

A Scale Adaptive Filtering Technique for Turbulence Modeling of Unsteady Flows in IC Engines

Federico Piscaglia, Andrea Montorfano, and Angelo Onorati
Politecnico di Milano

ABSTRACT

Swirling flows are very dominant in applied technical problems, especially in IC engines, and their prediction requires rather sophisticated modeling. An adaptive low-pass filtering procedure for the modeled turbulent length and time scales is derived and applied to Menter's original $k - \omega$ SST turbulence model. The modeled length and time scales are compared to what can potentially be resolved by the computational grid and time step. If the modeled scales are larger than the resolvable scales, the resolvable scales will replace the modeled scales in the formulation of the eddy viscosity; therefore, the filtering technique helps the turbulence model to adapt in accordance with the mesh resolution and the scales to capture. The novel turbulence model presented in this work will be called *Dynamic Length Scale Resolution Model* (DLRM), because of its capability to dynamically adapt its behavior according to the grid resolution and to consequently switch from modeling to resolving the turbulent length scales. Validation has been carried out both on a strongly swirling flow through a sudden expansion and on a simple IC engine geometry with one axial central valve; the model seems able to capture unsteady effects and to produce accurate time-averaged results (especially if compared to its standard RANS formulation) and looks particularly suitable when used with grids where turbulence would not be sufficiently resolved for an accurate LES.

CITATION: Piscaglia, F., Montorfano, A., and Onorati, A., "A Scale Adaptive Filtering Technique for Turbulence Modeling of Unsteady Flows in IC Engines," *SAE Int. J. Engines* 8(2):2015, doi:10.4271/2015-01-0395.

INTRODUCTION

There are many interesting strategies for unsteady turbulence modeling: Spalart [1] gave an overview and discussion about the advantages and limitations of many of these when applied to general problems. For industrial simulations of IC engines, unsteady RANS (Reynolds averaged Navier-Stokes) equations are established as a standard tool: the complete turbulence behavior is enclosed within appropriate turbulence model which takes into account all turbulence scales, from the largest eddies to the Kolmogorov scale. Turbulent length and time scales are estimated by dimensional considerations, and model transport equations are solved on reasonably coarse grids, that make this approach relatively cheap in terms of computational cost. Despite RANS is able to give a reasonable approximation of the wall shear stress, it is quite well known that the excessive predicted viscous behavior very often damps out the unsteady motion and the flow unsteadiness, because it overestimates the modeled turbulent length and time scales. Also, despite the uniqueness of a solution to the three-dimensional Navier-Stokes equations has yet to be mathematically proven [2], it is generally accepted that the solution is completely determined by the initial and boundary conditions: hence, unsteady RANS cannot account for randomness or independent events in the flow and it is characterized by simulation results that are perfectly repeatable, if constant boundary conditions (and the same computer) are used for the unsteady computation. In Large-Eddy Simulation (LES), the turbulent length scale is related to the computational grid and to the turbulent time scale from the resolved flow without additional equations; as a result, the approach is

potentially more accurate and it is able to provide the intrinsic unsteady character of the flow. For this reason, Large-Eddy Simulation has gained popularity and success; on the other hand, as the Reynolds numbers of most engineering flows are usually very large, LES of wall bounded flows very seldom represents an option for full scale industrial simulations, because the computational cost of LES scales near the walls is similar to Direct Numerical Simulation (DNS). In many LES conducted for wall- bounded flows, wall modeling based on logarithmic profile or other theoretical models are used to avoid the high resolution near the walls, so that the computational cost of LES for modeling the free-shear flows is only weakly dependent on the Reynolds number ($\sim Re^{0.4}$). In complex flows, such as in internal combustion engines, the validity of these theoretical models for LES at the walls is questionable and most of the times it does not improve the quality of the results. A very detailed overview about the use, the advantages and the limits of LES when applied to IC engines is discussed in [3]. Since the required grid resolution needs to be extremely high, LES simulation may become very expensive. Hybrid models are all based on the same idea to represent a link between RANS and LES, since they use the best of both worlds: they resolve the turbulence where possible and they model it elsewhere. As a consequence, they try to keep computational efficiency of RANS and the potential of LES to resolve large turbulent structures, even on coarser grids and with high Reynolds number. In the recent years, several hybrid LES/RANS methods have been proposed in the literature: examples are Detached Eddy Simulation (DES), Limited Eddy Simulation (LESs), hybrid

