

Development of a Comprehensive Validation Method for Dynamic Systems and Its Application on Vehicle Design

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ABSTRACT

Simulation based design optimization has become the common practice in automotive product development. Increasing computer models are developed to simulate various dynamic systems. Before applying these models for product development, model validation needs to be conducted to assess their validity. In model validation, for the purpose of obtaining results successfully, it is vital to select or develop appropriate metrics for specific applications. For dynamic systems, one of the key obstacles of model validation is that most of the responses are functional, such as time history curves. This calls for the development of a metric that can evaluate the differences in terms of phase shift, magnitude and shape, which requires information from both time and frequency domain. And by representing time histories in frequency domain, more intuitive information can be obtained, such as magnitude-frequency and phase-frequency characteristics. However, Most of the existing metrics only focus on time domain. In this paper, a validation method combining analysis in time and frequency domain is proposed. Two analytical cases are then utilized for the illustration of the proposed method and process. A real-world children restraint system design case is then conducted. The evaluation results are consistent with Subjective Matter Experts' judgments.

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INTRODUCTION

Computer Aided Engineering (CAE) has got its popularity in solving practical problems in various engineering disciplines, especially in automotive industry. Plenty of computer programs and models are developed to simulate dynamic systems. Before applied, the predictive capability of model needs to be evaluated. That calls for comparing CAE model outputs with test data in order to assess the validity of the CAE model for its intended usage, which called model validation. To accomplish the process of model validation, one of the crucial tasks is to select ideal metrics. There are several properties that an ideal metric should satisfy [1]. However, it is difficult to assess the validity of a model when dealing with time histories of dynamic systems, because such validation requires comprehensive evaluation of the curves' features, such as magnitude, phase and shape. Therefore, considering merely the discrepancy between test data and CAE outputs at each temporal point without taking phase error and shape error into account is not capable of presenting all the characteristics, which would lead to a dissatisfactory model validation as a result.

The study on model validation has attracted plenty of researchers' interest and a great number of metrics are available in recent years. Hills and Leslie [2] proposed a metric that normalize the discrepancies between experimental observations and model

predictions to calculate a relative error norm. Zhang and Mahadevan [3] validated the reliability of computational simulations based on a metric using Bayesian hypothesis testing. Ferson and Oberkampf [4] introduced a general validation metric that is popular in engineering, which usually called area metric. However, those metrics show the limitation in validation when encountering time histories data for they are not capable of recognizing the errors due to phase and shape. To tackle such drawback, researchers have been striving for metrics that can separate magnitude, phase and shape assessments correctly. Error Assessment of Response Time Histories (EARTH) were developed in recent years for dynamic system applications. It provides independent errors of phase, magnitude and slope but without an intuitive quantity to present the validity of a model. Considering its limitation, Zhan and Fu [5] investigated EARTH and proposed an enhanced version called Enhanced Error Assessment of Response Time Histories (EEARTH), which translates the three kinds of errors into a score between 0 and 100%, which is convenient for engineers to conduct decision making. Those two methods for time histories execute the process of model validation only in time domain. With respect to frequency domain, Jiang and Mahadevan [6] conduct model validation by decomposing the time history data into multiple time-frequency resolutions using a discrete wavelet packet transform method [7, 8]. However, model validation method that combine assessments in both time and frequency domain is rarely

found in the literatures. As frequency domain contains more intuitive information on phase, magnitude and frequency, this paper developed a comprehensive method for model validation of dynamic systems, which integrates both the analysis of features in time series and frequency characteristics. In time domain, area metric is employed to calculate the disagreement between two time series in the overall distributions, the time histories are then transformed into frequency domain for the purpose of getting the measurements of errors in phase, frequency and magnitude. As a result, combined measurement provides a more complete evaluation of the responses of interest.

The remainder of this paper is organized as following: firstly, the proposed framework of combined model validation method is introduced, followed by two analytical examples which is used to illustrate the proposed process, then the method is applied to a practical engineering case of children restrain system. Some conclusions are given in the end.

VALIDATION FOR TIME HISTORIES

The generalized procedure of implementing the proposed validation method for complicated dynamic systems is presented in [Figure 1](#). For the purpose of achieving comprehensive assessment of differences between time series, area metric is utilized to calculate the distribution character of each response. Meanwhile, original data are transformed from time domain into frequency domain to capture features of frequencies and corresponding magnitudes and phases.

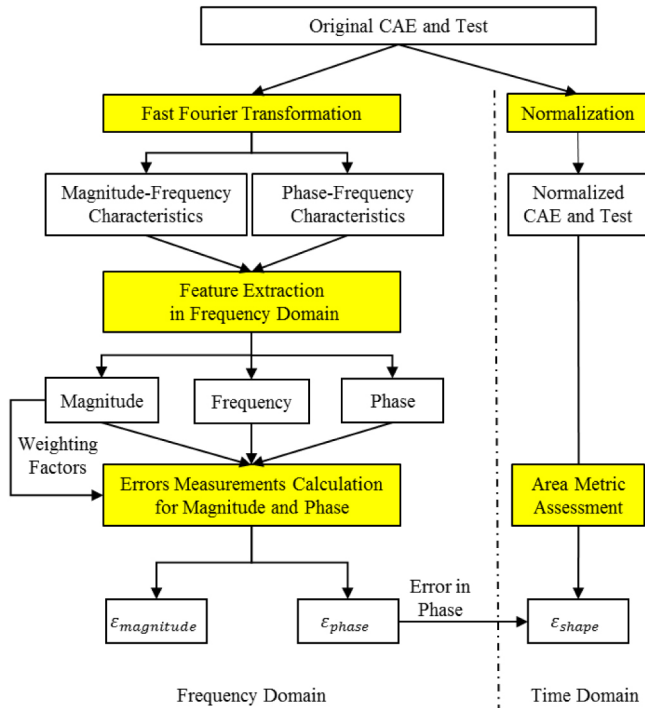


Figure 1. Flowchart of proposed validation method

Combined Metric for Dynamic Systems

To validate dynamic systems which generate complex functional data, it is not sufficient to merely measure the discrepancy in time series. The errors in shape and phase also need to be considered. But

in time domain, the features of time curves, such as phase and frequency, cannot be obtained intuitively. In this study, analysis in both time and frequency domain are associated to provide a comprehensive assessment of the magnitude error, phase error and shape error.

Area metric is a well-established metric in time domain. It is firstly proposed by Ferson [4] to investigate the disagreements of repeated tests and simulations in the statistical features. In this study, the normalized data are utilized to construct the distribution of samples. They are represented as non-decreasing step function with a steady step. For the data are in the type of discrete points, the distribution of data in a time curve is expressed as:

$$S_n(x) = \frac{\sum_{i=1}^n I(x_i, x)}{n} \quad (6)$$

where

$$I(x_i, x) = \begin{cases} 1 & x_i \leq x \\ 0 & x_i > x \end{cases} \quad (7)$$

Equation 6 implies that $S_n(x)$ has the function of presenting the number of points below a specific quantity x . After plotting the $S_n(x)$ of two data sets for comparison, area between the two CDFs (Cumulative Density Functions) can be calculated as the measurement of discrepancy, as denoted in [Equation 8](#).

$$d(S_n, S_n') = \int_{-\infty}^{\infty} |S_n(x) - S_n'(x)| dx \quad (8)$$

For example, given a real-world signal in the size of 2047×1 , a CAE model simulation should be validated with area metric. Such comparison is presented graphically in [Figure 2](#). The black line is CDF of test data while the red one denotes simulation. CDF of vibration signal shows the characteristic of normal distribution. When the length of step in vertical axis approaches infinite small, a smooth CDF curve is obtained. Thus area metric applied to time histories is capable of providing the distributions of two sets of samples. However, considering an extreme condition, when curves of CAE and test are mirror images of each other, the values of area metric would be the same while they are different in shape. Therefore, for more accurate error assessments, it is crucial to combine area metric with analysis in frequency domain.

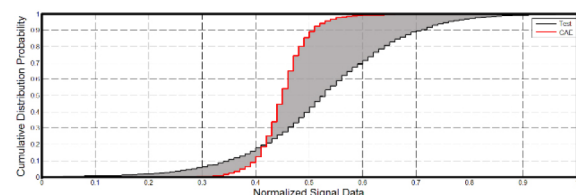


Figure 2. Illustration for area metric in time domain

In this paper, Fast Fourier Transform (FFT) is employed for data transformation. Original time histories are decomposed into plenty of signals of sine or cosine. By choosing the signals with large values in magnitude, the principal signals are determined. Figure 3 represents the same set of data in Figure 2 after transformed into frequency domain.

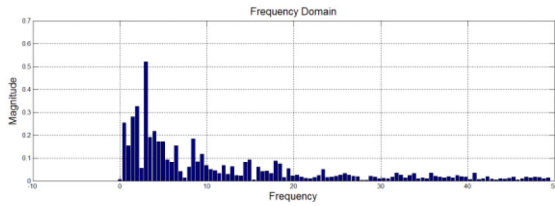


Figure 3. Decomposed data in frequency domain

Analysis in frequency domain provides information of frequency and corresponding amplitude and phase, based on which the errors in magnitude and phase for a certain response can be calculated with Equation 9 and Equation 10.

$$\varepsilon_{\text{magnitude}} = \sum_{i=1}^q \left(\frac{M_{\text{Test},i}}{\sum_{i=1}^q M_{\text{Test},i}} \left| \frac{M_{\text{CAE},i} - M_{\text{Test},i}}{M_{\text{Test},i}} \right| \times 100\% \right) \quad (9)$$

$$\varepsilon_{\text{phase}} = \sum_{i=1}^q \left(\frac{M_{\text{Test},i}}{\sum_{i=1}^q M_{\text{Test},i}} |P_{\text{CAE},i} - P_{\text{Test},i}| / 2\pi \times 100\% \right) \quad (10)$$

where q denotes number of frequency components, M_{CAE} and M_{Test} are the arrays of magnitudes and phases for each frequency. Combining above-mentioned measurement of area metric and computed $\varepsilon_{\text{phase}}$, error in shape can be obtained with Equation 11.

$$\varepsilon_{\text{shape}} = w_1 \times \text{Area} + w_2 \times \varepsilon_{\text{phase}} \quad (11)$$

where, w_1 and w_2 indicate the weighting factors for measures of area metric and $\varepsilon_{\text{phase}}$. There is no special requirements for those two factors except that their summation should be 1. The choosing for their values totally depends upon which of them matters more in a specific validation problem. As shown in Equation 11, this research is trying to establish relationship between assessment in both time domain and frequency domain. After assessing all the errors in magnitude, phase and shape with the proposed measurements, an overall evaluation for a response can be simply expressed as:

$$\varepsilon_{\text{response},i} = \varepsilon_{\text{magnitude},i} + \varepsilon_{\text{phase},i} + \varepsilon_{\text{shape},i} \quad (12)$$

where i indicates the index of response. Hence, an intuitive judgment on the fidelity of CAE model while incorporating three different features of time histories is presented.

ANALYTICAL CASE STUDIES

In this section, an analytical curve is used to illustrate the proposed process. Three groups of data, namely good, fair and poor, are generated based on the standard sine curve with 8 curves contained in each group, as shown in Figure 4.

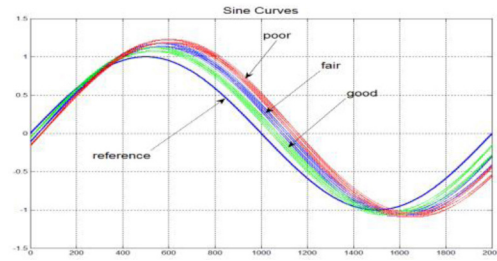


Figure 4. Sine curve and generated curves for comparison

The reference curve is regarded as test data in real world while others denote simulations in various levels of fidelity. Our purpose is to test if the proposed method is capable of distinguishing the three categories. The errors between given test and CAE data include difference in magnitude and phase while with minimum mismatch in shape.

24 curves are compared with the reference separately and their measurements in area metric and frequency characteristics are calculated. The measured areas values are provided in Table 1, which offer the guidance of difference in the overall distributions of samples. The smaller the quantity is, the better the curves match in shape. However, Table 1 shows that the limitation of the area metric. For example, assessment results of good-3 and good-4 are equal to that of fair-3 and fair-4, presented by bold quantities in Table 1. For better distinguishing the three groups of curves, additional analysis in frequency domain is necessary. Table 2 provides the measures in frequency domain for good-3, good-4, fair-3 and fair-4. Because the example is based on a sine curve, it is not necessary to assess all the frequency components but only the frequency with maximum magnitude.

Table 1. Area metric evaluations for sine curves, compared with reference

	good	fair	poor
1	0.0108	0.0141	0.0191
2	0.0112	0.0145	0.0196
3	0.0084	0.0084	0.0132
4	0.0088	0.0088	0.0138
5	0.0109	0.0143	0.0193
6	0.0112	0.0147	0.0199
7	0.0051	0.0086	0.0134
8	0.0055	0.0090	0.0140

Table 2. Analysis in frequency domain for sine curves

	reference	good-3	fair-3	good-4	fair-4
Magnitude	0.99	1.10	1.14	1.10	1.14
Phase	-0.49 π	-0.54 π	-0.57 π	-0.53 π	-0.57 π
Phase Shifting		-0.05 π	-0.08 π	-0.04 π	-0.08 π

As shown in Table 2, although response good-3 and fair-3 (or good-4 and fair-4) show no difference while measured by area metric in Table 1, the goods and the fairs show significant discrepancies in the phase shifting.

Moreover, a more realistic and content-rich wave is provided in Figure 5 for practical investigation