

# Validation Metric for Dynamic System Responses under Uncertainty

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## ABSTRACT

To date, model validation metric is prominently designed for non-dynamic model responses. Though metrics for dynamic responses are also available, they are specifically designed for the vehicle impact application and uncertainties are not considered in the metric. This paper proposes the validation metric for general dynamic system responses under uncertainty. The metric makes use of the popular U-pooling approach and extends it for dynamic responses. Furthermore, shape deviation metric was proposed to be included in the validation metric with the capability of considering multiple dynamic test data. One vehicle impact model is presented to demonstrate the proposed validation metric.

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## 1. INTRODUCTION

The key objective in research of model validation is to determine the degree to which the model is an accurate representation of the real world from the perspective of the intended uses of the model [1,2,3]. To date, majority of model validation work is for non-dynamic model performances [4,5,6,7,8,9,10,11]. New research challenges have to be addressed when model outputs are dynamic responses such as (i) how to characterize model uncertainty as a random process considering limited tests and model parameter uncertainty at available design configurations, (ii) how to build response surface for model uncertainty in the form of a random process, (iii) how to extend the validation metric for dynamic responses, and (iv) how to effectively consider dynamic model bias correction for reliability analysis and design. Resolving these challenges is imperative because accurate prediction of dynamic responses is critical in many applications especially for time-dependent reliability analysis. The contribution of this paper is to propose a validation metric for dynamic system responses. The rest of the paper is organized as follows. [Section 2](#) provides brief literature review of the U-pooling validation metric. [Section 3](#) introduces the proposed validation metric for dynamic responses. [Section 4](#) significantly enhances the computational efficiency for calculating the validation metric. Finally case studies and conclusion are presented in [sections 5](#) and [6](#), respectively.

## 2. REVIEW OF THE VALIDATION METRIC

Majority of available validation metric is designed for static system response such as the U-pooling [12] and Bayes factor [13]. Though metrics for dynamic responses are also available [14], they are specifically designed for vehicle impact application and uncertainties are not well considered in the metric. The proposed validation metric is related to the U-pooling metric, hence this metric is briefly reviewed in this section.

U-pooling metric was proposed by Ferson et al. [12] as a validation metric and has been adopted by many researchers in the study of model validation. The basic idea is to compare the cumulative distribution function (CDF) difference (i.e., the U-pooling value) between model prediction and test data in the standard Uniform space (or U-space) as shown in [Fig. 1](#). The smaller the area difference, the higher of the expected accuracy of the model prediction. For static responses, each test datum  $y_i$  corresponds to one  $u_i$  value which is calculated from the CDF value of the model prediction at the same design configuration (i.e.,  $u_i = F_{\hat{Y}}(y_i)$  where  $F(\cdot)$  is the CDF of  $\hat{Y}$ ). To adopt the U-pooling value for dynamic responses,  $F_{\hat{Y}}(y_i)$  should be calculated where  $y_i$  is a dynamic response from test and  $\hat{Y}$  is an arbitrary random process from model prediction.

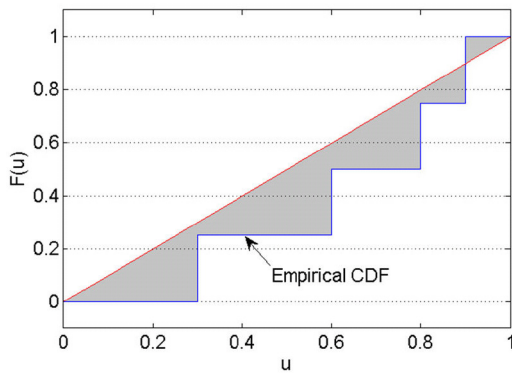


Figure 1. Illustration of the U-pooling value

### 3. VALIDATION METRIC FOR DYNAMIC RESPONSES

The proposed validation metric makes use of the U-pooling approach by addressing two technical components: i) extension of the U-pooling for dynamic responses, and ii) inclusion of the shape difference in the metric. The first component is addressed by discretizing the random process into  $n$  time steps such that the calculation of each  $u_i$  equivalents to identifying the  $n$ -dimensional joint CDF value for each random process realization of  $y_i$ . The second component is addressed by quantifying the shape deviation of the model prediction from the test. Technical details are elaborated as follows.

Under the assumption of Gaussian random process, the value of  $u_i$  can be estimated numerically using a number of algorithms since there is no closed form for  $n$ -dimensional normal joint CDF [15]. Considering an arbitrary random process, such calculation could be practically difficult. Generally, the sampling approach is able to approximate the  $u_i$  by counting the number of random process realizations less than or equal to the corresponding realization of  $y_i$  from the test as shown in Eq. (1).

$$u_i = F_{\hat{Y}}(y_i) \cong \sum_{k=1}^N M_k / N, \text{ where } M_k = \begin{cases} 1 & \hat{y}_{k,j} \leq y_{i,j} \quad j=1, \dots, n \\ 0 & \text{elsewhere} \end{cases} \quad (1)$$

where  $N$  is the total number of random process realizations from the model. It is worth nothing that above calculation is general but computationally expensive because each  $u_i$  estimation requires a magnitude of  $N \times n$  comparison operations. With several test observations of  $y_i$  ( $i=1, \dots, Q$ ), corresponding  $u_i$  can be calculated using Eq. (1) resulting in a U-pooling value as shown in Fig. 1.

Unlike static responses, shape difference of dynamic responses is another important attribute between model prediction and test, which is not able to be accounted for in Eq. (1). In other words, the same  $u_i$  value could represent different quality of the response matching as shown in Fig. 2a. Therefore, it is desirable to include the shape difference in the validation metric for dynamic system responses.

Under the assumption that model prediction is valid and truly represents the real physical systems, test observations equivalent to corresponding random process realizations from the model prediction. Therefore, the expected shape deviation between test observations and corresponding realizations from the model should be as small as possible. Otherwise, the assumption is not valid and the magnitude of such shape deviation indicating the level of disagreement between model prediction and test. Two remaining challenges need to be addressed to include the shape difference in the validation metric: i) quantification of the shape deviation, and ii) identification of the corresponding realizations from the model.

Quantification of the shape deviation is proposed to be conducted in a normalized region as shown in Fig. 2b so that its magnitude is at the same level as the U-pooling value. Assign  $S_i(y, \hat{y})$  as the shape deviation between  $y_i$  and the corresponding random process realization from the model, its value is the shaded area as shown in Fig. 2b. The corresponding realizations from the model based on test observations can be identified using Eq. (2).

$$\hat{y}_i = F_{\hat{Y}}^{-1} \left[ \frac{i}{Q+1} \right] \quad (2)$$

where  $Q$  is the number of test observations;  $i$  is the order of the test observations; and  $F_{\hat{Y}}^{-1}(\bullet)$  is the inverse CDF of  $\hat{Y}$ . For example, one test observation corresponds to  $F_{\hat{Y}}^{-1}(0.5)$  indicating the median of the random process or the mean for Gaussian random process. Two test observations correspond to  $F_{\hat{Y}}^{-1}(0.33)$  and  $F_{\hat{Y}}^{-1}(0.67)$ , respectively. Identification of the corresponding realizations meeting the CDF values can be conducted from the marginal distributions.

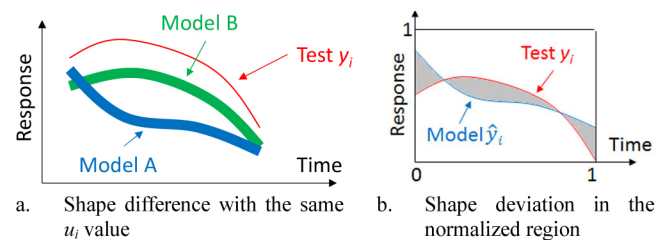


Figure 2. Illustration of the shape difference for dynamic responses

The proposed validation metric for dynamic responses is formulated in Eq. (3).

$$\Psi = U(u_1, \dots, u_i, \dots, u_Q) + \frac{1}{Q} \sum_{i=1}^Q S_i(y, \hat{y}) \quad (3)$$

where the 1<sup>st</sup> part is the U-pooling value and the 2<sup>nd</sup> part is the average of shape deviation. The smaller value of  $\Psi$  indicates the better model accuracy based on available test observations. It is worth noting that calculation of the proposed metric is computationally expensive for each  $u_i$ . Therefore, efficient approaches are proposed for computing the U-pooling in Section 4.