LES-RANS, or second generation unsteady RANS models, including partially averaged NavierStokes (PANS) and scale-adaptive simulation (SAS). A recent application of PANS to IC engine simulation has been shown in [4]. There is obviously the need in IC engines simulation for a turbulence model that can distinguish between what can be resolved and what cannot, at the same time as it produces an accurate estimate of the wall shear stress. Very Large Eddy Simulation (VLES) is another kind of hybrid method which starts to expand as a promising compromise for simulation of industrial flow problems with reasonable computational time and costs. In VLES large turbulence structures are resolved by an unsteady simulation and the minor structures are modeled with an adequate turbulence model. Compared to the LES and similarly to the other hybrid models, the main difference is that with VLES a smaller part of the turbulence spectrum is resolved and the influence of a larger part of the spectrum has to be expressed with the model. In VLES models, the additional requirement is an appropriate filtering technique which distinguishes between resolved and modeled part of the turbulence spectrum. The filtering procedure provides the adaptive characteristic of the VLES models, enabling them to be applied for the whole range of turbulence modeling approaches from the RANS to the DNS; also, it limits the influence of the statistical turbulence model on the unsteady mean flow field.

A VLES model, sometimes also called Variable Resolution (VR) approach, to predict stochastic flows in internal combustion engines is presented. The filtering approach used may be considered converse to the filtering approach that is used in LES. Instead of solving the filtered equations to avoid the computation of small scales, the modeled length and time scales are filtered in order to suppress their negative influence on the unsteady flow field. This approach has been originally developed by Willems [5] and is similar to other approach of Speziale [6, 7] and Fasel [8], because the filter is applied directly to the Reynolds stress tensor and the turbulence model is left unchanged. Similarly to [9], the functional form of the filter is derived from the relation between filtered and non-filtered time scales. In this work, a novel formulation of the functional form of the filter function, including time resolution in the criteria to maintain phase coherence, is proposed and it is applied to compressible flows. The modeled length and time scales are compared with what it can be resolved by the computational grid and time step. If the modeled scales are larger than the resolvable scales, resolvable scales will replace the modeled scales in the formulation of the eddy viscosity. To distinguish between large- and small-scale turbulence, the upper limit of the length scales of non-resolved turbulence is made proportional to the local grid spacing or the product of the local velocity magnitude and the time step of the simulation; there is no lower limit because the mean non-resolved turbulent length scale may be much smaller than the local grid spacing, especially near the walls. This allows for a much coarser grid resolution than in LES; also, the filter will allow large-scale unsteady structures in the flow, and the model will still produce a wall shear stress comparable to what is produced by the standard RANS model. The filtering technique has been applied to a compressible formulation of the $k - \omega$ SST model [10] including the optional sink term for rough walls [11]. Model validation has been performed against experimental measurements on a strongly swirling flow through a sudden expansion [12] and on a flow around a poppet valve [13].

TURBULENCE MODELING

The compressible formulation of the $k - \omega$ SST model [10] including the optional sink term for rough walls [11] has been used as the basic turbulence model in this work. The main idea behind this choice is to retain the robust and accurate formulation of the RANS model in the near wall regions and in zones of the freestream region where the mesh resolution is not sufficiently high for the direct solution of the main turbulent scales. The model is a two-equation eddy viscosity model, and it is coupled to the averaged NavierStokes equations by the eddy-viscosity assumption:

$$-\tau_{ij} = 2\mu_t S_{ij} - \frac{2}{3}\rho k \delta_{ij} \quad (1)$$

where τ_{ij} is the viscous stress tensor and S_{ij} is the strain rate tensor, defined as:

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2)$$

The Boussinesq assumption introduces the concept of a turbulent eddy viscosity, μ_t , and it is particularly suitable when the influence of turbulence on the mean flow is dominated by a mixing process. The eddy viscosity has the same dimension as the viscosity of the fluid and is assumed to be proportional to a function of the local turbulent length and time scales:

$$\mu_t \sim L_t^2 / T_t \quad (3)$$

The turbulent length and time scales are unknown local properties of the turbulent flow and must be modeled. If the modeled turbulent kinetic energy k and specific dissipation rate ω are solved, a measure of the turbulent length and time scales can be estimated.

