\ \vdots \\ A_m^{ik_n-} \end{bmatrix} = T^i \begin{bmatrix} A_m^{ik_1+} \\ A_m^{ik_2+} \\ \vdots \\ A_m^{ik_n+} \end{bmatrix} + b^i, \quad (2)$$

Where T^i is the transfer matrix of the subsystem i , and the vector b^i is equal to zero if the subsystem i is not active, i.e. it does not generate acoustic energy. The transfer matrix of each component adds g_{ik} constraints per section S_{ik} , and g_e constraints per section S_e .

To enforce continuity of the acoustic pressure and velocities at the junction between two subsystems, the amplitudes of the outgoing duct modes of section S_{ik} must be equal to the amplitude of the incoming duct modes of section S_{ki} ,

$$A_m^{ik-} = A_m^{ki+}, \quad (3)$$

adding again g_{ik} constraints per section S_{ik} . In order to close the system, only additional g_e constraints per entry/exit section S_e are needed. Those are introduced using the general expression below:

$$A_m^{e+} = R A_m^{e-} + \bar{A}_m, \quad (4)$$

with R a complex reflection coefficient assumed constant for all modes (and equal to zero for anechoic condition), and $\bar{A}_m$ incoming modal amplitudes representing the engine excitation, for example.

The modal amplitudes are chosen either to ensure an equal distribution of acoustic power for all cut-on modes, or set to zero except for the plane mode.

The system of equation obtained is sparse and of relatively small size (the modal amplitudes), it can be solved by classical full or sparse direct solvers.

Choice of Number of Modes per Section

Since the duct modes are a full basis, it is possible to show that the method converge to the unsplit solution when g_{ik} increases. In practice, it is sufficient to keep only the propagating modes (k_m^{ik} real, cut-on modes) and the first evanescent modes (k_m^{ik} imaginary, cut-off modes) because the strongly evanescent modes decrease quickly enough in the direction perpendicular to the section so that they do not affect the global solution.

Evaluation of the Transfer Matrix T^i

The transfer matrix of complex subsystems can be evaluated by a FEM code supporting duct mode excitations and in this paper this is done by means of the Actran software, R15 [5].

However, for very simple subsystems, it is possible to derive the transfer matrix analytically. In this work, this is done for straight ducts, that simply introduce a phase shift between their incoming and outgoing modes:

$$\begin{bmatrix} A_m^{ik_1-} \\ A_m^{ik_2-} \end{bmatrix} = \begin{bmatrix} 0 & e^{jk_m^{ik_2+} L_i} \\ e^{jk_m^{ik_1+} L_i} & 0 \end{bmatrix} \begin{bmatrix} A_m^{ik_1+} \\ A_m^{ik_2+} \end{bmatrix}, \quad (5)$$

with L_i the length of the subsystem i . Note that contrary to plane mode, it is crucial to keep the axes $\mathbf{e}_x$, $\mathbf{e}_y$ parallel between the two end sections of the duct subsystem, to avoid phase errors.

Finally, in this paper also curved ducts are evaluated by means of an analytical representation by using an equivalent mean duct length. In this case strict parallelism of local coordinate systems is not possible, but they are chosen as parallel as possible.

EXHAUST SYSTEM MODEL

The main objective of this paper is to validate the Transfer Matrix Method for higher order modes previously described. For this purpose, an exhaust duct with a geometric configuration typically used in motorcycles is considered and it is depicted in Figure 2. The selected geometry features four ducts with an initial strong curvature and merging together in pairs into two straight ducts thanks to two bifurcations. The latter two straight ducts further merge into a single duct by means of another bifurcation. Finally, the model terminates with a muffler which radiates the noise in the far field.

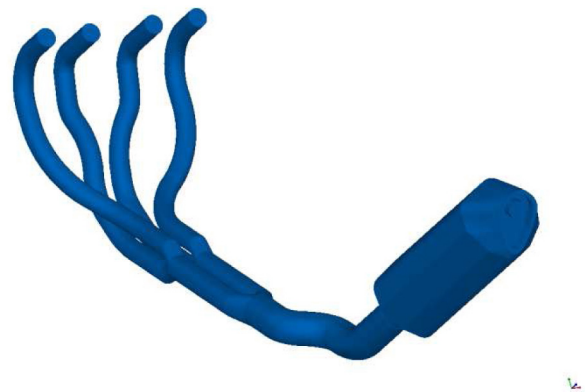


Figure 2. Exhaust model.