

Suspension Kinematic/Compliance Uncertain Optimization Using a Chebyshev Polynomial Approach

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ABSTRACT

The optimization of vehicle suspension kinematic/compliance characteristics is of significant importance in the chassis development. Practical suspension system contains many uncertainties which may result from poorly known or variable parameters or from uncertain inputs. However, in most suspension optimization processes these uncertainties are not accounted for. This study explores the use of Chebyshev polynomials to model complex nonlinear suspension systems with interval uncertainties.

In the suspension model, several kinematic and compliance characteristics are considered as objectives to be optimized. Suspension bushing characteristics are considered as design variables as well as uncertain parameters. A high-order response surface model using the zeros of Chebyshev polynomials as sampling points is established to approximate the suspension kinematic/compliance model. For the uncertain optimization problem, a double-loop optimization process is used, in which the inner loop is to calculate the bounds of the interval design functions and the outer loop is to achieve optimum suspension kinematic/compliance characteristics. Numerical results show the effectiveness of the proposed optimization method.

KEYWORDS: uncertain optimization, suspension, kinematic/compliance, Chebyshev polynomials

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INTRODUCTION

Kinematic and compliance (K&C) characteristics of the vehicle suspension are key factors from which many aspects of the ride and handling performances of a vehicle can be estimated and explained [1]. For this reason, there have been many optimization schemes for suspension K&C. Yi et al. [1] established a new methodology in which both kinematic and compliance characteristics targets could be met by separating the whole optimization problem into partial ones. Yahya et al. [2] presented a model validation methodology on suspension and steering of a heavy commercial vehicle. However in these studies the design variables are usually the locations of hard points but the compliance elements characteristics such as bushings are not considered in the optimization process.

Vehicle suspensions can be regarded as interconnections of rigid bodies with kinematic joints and compliance elements such as springs, bushings, and stabilizers [3]. The compliance elements can affect the suspension performance significantly [4] as well as steering subsystem [5]. With bushings more and more widely used in suspensions, they can influence the full vehicle performance, especially handling performance [6]. Li et al. [7] presented an optimization process of the elastomeric bushings in which the vehicle

handling and ride performance could be improved and the durability of suspension component could be kept from getting worse. It proved again that bushings had an important effect on the performances of full vehicles. Therefore the characteristics of the bushings should be considered as design variables in the suspension K&C optimization.

Most works in suspension optimization are based on the assumption that the suspension parameters are deterministic. Actually a number of problems in real world are too complex to be defined deterministically due to the lack of sufficient information, which makes the deterministic assumption not always valid. Therefore there is an increasing demand to consider the impact of uncertainties quantitatively in the optimization of suspensions. Weng [8] focused on the methodology development and application of robust optimization of designing automotive suspension systems in frequency domain, considering sprung mass and suspension stiffness as uncertain variables. Ma et al.[9] explored an interval method to describe the uncertainty suspension kinematic characteristics. This method uses Newton iteration, which means the derivative of the model have to be calculated. Kang et al.[10] took the bushing compliance uncertainty into account when optimizing the suspension kinematic behaviors. Wu et al.[11] proposed a new design optimization framework for suspension systems considering the

kinematic characteristics in the presence of uncertainties. However, the design variables are locations of hard points and only kinematic characteristics are investigated.

It can be seen that the number of studies in which bushing characteristics or uncertain factors are considered is relatively small. To this end, suspension K&C will be optimized in this study, in consideration of bushing characteristics and uncertain parameters simultaneously.

BASIC THEORY

In this section the approximation using the Chebyshev polynomial theory is presented firstly, and then a new Chebyshev metamodel is proposed. More details can be found in Ref. [11].

Chebyshev Polynomial Approximation Theory

For a continuous function $f(x)$ in $[a, b]$, there will exist a polynomial $p(x)$ which can uniformly approximate to the function $f(x)$ in $[a, b]$.

For $\forall \varepsilon > 0$, we have

$$\|f(x) - p(x)\| < \varepsilon, x \in [a, b] \quad (1)$$

Let $p_n(x)$ denote a set of polynomials of degree not bigger than n . For any nonnegative integer n there exists a unique polynomial $p_n^*(x)$ in $p_n(x)$, such that

$$\|f(x) - p(x)\|_{\infty} \geq \|f(x) - p_n^*(x)\|_{\infty}, x \in [a, b] \quad (2)$$

The function $f(x)$ can be approximated by the truncated Chebyshev series of degree n as follows

$$f(x) \approx p_n(x) = \frac{1}{2} f_0 + \sum_{i=1}^n f_i C_i(x) \quad (3)$$

where f_i are the constant coefficients and $C_i(x)$ are the Chebyshev polynomials

$$C_i(x) = \cos i\theta \quad (4)$$

where $\theta = \arccos((2x - (b + a)) / (b - a)) \in [0, \pi]$. To simplify the notes, we define $x \in [-1, 1]$ and other domain can be transformed to the domain $[-1, 1]$ via the linear transformation. The Chebyshev polynomials can also be denoted as following recurrence formula.

$$\begin{cases} C_0(x) = 1 \\ C_1(x) = x \\ C_n(x) = 2xC_{n-1}(x) - C_{n-2}(x); n \geq 2 \end{cases} \quad (5)$$

Based on the orthogonality of the Chebyshev polynomial, the constant coefficients f_i can be calculated by

$$\begin{aligned} f_i &= \frac{2}{\pi} \int_{-1}^1 \rho(x) f(x) C_i(x) dx \\ &= \frac{2}{\pi} \int_0^\pi f(\cos \theta) \cos i\theta d\theta, i = 0, 1, 2, \dots, n \end{aligned} \quad (6)$$

where $\rho(x) = 1 / \sqrt{1 - x^2}$ is the weight function.

The integrals of equation (6) can be calculated using numerical integral methods, given by

$$\begin{aligned} f_i &= \frac{2}{\pi} \int_{-1}^1 \frac{f(x) C_i(x)}{\sqrt{1 - x^2}} dx \approx \frac{2}{\pi} \frac{\pi}{m} \sum_{q=1}^m f(x_q) C_i(x_q) \\ &= \frac{2}{m} \sum_{q=1}^m f(\cos \theta_q) \cos i\theta_q \end{aligned} \quad (7)$$

where m denotes the order of the integral formula and x_q are the interpolation points, namely the zeros of the Chebyshev polynomials of degree m , shown as

$$x_q = \cos \theta_q, \theta_q = \frac{2q-1}{m} \frac{\pi}{2}, j = 1, 2, \dots, m \quad (8)$$

For multi-dimensional problems, the Chebyshev polynomials are the tensor product of each one-dimension polynomials. A k -dimensional Chebyshev polynomial of $\mathbf{x} \in [-1, 1]^k$ is given by

$$C_{n_1, n_2, \dots, n_k}(x_1, x_2, \dots, x_k) = \cos(n_1 \theta_1) \dots \cos(n_k \theta_k) \quad (9)$$

The function $f(\mathbf{x})$ can also be approximated as truncated Chebyshev series of degree n as

$$f(\mathbf{x}) \approx \sum_{i_1=0}^n \dots \sum_{i_k=0}^n \left(\frac{1}{2}\right)^p f_{i_1, \dots, i_k} C_{i_1, \dots, i_k}(\mathbf{x}) \quad (10)$$

where p is the number of subscript equals to zero. Similarly the multi-dimensional coefficients $f_{i_1, \dots, i_k}$ can be expressed as

$$\begin{aligned} f_{i_1, i_2, \dots, i_k} &= \left(\frac{2}{\pi}\right)^k \times \\ &\int_0^\pi \dots \int_0^\pi f(\cos \theta_1, \dots, \cos \theta_k) \cos i_1 \theta_1 \dots \cos i_k \theta_k d\theta_1 \dots d\theta_k \end{aligned} \quad (11)$$

Use the numerical integral methods for equation (11), we have

$$\begin{aligned} f_{i_1, i_2, \dots, i_k} &\approx \left(\frac{2}{m}\right)^k \times \\ &\sum_{q_1=1}^m \dots \sum_{q_k=1}^m f(\cos \theta_{q_1}, \dots, \cos \theta_{q_k}) \cos i_1 \theta_{q_1} \dots \cos i_k \theta_{q_k} \end{aligned} \quad (12)$$