Derivation of the Filter Function

A low-pass filtering operation is applied to the turbulence model in order to allow the existence of resolvable turbulent scales in the solution of the flow field. Similarly to [9], the form of the filter function is derived from a dimensional analysis and the filter is applied to the turbulent length and time scales, rather than to the turbulent kinetic energy [5]. As it will later discussed, this allows to apply the approach to any URANS model with minor modifications. The local modeled turbulent length scale can be directly evaluated by the turbulent kinetic energy and dissipation, as follows:

$$L_t \sim k^{1/2} / \omega \quad (4)$$

$$T_t \sim 1 / \omega \quad (5)$$

In the approach presented in this work, the filtering operation is based on the comparison between the modeled and the resolved turbulent length scales. The upper limit of the modeled turbulent length scale

corresponds exactly to the lower limit of the resolved turbulent length scales. The largest length scale that needs to be a part of the eddy viscosity formulation is:

$$\ell_t = \min\{L_t, \Delta_f\} \quad (6)$$

where Δ_f is evaluated by comparing a function of the local flow velocity U and an equivalent LES filter size Δ_{eq} :

$$\Delta_f = \max(\alpha|U|\delta t, \Delta_{eq}) \quad (7)$$

In Eq. (7), the product $\alpha|U|\delta t$ is a measure of the shortest distance over which a fluid particle can be traced in an unsteady computation and the maximum length scale that needs to be a part of the eddy viscosity formulation. In this sense, the computational timestep influences the lower limit for the resolved time scale and includes the implicit relationship between space and time in the filtering operation. Introducing the maximum flow Courant-Friedrichs-Lewy (CFL) number:

$$CFL_{max} = \frac{U_{max} \delta t}{\delta x_{min}} \quad (8)$$

the time integration step is usually limited to:

$$\delta t \leq \frac{CFL_{max} \delta x_{min}}{|U_{max}|} \quad (9)$$

where CFL_{max} is usually chosen on the basis of the numerical method and of the level of unsteadiness of the problem.

Being the local CFL number defined on cell i :

$$CFL_i = \frac{|U_i| \delta t}{\delta x_i} \quad (10)$$

the criterion (9) ensures that for each cell the local CFL number CFL_i is lower than the maximum CFL:

$$\beta = \frac{CFL_i}{CFL} \leq 1 \quad (11)$$

In general, there is no way to mathematically distinguish between turbulence and unsteadiness. For this reason, it is very difficult to estimate the minimum length scale that the turbulence model is able to capture, since it depends both on the spatial resolution and on the temporal correlation between timesteps, which is strictly related to the CFL used by the numerical solver. A conservative way of thinking would lead to the conclusion that for each computational cell the smallest turbulent length scale that can be captured by the turbulence model is $(|U| \delta t)_{MAX}$

$$\begin{aligned} |U| \delta t|_{MAX} &= CFL \cdot \delta x \\ &= \frac{CFL_i}{\beta} \delta x \\ &= \alpha CFL_i \delta x \\ &= \alpha |U| \delta t \end{aligned} \quad (12)$$

where

$$\alpha = \frac{1}{\beta} = \frac{CFL}{CFL_i} \quad (13)$$

In [9], numerical experiments suggested that the optimum multiplier α to the product $|U| \delta t$ should be 3 in order to properly resolve the turbulent structures. In the present paper, the coefficient α is dynamically calculated by the model from the local and the overall CFL number, which varies in time and space; this is done to correlate the temporal scales of the vortices and the spatial resolution of the grid in the filter operation and it is particularly important in engine flows, where cell size and flow conditions significantly vary between the near valve region and the other regions of the mesh. A dynamic procedure for run-time calculation of the integral length scale allows for a more correct estimation of the minimum integral length scale that can be locally captured by the model. In Eq. (7), the (estimated) upper limit of the dissipation range scales is compared to the length of the resolved scales through the Length Scale Resolution (LSR) parameter [14]. LSR provides a relationship between the actual resolved energy level and the corresponding lower limit of the inertial sub-range and it is defined as:

$$LSR = \frac{\Delta}{\ell_{di}} \quad (14)$$

where Δ is the local filter size and ℓ_{di} is the lower limit of the inertial sub-range [15]:

$$\ell_{di} \approx 60\eta \quad (15)$$

and η is the Kolmogorov scale:

$$\eta = \nu^{3/4} \varepsilon^{-1/4} \quad (16)$$

Where the LSR value is equal to 1 all the turbulent scales up to the dissipation range are resolved. From the definition of Eq. (14), the evaluation of the actual resolved energy level is directly linked to the local filter size; in this way the adopted mesh size is related to the local energy resolution all over the computational domain. In [16], it was found that

$$LSR \leq 5 \quad (17)$$

is the upper limit to guarantee a reasonable resolution is space for LES at an affordable computational cost. From Eq. (17) and (14), it follows that the maximum equivalent filter width Δ_{eq} to use in Eq. (7) is:

$$\Delta_{eq} \leq LSR \cdot \ell_{di} \quad (18)$$

The upper limit on the modeled length and time scales ℓ_t can also be defined in terms of the filtered (non-resolved) variables:

$$\ell_t \sim \hat{k}^{1/2} / \hat{\omega} \quad (19)$$

$$t_t \sim 1 / \hat{\omega} \quad (20)$$

The specific dissipation rate ω is related to the dissipation rate ε by the relation:

$$\omega = \frac{\varepsilon}{\beta^* k} \quad (21)$$

where $\beta^* = 0.09$. The filtered specific dissipation can be written as

$$\hat{\omega} = \frac{\hat{\varepsilon}}{\beta^* \hat{k}} \quad (22)$$

The dissipation rate is never resolved in anything cheaper than a DNS, hence

$$\hat{\varepsilon} = \varepsilon \quad (23)$$

and from Eq. (22) and (23) it follows:

$$\hat{\omega} = \frac{\omega k}{\hat{k}} \quad (24)$$

and from Eqs. (4), (19) and (22) an expression for the filtered turbulent kinetic energy can be found:

$$\hat{k} = g(\ell_t, L_t) k \quad (25)$$

where the equality follows from the assumption that the constants of proportionality in Eq. (4) and (19) are equal. The filter function results to be defined as:

$$g \equiv (\ell_t / L_t)^{2/3} \quad (26)$$

in which ℓ_t is computed from Eq. (6) and the modeled turbulent scale L_t is calculated as:

$$L_t \simeq \frac{k^{1/2}}{\beta^* \omega} \quad (27)$$

In regions where turbulence cannot be resolved, i.e. where $L_t < \Delta_f$ in Eq. (6), the filter function is equal to unity and in Eq. (25) $\hat{k} = k$. However, there is no need to compute the filtered turbulent variables explicitly. A filtered eddy viscosity can be constructed directly from the non-resolvable turbulent length and time scales:

$$\hat{\mu}_t \sim \ell_t^2 / t_t \quad (28)$$

It follows from Eqs. (19), (20), (24) and (25) that

$$\hat{\mu}_t = g^2 \rho \frac{k}{\omega} \quad (29)$$

where the equality must hold in order to recover the original (non-filtered) eddy viscosity formulation, $\mu_t = \rho k / \omega$, in regions where the filter is inactive. Note that the only modification to the original eddy viscosity formulation is the factor g^2 , i.e., the square of the filter function.

The filter function can be derived and applied using any two-equation turbulence model: it will always have the same functional form reported in Eq. (26) and it will always appear as squared when used in the formulation of the eddy viscosity; also, it has an explicit grid dependence in the eddy viscosity formulation through the parameter ℓ_t in Eq. (26) when $\Delta_f < L_t$ in Eq. (6).

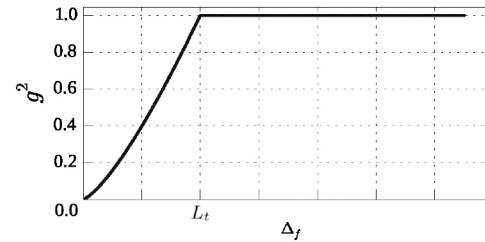


Figure 1. The filter function $g^2(\Delta_f)$ is clipped to 1 as Δ_f is equal to the integral length of the modeled scales L_t and it tends to zero with a minimum as the grid size tends to the fine grid limit.

The definition of Eq. (26) is thought to allow the filter function to go naturally to zero as the grid spacing tends to zero and as the grid size tends to the fine grid limit:

$$\lim_{\Delta_f \rightarrow 0} g^2 = 0 \quad (30)$$

Also the derivative of g^2 with respect to the filter width:

$$\frac{\partial (g^2)}{\partial \Delta_f} = \frac{4}{3} \left(\frac{\Delta_f}{L_t} \right)^{1/3} \quad (31)$$

must tend to zero as $\Delta_f \rightarrow 0$: