

Buckling Analysis of Uncertain Structures Using Imprecise Probability

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ABSTRACT

In order to ensure the safety of a structure, adequate strength for structural elements must be provided. Moreover, catastrophic deformations such as buckling must be prevented. Using the linear finite element method, deterministic buckling analysis is completed in two main steps. First, a static analysis is performed using an arbitrary ordinate applied loading pattern. Using the obtained element axial forces, the geometric stiffness of the structure is assembled. Second, an eigenvalue problem is performed between structure's elastic and geometric stiffness matrices, yielding the structure's critical buckling loads. However, these deterministic approaches do not consider uncertainty the structure's material and geometric properties.

In this work, a new method for finite element based buckling analysis of a structure with uncertainty is developed. An imprecise probability formulation is used to quantify the uncertainty present in the mechanical characteristics of the structure. For each element, independent variations are considered. Using the developed method, entitled Imprecise Monte-Carlo Buckling Analysis (IMBA), sharp bounds on critical buckling loads at different probability levels are obtained. This method establishes a framework for handling uncertainty in structural stability. A numerical example problem is presented that illustrates the capabilities of the new method along with discussions on their computational efficiency.

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INTRODUCTION

Critical buckling loads of a structure are considered as fundamental criteria for stability of the structure. In deterministic structural analyses, functional or numerical schemes are applied to obtain critical buckling loads and their corresponding mode shapes. Finite element analysis is one scheme that obtains these values by performing a generalized eigenvalue problem on the structure's elastic and geometric stiffness matrices. Throughout deterministic buckling analyses, uncertainties in structural parameters (including geometric and material properties) are not considered.

There are two distinct types of uncertainty: aleatoric and epistemic. Aleatoric uncertainty is due to inherent randomness in a system. This type of uncertainty is irreducible with higher precision or further data acquisition. Epistemic uncertainty is due to lack of knowledge, modeling errors, and/or insufficient data. This uncertainty, also known as reducible uncertainty, can be reduced with further investigations, more data, and updated models. Aleatoric uncertainties are traditionally treated using theories of probability, whereas epistemic uncertainties are generally treated using possibilistic methods such as interval or fuzzy analyses which provide bounds on the uncertain quantities. However, both of these methods of characterizing uncertainty have shortcomings. Probabilistic

approaches assume a well-defined Probability Density Function (PDF) which may not correspond to the true PDF of the physical system. On the other hand, possibilistic approaches do not consider the distribution of the likelihood of occurrence of the parameters within the provided bounds, similar to the membership function.

One method for handling uncertainty in a system with no assumption of a well-defined PDF is imprecise probability. This method involves setting bounds on the Cumulative Distribution Function (CDF), between which the true CDF exists.

In this work, a new method for reliable buckling analysis of structures using imprecise probability is developed. Independent uncertainties in the elastic stiffness for each element are considered. This method, entitled Imprecise Monte-Carlo Buckling Analysis (IMBA), obtains bounds on natural circular frequencies at different probability levels based on modified and hybrid applications of both interval analysis and Monte-Carlo simulation. This method applies the scheme for bounding critical buckling loads in the presence of interval uncertainty on various levels of probability leading to establishment of a framework for handling incomplete information in the stability of structures.

OVERVIEW

First, a historical background and review of imprecise probability and structural stability are presented. Next, a review of deterministic and interval buckling analysis is presented, followed by the definition of the imprecise probability eigenvalue problem, leading to the development of IMBA. Finally, a numerical example problem is presented that illustrates the capabilities of the new method followed by concluding remarks.

IMPRECISE PROBABILITY STRUCTURE

Imprecise probability allows a single function interval (epistemic) representation of uncertainty with a completely defined CDF (aleatoric) under the same framework in which, $\bar{F}$ and F are the upper and lower bounds of all possible CDF's, respectively (Figure 1).

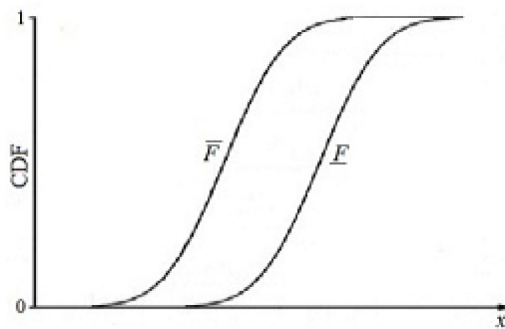


Figure 1. General imprecise probability structure.

HISTORICAL BACKGROUND

An early framework for handling imprecise probability is the Dempster-Shafer approach, which considers interval uncertainty on a deterministic CDF. Williamson and Downs [9] developed a method for combining imprecise probabilities using the four basic arithmetic operations, known as dependency bounds convolutions. These methods are enveloping methods in order to retain robustness in calculations, while avoiding overestimation on bounds due to dependency between different imprecise probabilities. Ferson and Donald [4] developed a method for handling imprecise probability, referred to as Probability Bounds Analysis (PBA). Another method for handling imprecise probability is to discretize the upper and lower CDFs into probability boxes (p-boxes). While the development of Dempster-Shafer (DS) structures has been confirmed several times, the methods proposed for combining multiple DS structures has shown complexities. Ferson et al. [5] suggested that the concept of Dempster-Shafer structures and p-boxes are in fact highly related and outlined procedures for converting from one to the other. They also expanded methods for combining multiple DS structures or p-boxes, for operations including: enveloping, intersection, unweighted and weighted mixtures, averaging, and logarithmic pooling. Moreover, they outlined the various desired properties of any method of combining multiple p-boxes or DS structures, summarized which methods satisfied these properties, and concluded that the best method for combining is enveloping, which is used in this work. Modares et al. [6] showed a method of dynamic analysis using finite element method for mechanical uncertainties defined by interval

variables, which yields exact bounds on a system's natural frequencies. This method utilizes the monotonic behavior of eigenvalues of symmetric matrices subject to non-negative definite perturbations. Moreover, Zhang et al. [10] outlined a method for extending imprecise probability methods to finite element analysis with p-box uncertainty in applied loading in static problems.

REVIEW OF DETERMINISTIC BUCKLING ANALYSIS

As discussed before, buckling analysis using the linear finite element method consists of two main steps. First, a parametric static analysis is performed using an arbitrary ordinate of applied load to compute the displacement pattern as:

$$[K_e]\{U\} = \{P\} \quad (1)$$

in which $[K_e]$ is the elastic stiffness matrix, $\{U\}$ is the displacement vector and $\{P\}$ is the force vector. Similarly:

$$(\sum_{i=1}^n [\bar{K}_{e_i}])\{U\} = \{P\}$$

in which $[\bar{K}_{e_i}] = [L_i][K_{e_i}][L_i]^T$ is the deterministic element contribution to the global elastic stiffness matrix for member i , where $[L_i]$ is the element Boolean connectivity matrix.

Using the computed displacement vector, the internal forces can be obtained as a function of the load ordinate. Using the element internal axial forces, f_i , the geometric stiffness of the structure is developed, which represents the pre-compression load's effects on the total stiffness of the structure as:

$$[K_g] = \sum_{i=1}^n [L_i](f_i[K_{g_i}])[L_i]^T$$

or:

$$[K_g] = \sum_{i=1}^n f_i [\bar{K}_{g_i}]$$

in which $[\bar{K}_{g_i}] = [L_i][K_{g_i}][L_i]^T$ is the deterministic force independent element contribution to the global geometric stiffness matrix.

The second step is solving a generalized eigenvalue problem between the elastic, $[K_e]$, and geometric, $[K_g]$, stiffness matrices of the structure in order to find the critical buckling load, λ , in terms of the geometric and material characteristics of the structure as:

$$([K_e] - \lambda[K_g])\{U\} = \{0\} \quad (2)$$

REVIEW OF INTERVAL BUCKLING ANALYSIS

As in deterministic buckling analysis, the first step of interval buckling analysis is a parametric interval static analysis using an arbitrary ordinate of applied load: