

Error Reduction in Spatial Robots Based on the Statistical Uncertainty Analysis

S. Khodaygan
Sharif University of Technology

M. Hafezipour
K.N. Toosi University of Technology

ABSTRACT

Kinematic accuracy of the robot end-effector is decreased by many uncertainties. In order to design and manufacture robots with high accuracy, it is essential to know the effects of these uncertainties on the motion of robots. Uncertainty analysis is a useful method which can estimate deviations from desired path in robots caused by uncertainties. This paper presents an applied formulation based on Direct Linearization Method (DLM), for 3D statistical uncertainty analysis of open-loop mechanisms and robots. The maximum normal and parallel components of the position error on the end-effector path are introduced. In this paper, uncertainty effects of both linear and angular variations in performance of spatial open-loop mechanisms and robots are considered. Based on the relations for the percent contributions of manufacturing variables, for the position error reduction, the tolerances that have the most significant effects on the commutated uncertainty zone of the end-effector position can be modified. The proposed method is illustrated using a spatial manipulator with three-revolute joints and verified with a Monte Carlo simulation method. Finally, normal and parallel distances to end-effector path are determined as error bands for all over range of motion. The results of applying this method demonstrate that estimating the position error in mechanisms and robots can be done efficiently and precisely.

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INTRODUCTION

Applications of robots have been expanded over the five past decades and demand for robots with high accuracy has been growing over the past ten years, especially robots used in aerospace and medicine sectors have to work with high accuracy. Since undesirable motion of end-effector can produce hazardous and costly results, designers use uncertainty analysis to create products that are reliable and economical. The goals of mechanical systems tolerancing are to produce designs that can be assembled and that function correctly despite manufacturing variation [1]. The positional and directional tolerances of the robot end-effectors may originate from five factors that cause robot errors: environmental (such as temperature or the warm-up process), parametric (for example, kinematic parameter variation due to manufacturing and assembly errors, influence of dynamic parameters, friction and other nonlinearities, including hysteresis and backlash), measurement (resolution and nonlinearity of joint position sensors), computational (computer round-off and steady-state control errors) and application (such as installation errors) [2,3]. Different researches have been done to contain effect of tolerances in the analysis/design of a system. In previous attempts the probabilistic methods was presented. In [4, 5] joint clearance and link tolerance are supposed to be random errors. In order to represent these errors, appropriate probability density functions were assumed and a four-bar mechanism was analyzed by use of effective link lengths. Rao and Bhatti [6] performed sensitivity analysis by way of a

probabilistic approach. A general probability density function (pdf) of the endpoint of planar robots was presented in [7]. Kim, Song, and Kang [8] calculated kinematical reliability for the positioning and orientation repeatability analytically based on an advanced first-order second moment (AFOSM) method. Direct Linearization Method (DLM) which is a method for simplification of definition of kinematic error has been employed in [9]. In this method equations of motion are linearized by using Taylor's series expansion. In order to clarify the influence of loading on variation of the assembly specification, finite elements method analysis was added to DLM in [10]. Binaud, Caro, and Wenger [11, 12] used the linearization method for sensitivity analysis of planar parallel manipulators. Kinematic accuracy analysis of the mechanisms and robots based on screw theory were proposed by [13, 14, 15]. Using the interval linearization method for the sensitivity analysis of serial and parallel manipulators was proposed by [3, 16]. Khodaygan and Movahhedy [17, 18] presented a new method for tolerance analysis of mechanical assemblies with asymmetric tolerances based on uncertainty and accumulation models. According to this scheme, tolerance components were described as fuzzy numbers with their membership functions constructed using the statistical distributions of manufactured variables. A feature based approach for tolerance analysis of the mechanical assemblies established upon modal interval mathematics and small degrees of freedom (MI-SDOF) concepts was introduced in [19]. Khodaygan introduced an efficient approach for analysis and compensation errors in the manufacturing tool

system [20]. To eliminate the effects of the locating errors on the machined features, the location and orientation of the cutting tool in the cutter location data are corrected. Estimation of the probability distribution of the motion error by assumption dimension variables as truncated random variables has been done in Ref. [21, 22].

In this paper, estimation of the statistical position uncertainty in the spatial open loop mechanisms and robots based on Direct Linearization Method (DLM) is presented. Since equations of motion are essential for tolerance analysis, these equations are determined by using Denavit - Hartenberg convention. These equations are linearized by using Taylor's series expansion and limits of error are determined by an ellipsoid surface. For precise analysis of uncertainties in the position error of the end-effector, the corresponding error is decomposed into two main categorizes; the maximum normal and parallel. The normal and parallel distance to end-effector path error is considered as maximum error for whole of motion. Applicability of the presented method to a spatial robot is demonstrated using an RRR spatial manipulator and the computational results are compared with results of Monte Carlo simulations.

GENERAL APPROACH FOR THE ESTIMATION OF POSITION ERROR

In this section fundamental concepts and steps for the proposed method are presented. Generally, the proposed method for 3D statistical uncertainty analysis of the spatial robots based on direct linearization method consists of the following steps;

1. Kinematic modeling of the spatial robot
2. Simplification of kinematic equations using DLM
3. Estimation of the position error
4. Determination of the maximum error

Kinematic Modeling of the Spatial Robot

In order to perform statistical uncertainty analysis and estimate the position error of the robots manipulator end-effector, equations which describe the robot motion should be known. The connection between the end-effector motion and the joint displacements is explained by kinematic equations. Equations are solved based on forward kinematic approach. Therefore, the unknown orientation and position of the end-effector from known values for the joint variable (angles in revolute joints or extension in prismatic joints) can be estimated. The inverse kinematic approach is not as simple as forward kinematic approach. Since kinematic equations are nonlinear, their solution are not always in a closed form, so these equations should be solved by the numerical methods [23].

The Denavit - Hartenberg Convention

The position and orientation of the end-effector can be determined based on solving the forward kinematic equations. In order to express these equations in a reference coordination system, it is necessary to attach a frame to each link of manipulator. Although the choice of these frames is arbitrary, their systematic attachment can be useful. A commonly used convention for selecting frames of reference in

robotic applications is the Denavit - Hartenberg, or D-H convention. In this convention, each homogeneous transformation ${}^{i-1}A_i$ as a transformation matrix can be given as [24];

$${}^{i-1}A_i = \begin{pmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

where S and C are sine and cosine, respectively. The Denavit-Hartenberg parameters of i th link (a_i , θ_i , α_i and d_i) are shown in Figure 1.

The overall transformation matrix is defined by multiplying these homogeneous transformation matrices;

$${}^0A_n = {}^0A_1 {}^1A_2 \cdots {}^{n-1}A_n \quad (2)$$

Finally, the forward kinematics of the robot manipulator can be determined by using Eq.(3)

$${}^0P_E = {}^0A_n {}^n P_E \quad (3)$$

where ${}^0P_E$ and ${}^iP_E$ are positions of end-effector with respect to the reference and the i th frame, respectively.

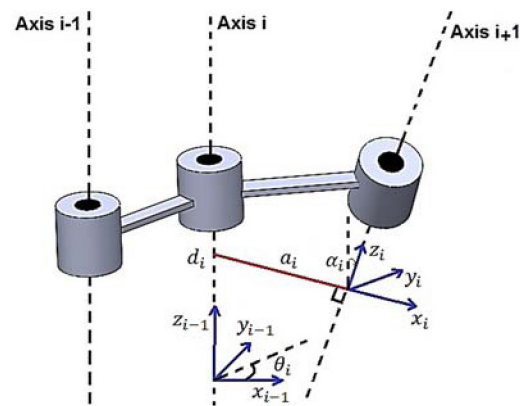


Figure 1. A schematic of Denavit-Hartenberg parameters

Error Analysis

The main goal of the position error analysis in kinematic linkages is usually to find error bands around an ideal path [9]. The end-effector errors are caused by different factors. These errors represent the deviation of the end-effector with respect to the ideal path that determined by forward kinematic equations. In this paper two kinds of error are assumed;

1. The dimensional errors in link lengths due to manufacturing processes.
2. The dimensional errors in joint angles as a result of different factors like the clearances and the sensors errors.

Direct Linearization Method

In order to find the error band, the motion equation of mechanism should be determined. These equations were expressed by Eq. (3). These equations are then linearized using a first-order Taylor's series expansion and written as;

$$\{\Delta P\} = [C]\{\Delta X\} + [D]\{\Delta U\} \quad (4)$$

where $\{X\}$ is the vector of independent variables and $\{U\}$ is the vector of dependent variables. $[C]$ and $[D]$ are matrices which represent first-order partial derivatives of Eq. (3) with respect to the independent and dependent variables, respectively. $[C]$ and $[D]$ can be expressed as:

$$[C] = \frac{\partial P_i}{\partial X_j} \quad (5)$$

$$[D] = \frac{\partial P_i}{\partial U_j} \quad (6)$$

where P_i is $\{P_x, P_y, P_z\}$ the position of end-effector. There is not any dependent variable in the open loop mechanism, because joint variables are inputs of control system and these variables are independent. So Eq. (4) can be written as;

$$\{\Delta P\} = [S_{ij}]\{\Delta X\} \quad (7)$$

where $[S_{ij}]$ is the sensitivity matrix. This matrix correlates the position of end-effector to the independent variables and includes the kinematic of mechanism.

Bivariate Normal Model of the Position Error Using DLM

After finding the sensitivity matrix, the effect of each independent variable on the position of the end-effector can be estimated using the statistical approach. The statistical position error can be estimated based on the following equation [9];

$$\sigma_{pi}^2 = Var(P_i) = \sum_{j=1}^n (S_{ij})^2 (\sigma_j)^2 \quad (8)$$

where σ_j is the standard deviation of the j th independent variable, $Var(P_i)$ is the position variance of the i th dimension. Since in this work, the errors are estimated based on the 3-sigma scenario, the standard deviation is one third of the symmetric tolerance value. In assemblies or kinematic linkages, the position error is generally not independent with respect to the different axes, meaning you cannot have an error in the x direction without introducing error in the y direction [9]. This correlation between independent variable decreases accuracy of DLM method and Eq. (8) cannot be useful for estimation of the position error. In order to estimate position error

with more accurate, presentation a model which can account for this correlation is necessary. Bivariate normal model of the position error using DLM, can explain this correlation.

Use of covariance is necessary for extending the DLM method to a bivariate normal model. In the case of multivariate distribution, the variance of manufacturing variables is expressed as the covariance matrix V , which presents the variance of each manufacturing variable along with the correlation between the variables [25]. In the mechanism without correlation between independent variable the covariance matrix can be written as;

$$[V] = \begin{pmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_n^2 \end{pmatrix} \quad (9)$$

The covariance matrix for mechanism with correlation between independent variable was presented by Brown [26] as;

$$[\Sigma] = \begin{pmatrix} V_x & V_{xy} & V_{xz} \\ V_{yx} & V_y & V_{yz} \\ V_{zx} & V_{zy} & V_z \end{pmatrix} \quad (9)$$

where $[\Sigma]$ can be written as follow;

$$[\Sigma] = [S_{ij}][V][S_{ij}]^T \quad (10)$$

The matrix $[\Sigma]$ has three eigenvalue and three eigenvectors. The eigenvalues of any real symmetric matrix are all real (we omit the proof). There always exist three eigenvectors that are mutually perpendicular. Eigenvectors and eigenvalues can be calculated by solving below equation

$$[\Sigma]\vec{n} = \lambda\vec{n} \quad (11)$$

where $\vec{n}$ is an eigenvector and λ is the corresponding eigenvalue. The surface of equal probability represents an ellipsoid. The principal diameters of this ellipsoid can be found using

$$D_k = 2\rho_k = 2\sqrt{\lambda_k} \quad k = 1, 2, 3 \quad (12)$$

The surface of equal probability can be presented by the following equation [9];